The bright side of Coulomb blockade:

Radiation from a Josephson junction in the single Cooper pair regime

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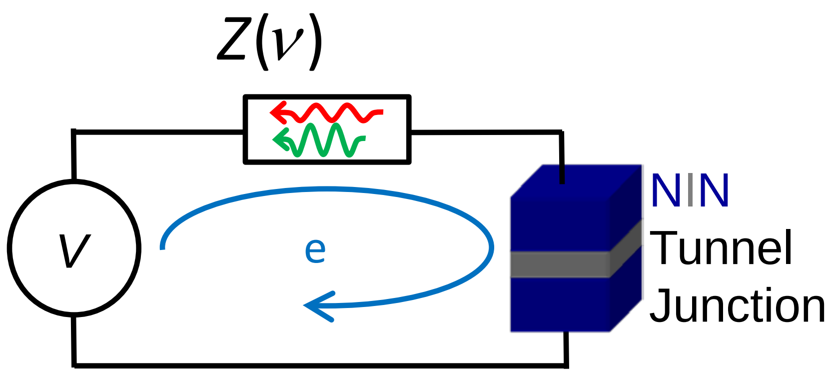
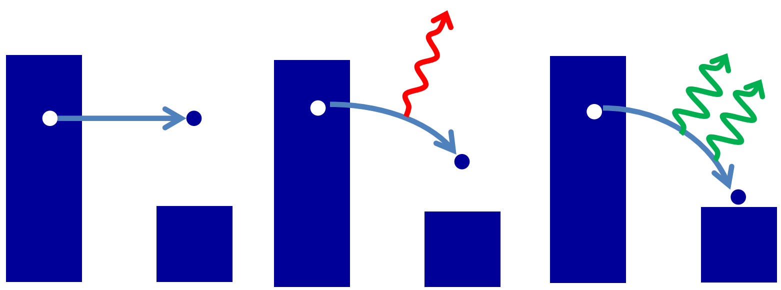
SCOPE

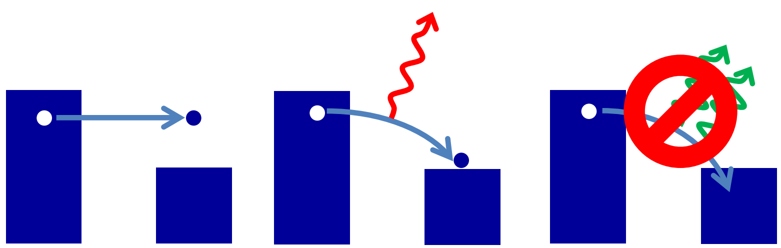
Single COoper Pair Electronics

SEVENTH FRAMEWORK PROGRAMME

Dynamical Coulomb blockade



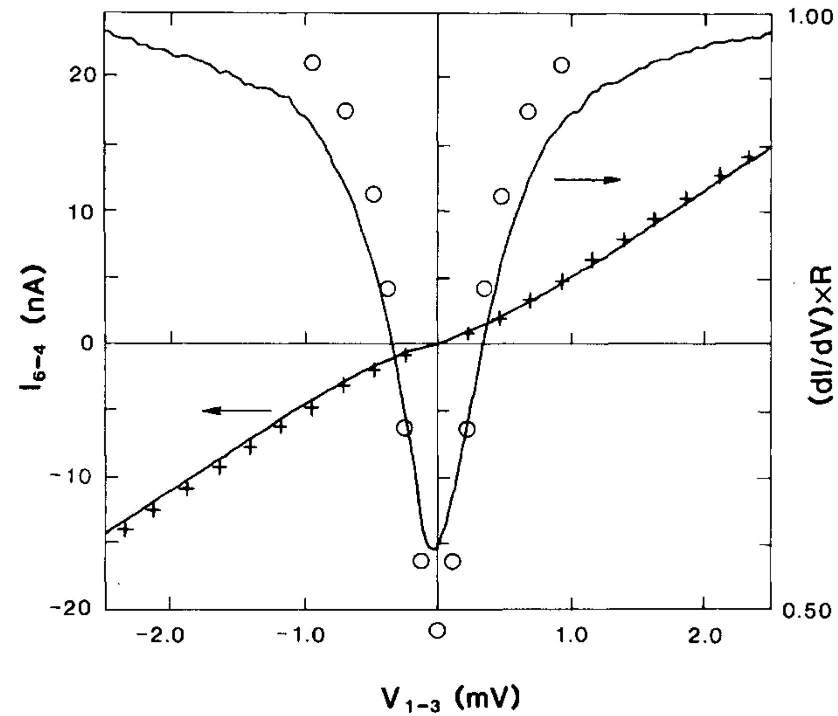




Energy balance

$$eV = E_{\text{hole}} + E_{\text{electron}} + E_{\text{photon}}$$

Delsing *et al.*, PRL **63**, 1180 (1989)



$P(E_{\text{photon}})$: probability to emit E_{photon} into $Z(\nu)$

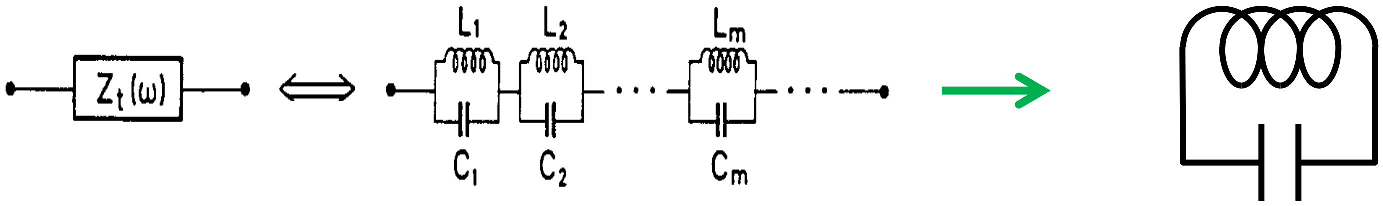
$$\mathrm{Re}[Z(\nu)] \ll h/e^2 \Rightarrow$$

$$P[Z](E_{\text{photon}} = h\nu) \approx \frac{2}{h\nu}\frac{e^2}{h}\mathrm{Re}[Z(\nu)]$$

Ingold & Nazarov, arxiv:0508728 (1992)

A simpler system

- Environment: single mode



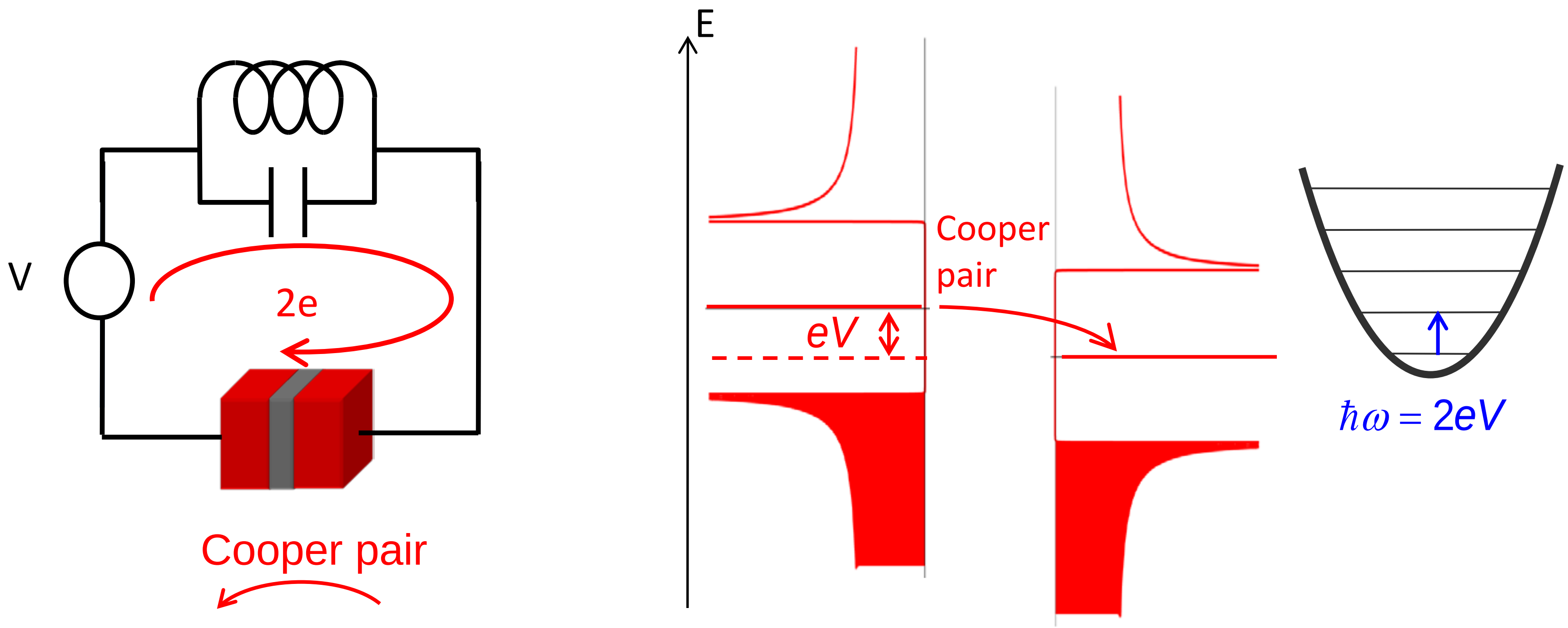
- No quasiparticles: use Josephson junction polarized below the gap voltage

NIN
Tunnel
Junction

SIS
Josephson
Junction

$$\Rightarrow 2eV = E_{\text{photon}}$$

A simpler system



$$H = h\omega(a^{\dagger}a + 1/2) - E_J \cos\varphi$$

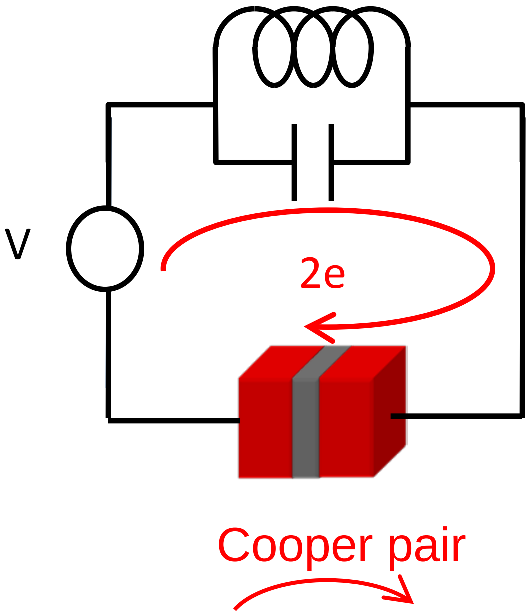
$$\varphi = 2eVt/\hbar + \sqrt{r}(a^{\dagger} + a)$$

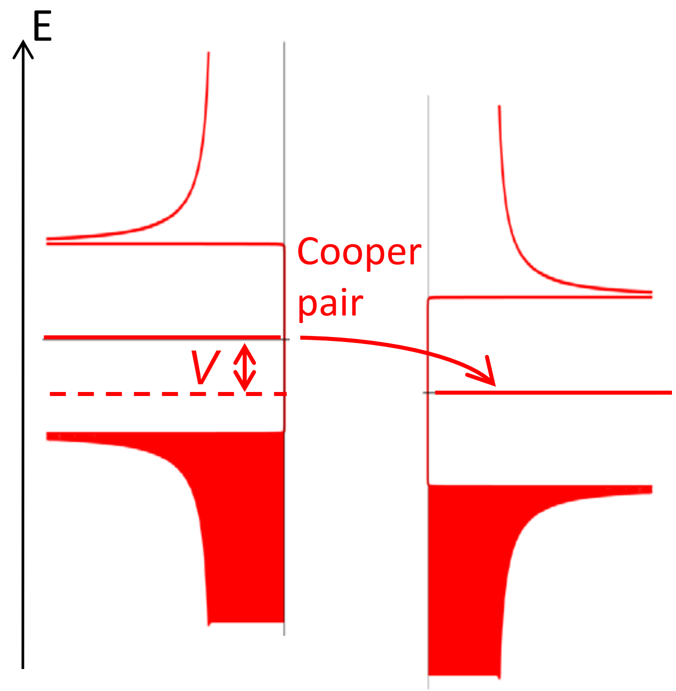
$$r = \frac{\pi Z}{h/4e^2} \qquad Z = \sqrt{\frac{L}{C}}$$

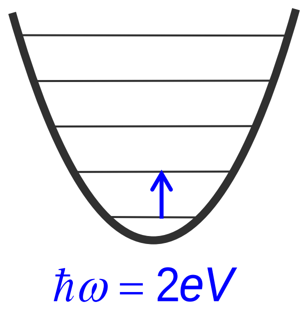
H. Pothier,
Ph. D. dissertation (1991)

+ Fermi golden rule calculation

A simpler system





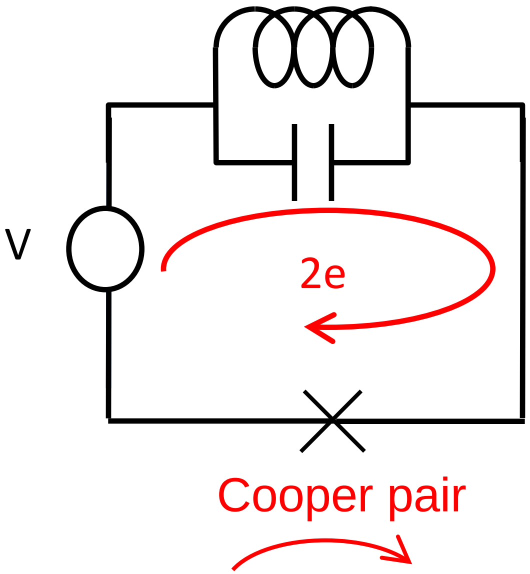


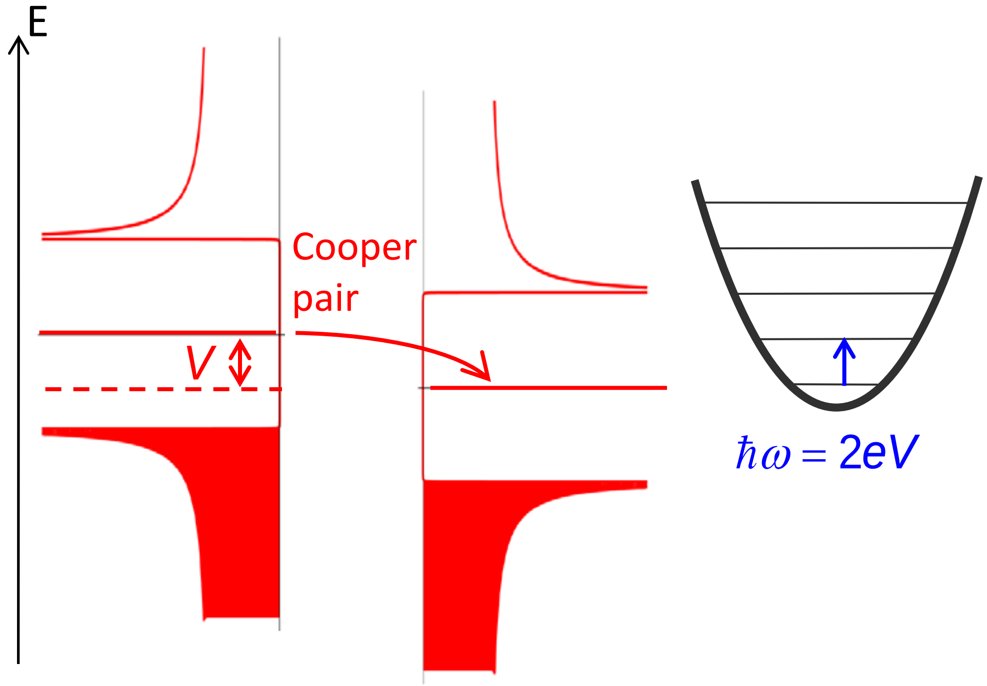
$$\Gamma^{2e}_{\rightarrow}(V) = \frac{\pi E_J^2}{2\hbar} \sum_n \left| \langle n | e^{i\varphi} | 0 \rangle \right|^2 \ \delta(2eV - n\hbar\omega)$$

$$= \frac{\pi E_J^2}{2\hbar} \sum_n \frac{\exp(-r)\, r^n}{n!} \ \delta(2eV - n\hbar\omega)$$

$$\Gamma^{h\nu}_{\rightarrow}(V = n\hbar\omega / 2e) = n\Gamma^{2e}_{\rightarrow}$$

A simpler system

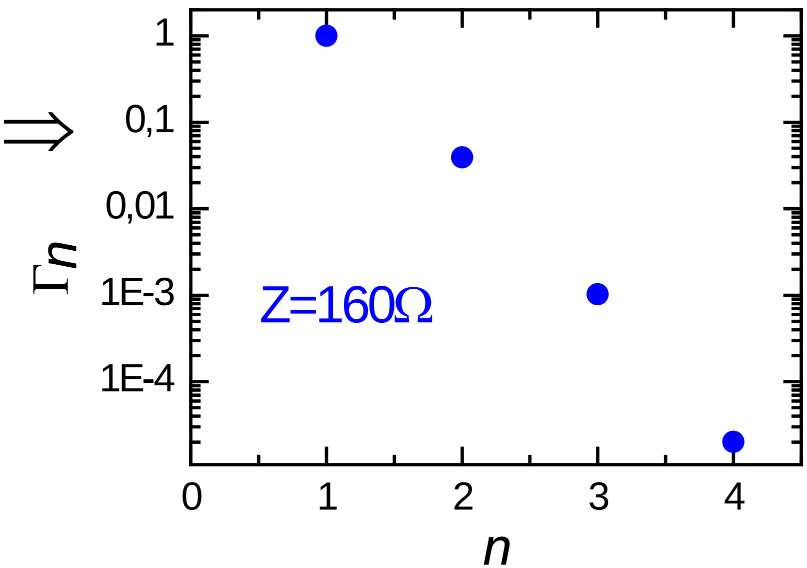




$$\Gamma_{\rightarrow}^{2e}(V) = \frac{\pi E_J^2}{2\hbar}\sum_n \frac{\exp(-r)\, r^n}{n!}\,\delta(2eV - n\hbar\omega)$$

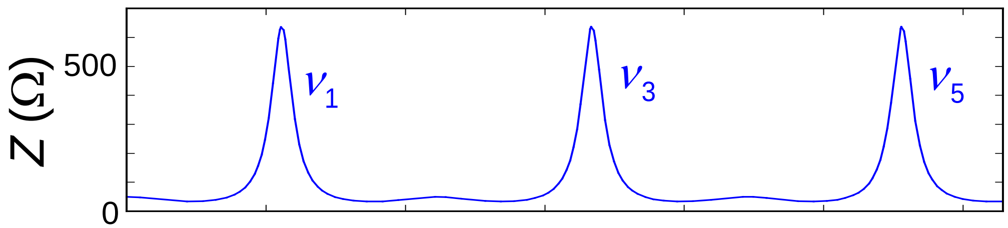
$$Z = 160\,\Omega \Rightarrow r \simeq 0.08$$

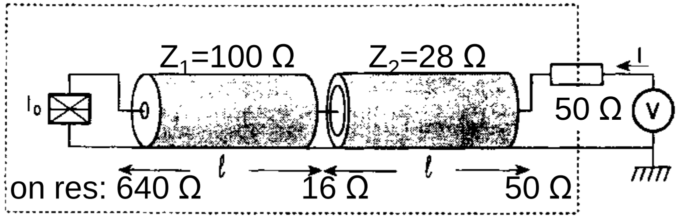
$\Rightarrow$

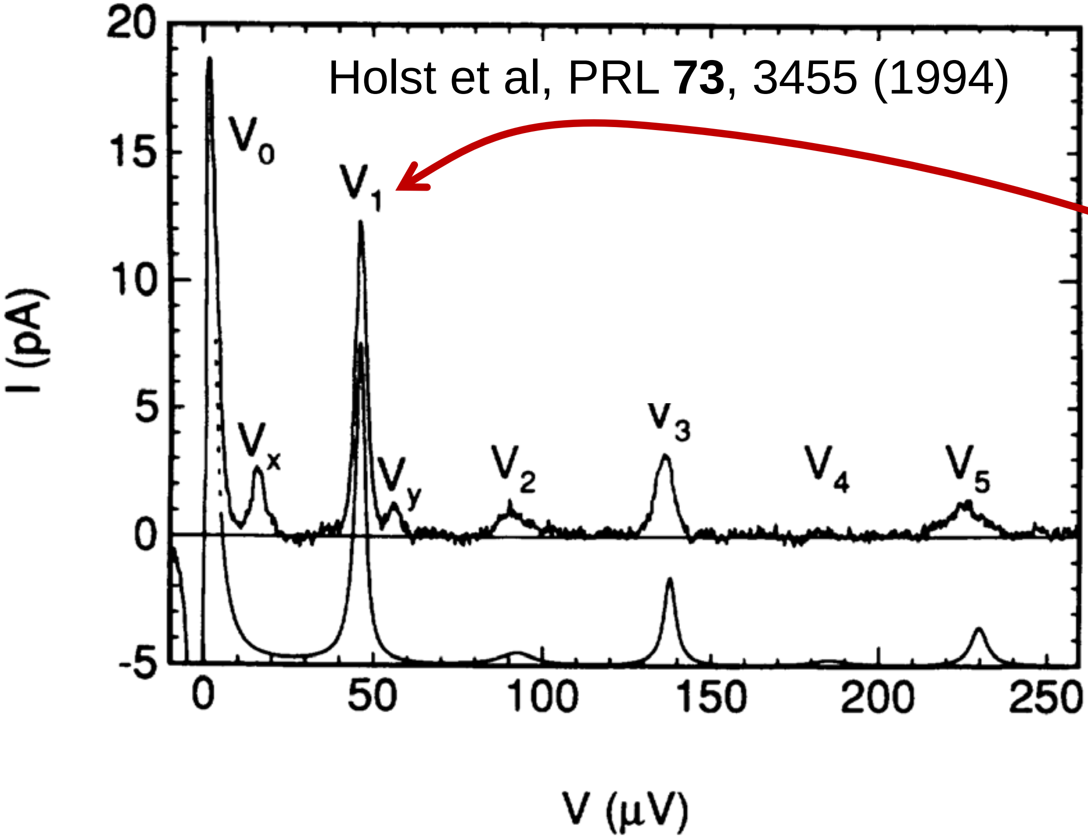


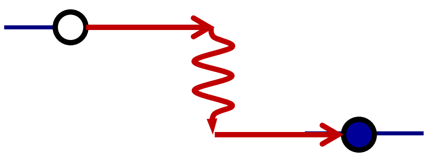
Josephson junction and resonator

$$\nu_1 = 25\ \text{GHz},\ Q \approx 5,\ Z_1 \approx 120\ \Omega \ll h/4e^2$$



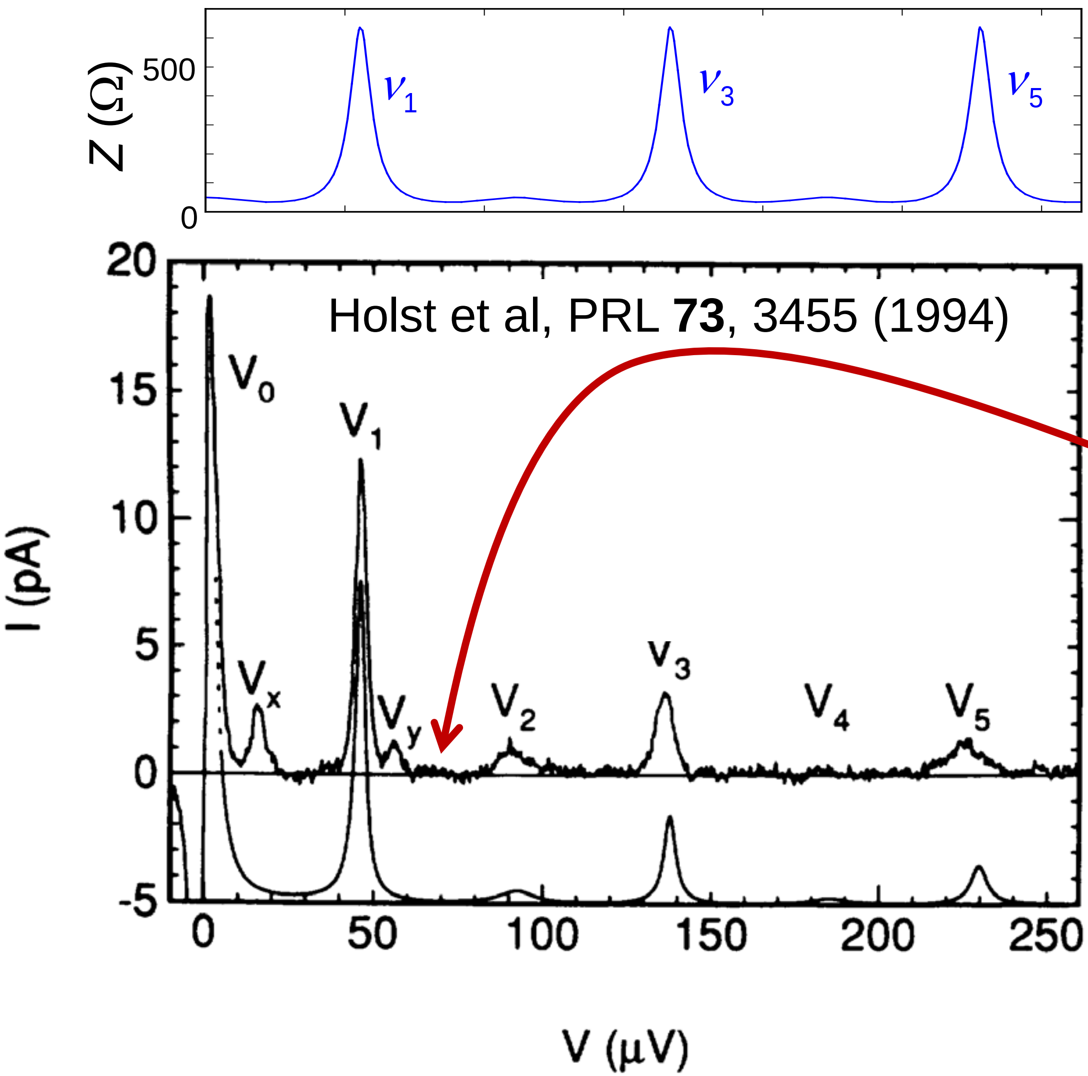


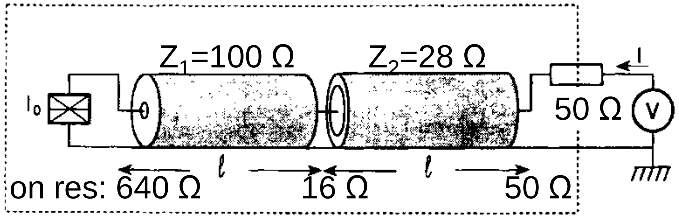


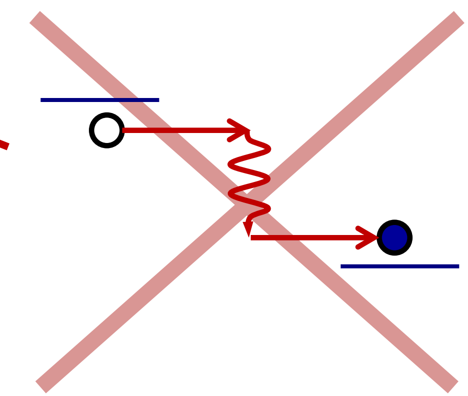


Josephson junction and resonator

$\nu_1 = 25$ GHz

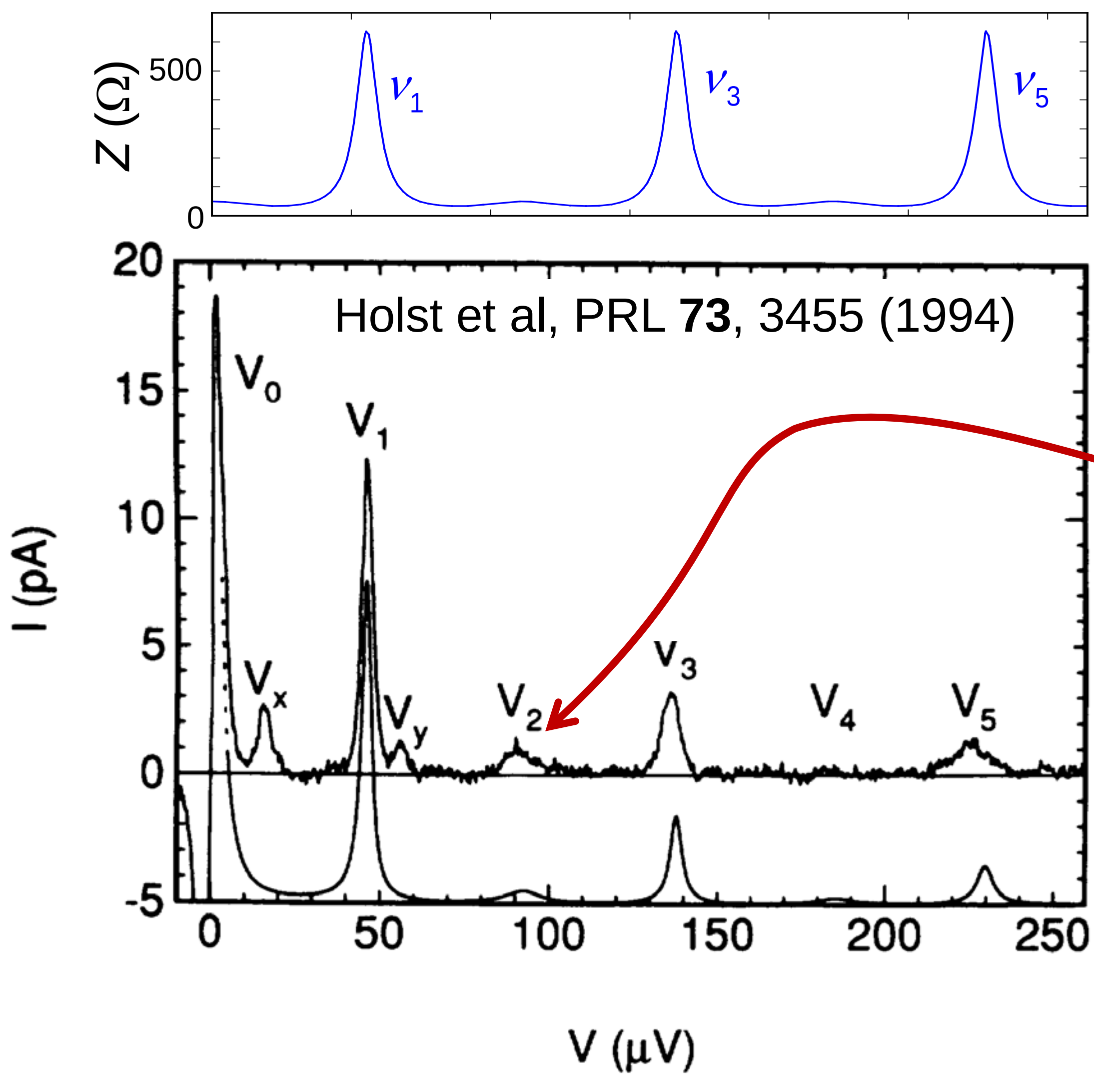


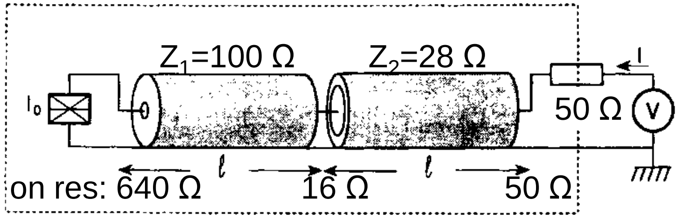


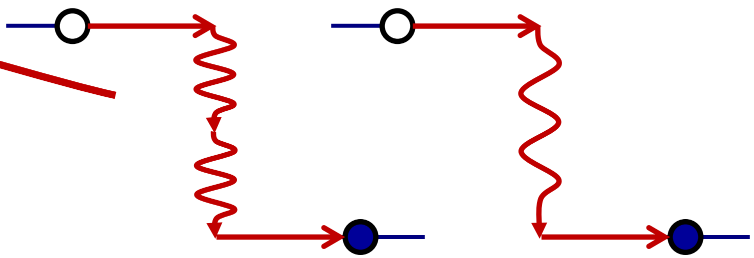


Josephson junction and resonator

ν_1 = 25 GHz

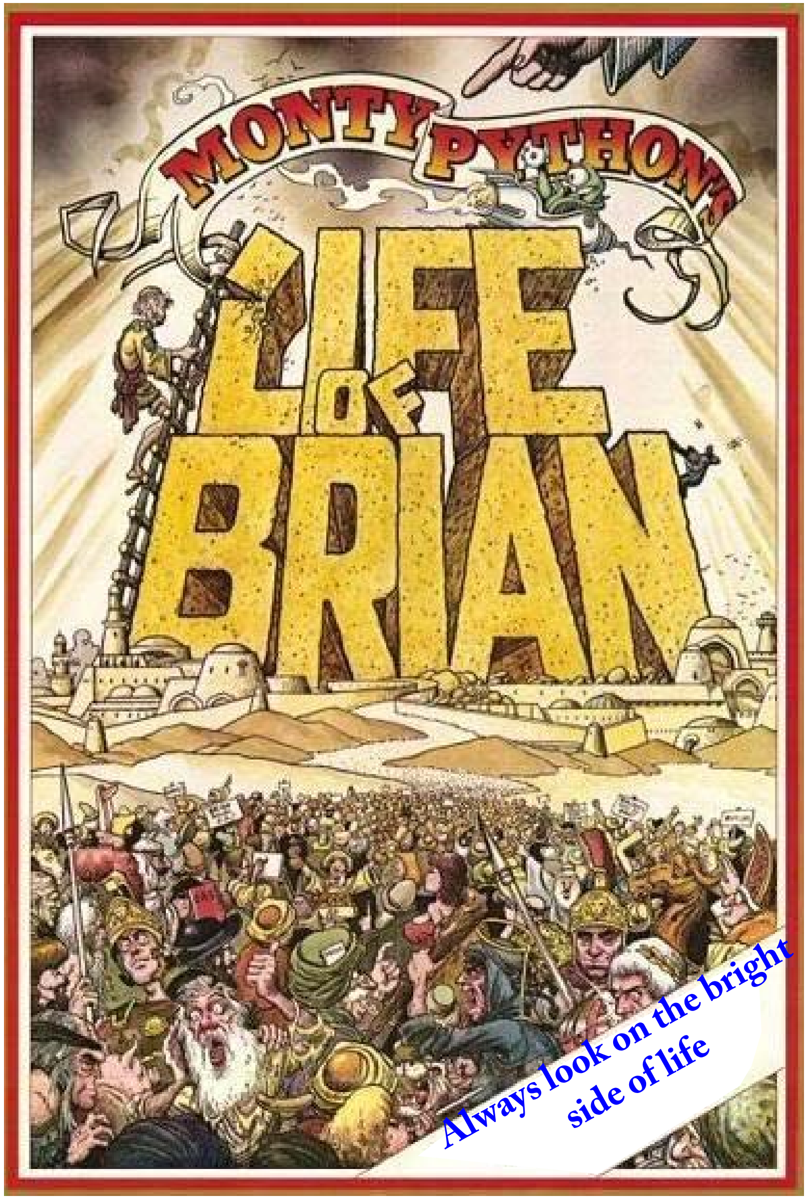






Two photon processes weak because $Z_1 \ll h/4e^2$

Goal



Dynamical Coulomb blockade:

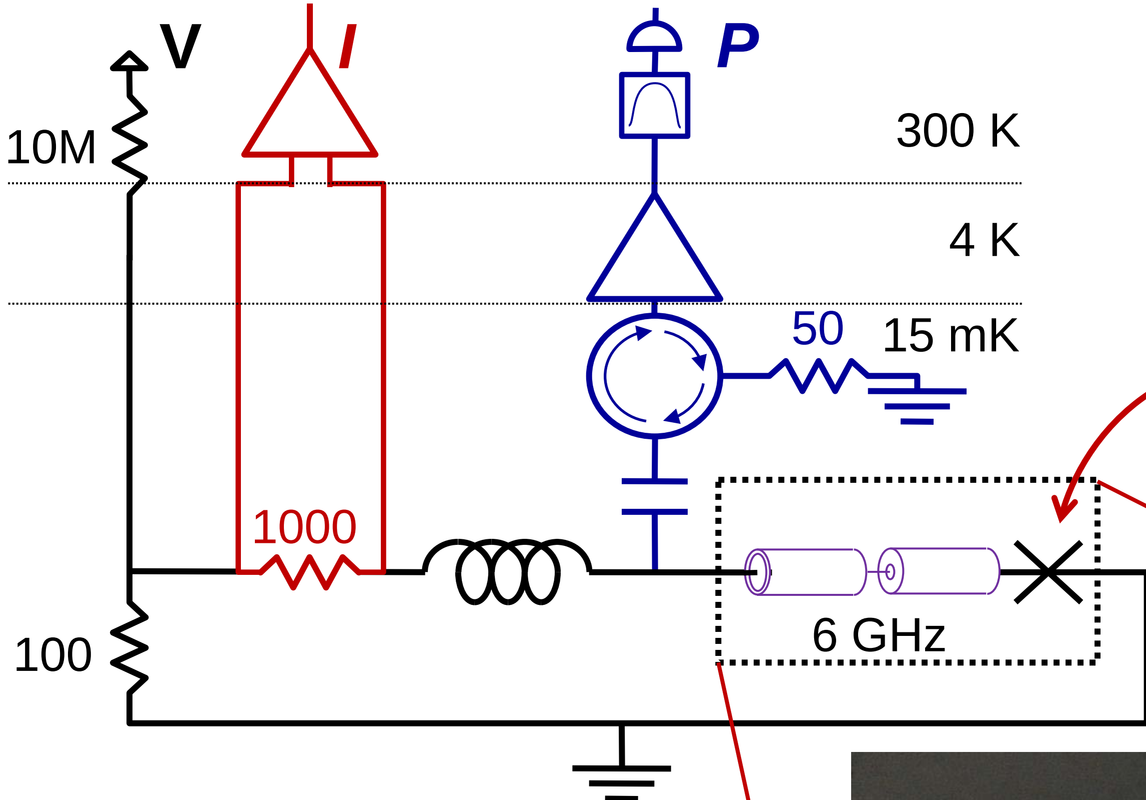
- Effect due to photons
- DC side well established
- But no one has seen photons

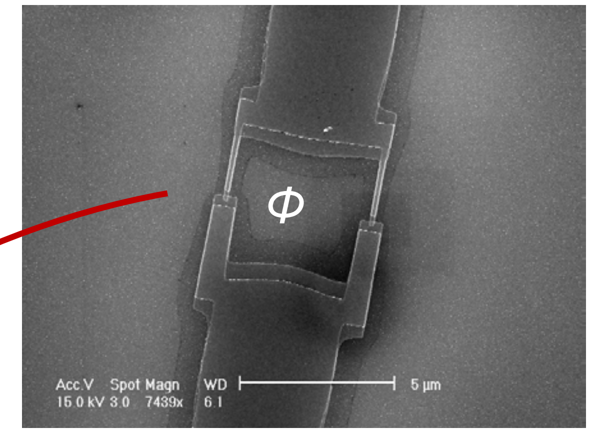
Look on the bright side of….

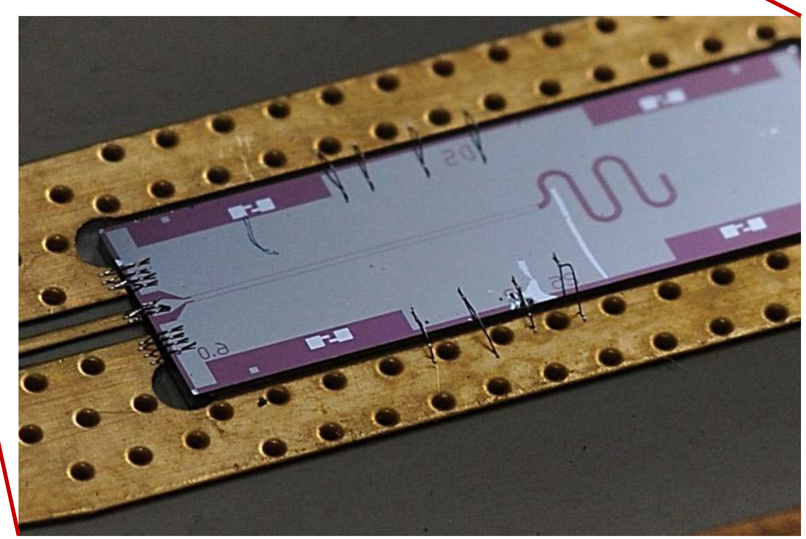
Coulomb blockade

Setup

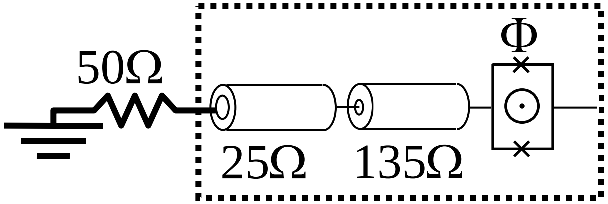
$$E_J(\Phi) = E_J^0 \left|\cos(\pi \,\Phi / \Phi_0)\right|$$

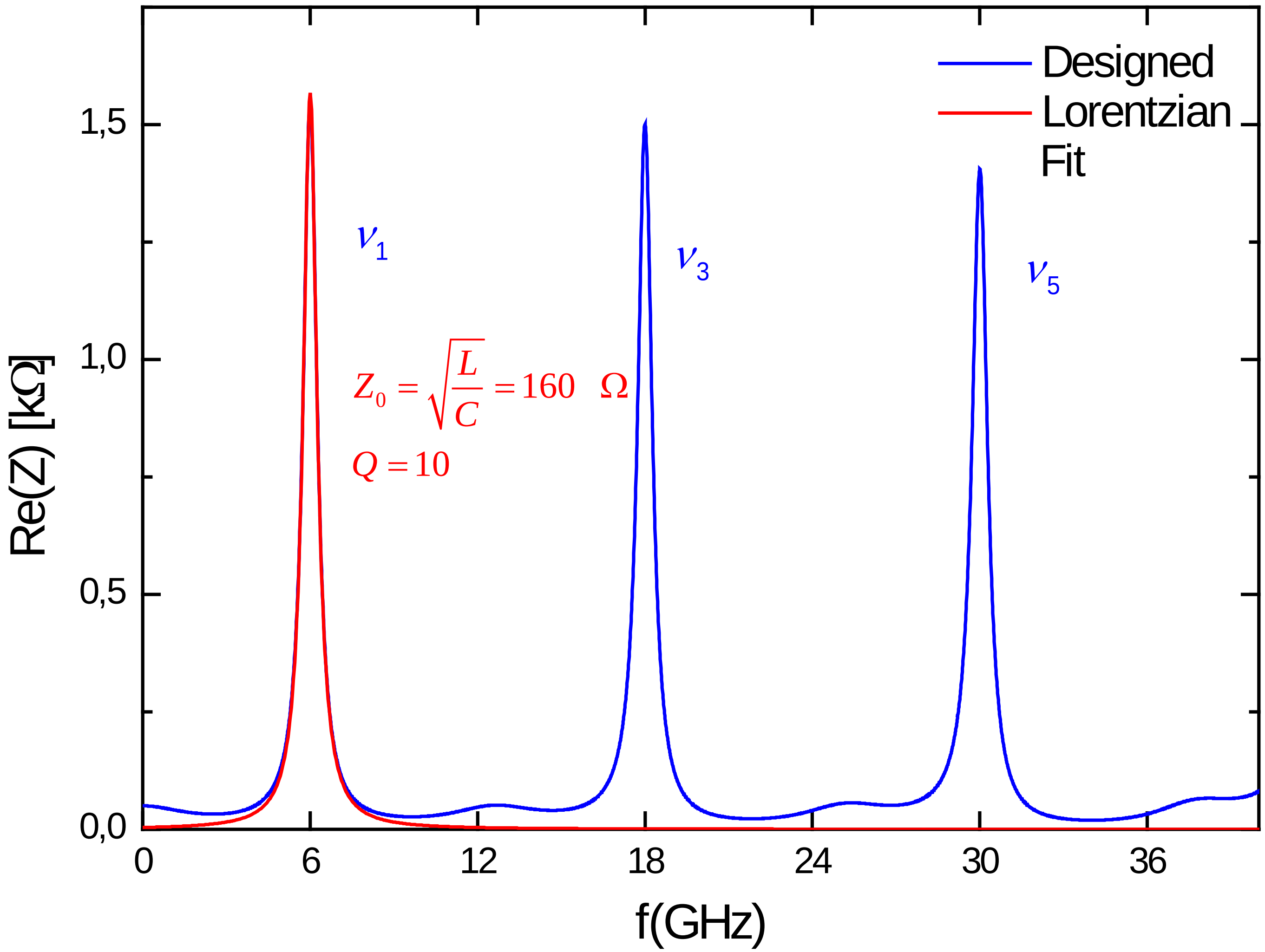






Quarter-wave resonator



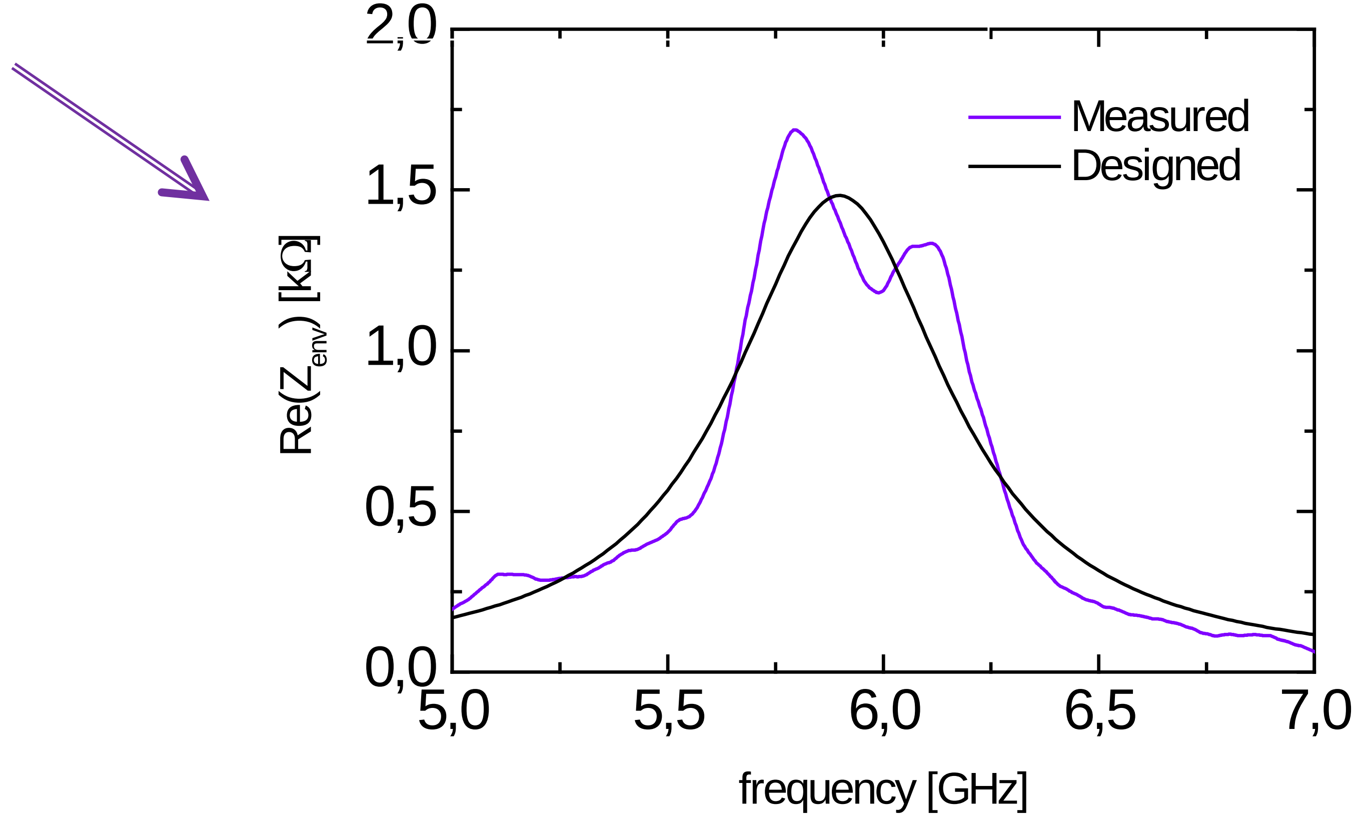


Calibration of the detection impedance

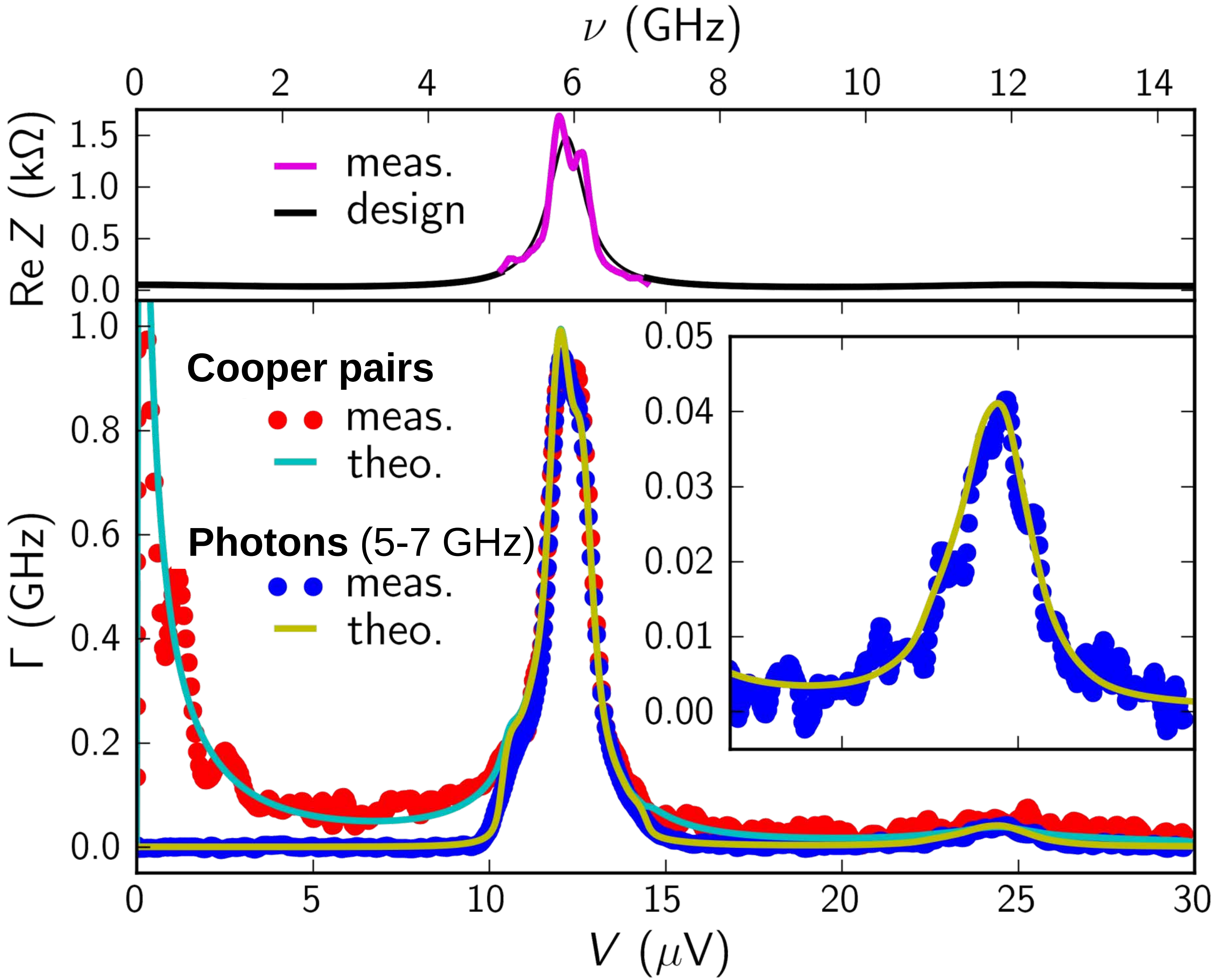
Apply $eV \gg \Delta,\ h\nu,\ k_B T \;\Rightarrow$ quasi-particle shot noise

$$S_{II} = 2eI \;\Rightarrow P = 2eV \int \frac{\mathrm{Re}[Z(\nu)]R_t}{\left|R_t + Z(\nu)\right|^2} d\nu \Rightarrow \mathrm{Re}[Z(\nu)]$$

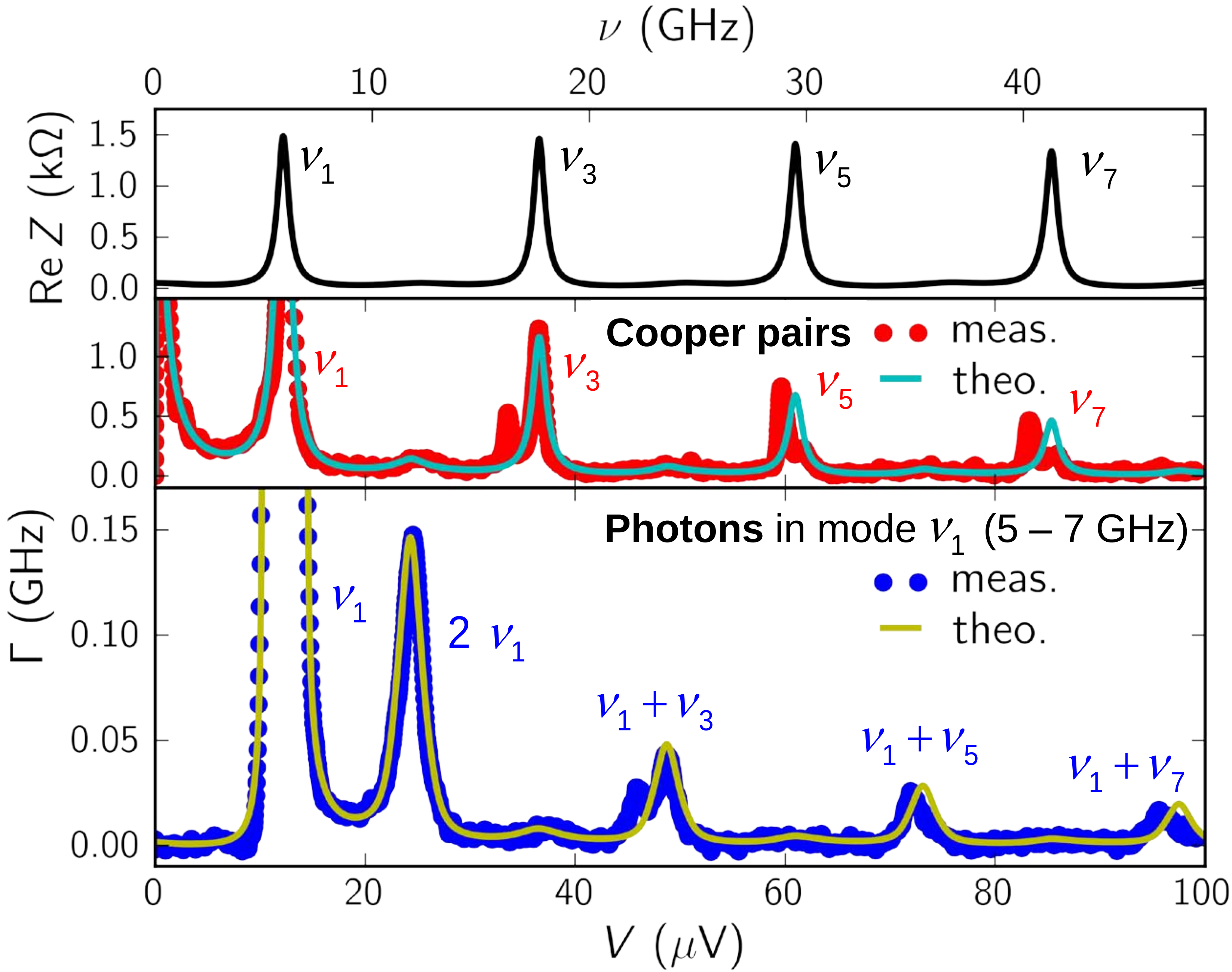
$$R_t = 18\ \mathrm{k}\Omega$$



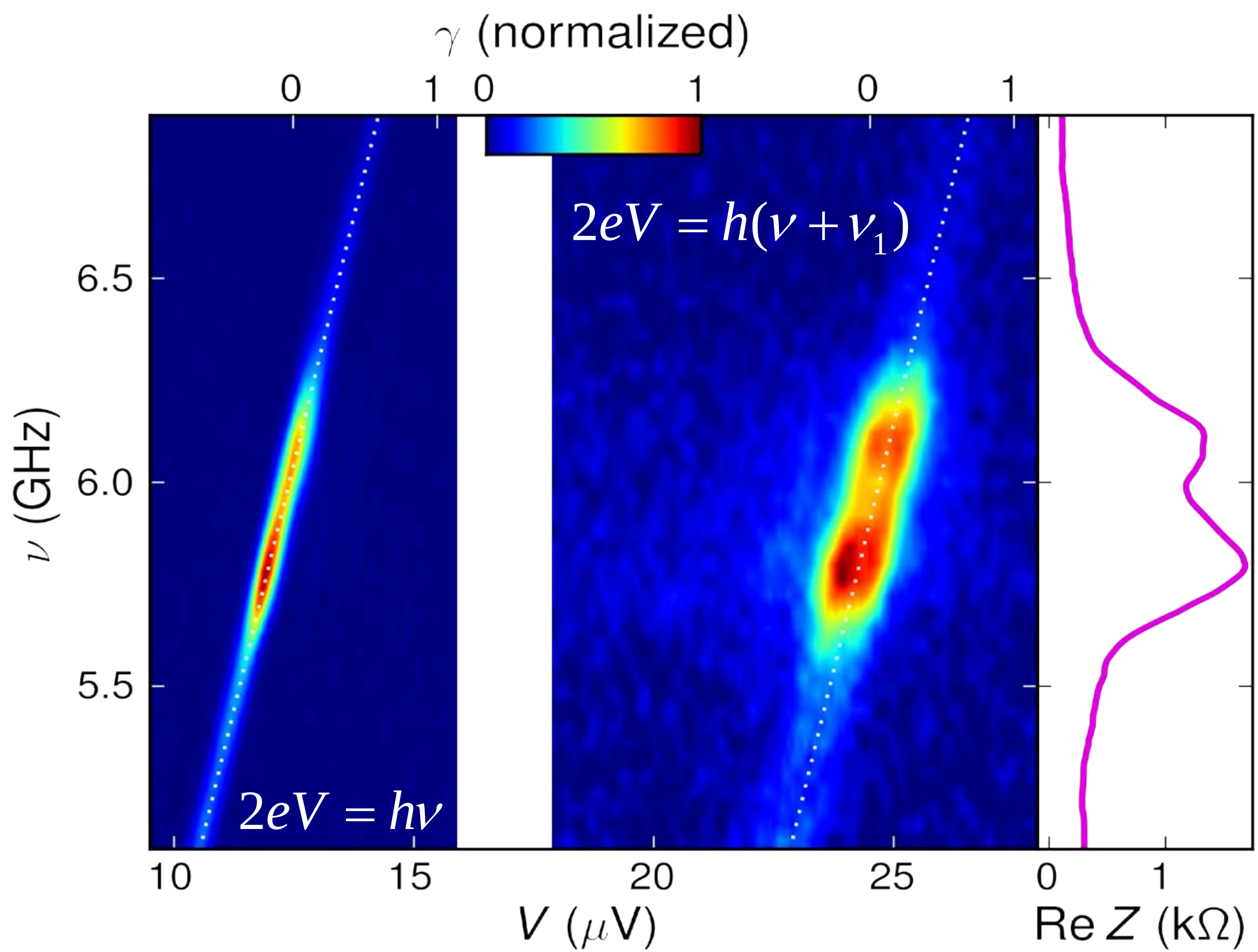
Cooper pair and photon rate match



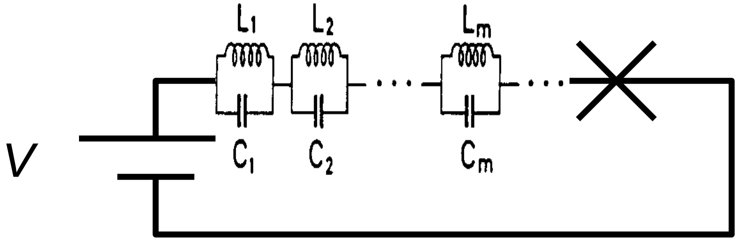
Second order processes



Spectral properties of emitted radiation



Coulomb blockade with an arbitrary environment



$$H = \sum_i h\nu_i (a_i^\dagger a_i + 1/2) - E_J \cos\varphi$$

$$\varphi = \frac{2e}{\hbar} Vt + \sum_i \sqrt{\pi \tfrac{4e^2}{h} Z_i} (a_i^+ + a_i)$$

Cooper pair rate:

$$\Gamma = \frac{\pi}{2\hbar} E_J^2 \sum_{n_1, n_2, \ldots} \left| \langle n_1, n_2, \ldots | e^{i\varphi} | 0 \rangle \right|^2 \delta\left(2eV - \sum_i n_i h\nu_i \right)$$

$$= \frac{\pi}{2\hbar} E_J^2 P(2eV)$$

$$\approx \frac{\pi}{\hbar} E_J^2 \frac{4e^2}{h} \frac{\mathrm{Re}\, Z(2eV/h)}{2eV}$$

Alternate calculation of P(E):
Ingold & Nazarov, arxiv:0508728 (1992)

Photon rate at $\nu=\nu_m$: exclude mode m from sum

$$\Gamma_{m,n_m} = \frac{\pi}{2\hbar} E_J^2 \left| \langle n_m | e^{i\varphi_m} | 0_m \rangle \right|^2 \sum_{\ldots n_{m-1}, n_{m+1}, \ldots} \left| \langle \ldots n_{m-1}, n_{m+1}, \ldots | e^{i(\varphi - \varphi_m)} | 0 \rangle \right|^2 \delta\left(2eV - \sum_{i \neq m} n_i h\nu_i - n_m h\nu_m \right)$$

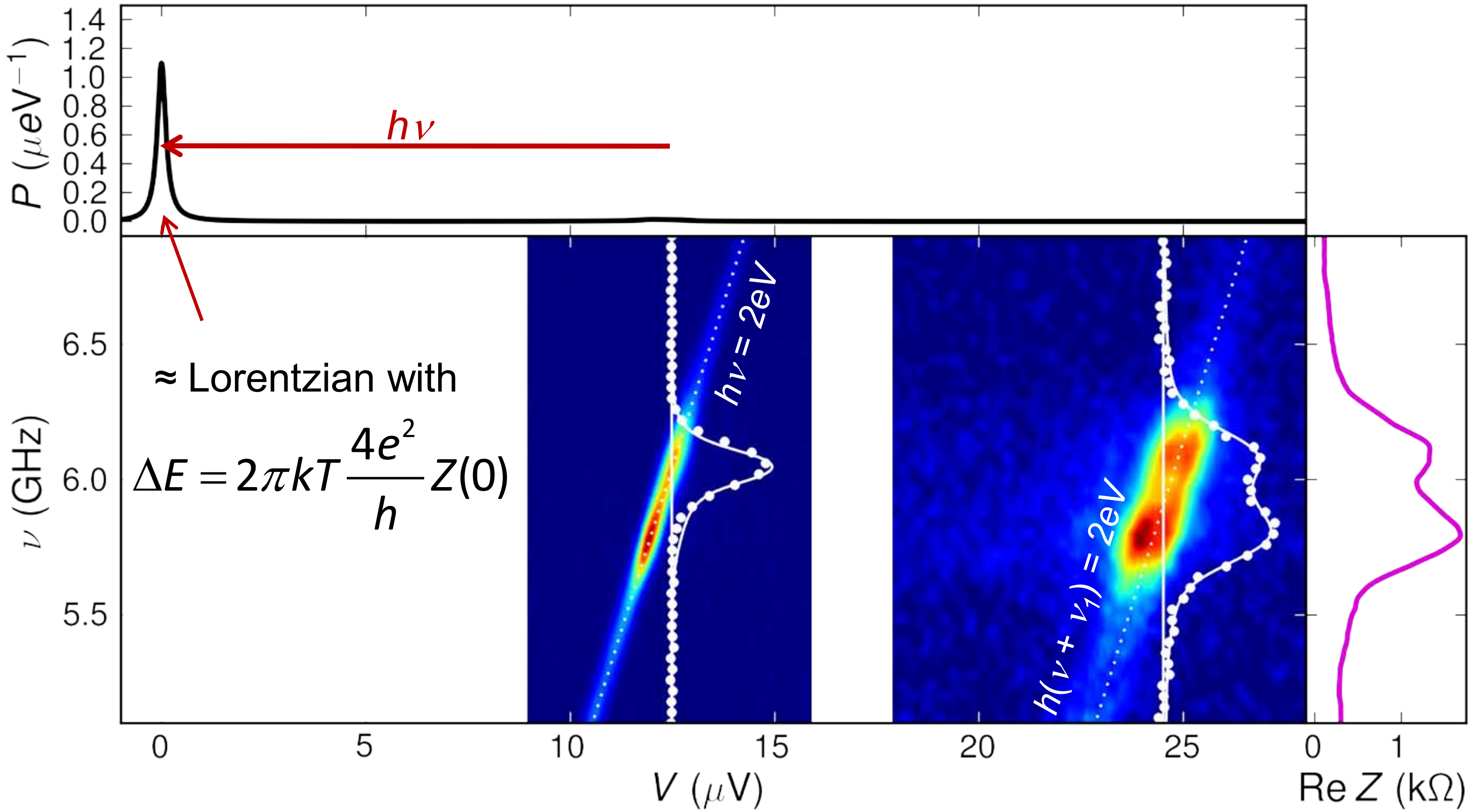
Photons emitted in $\delta\nu_m \to 0$:

$$\frac{\delta\Gamma}{\delta\nu} = 2 \frac{4e^2}{h} \frac{\mathrm{Re}\, Z(\nu)}{\nu} \times \frac{\pi}{2\hbar} E_J^2 P(2eV - h\nu)$$

probability of photon emission at ν

tunneling rate while absorbing rest of energy

Spectral properties of emitted radiation



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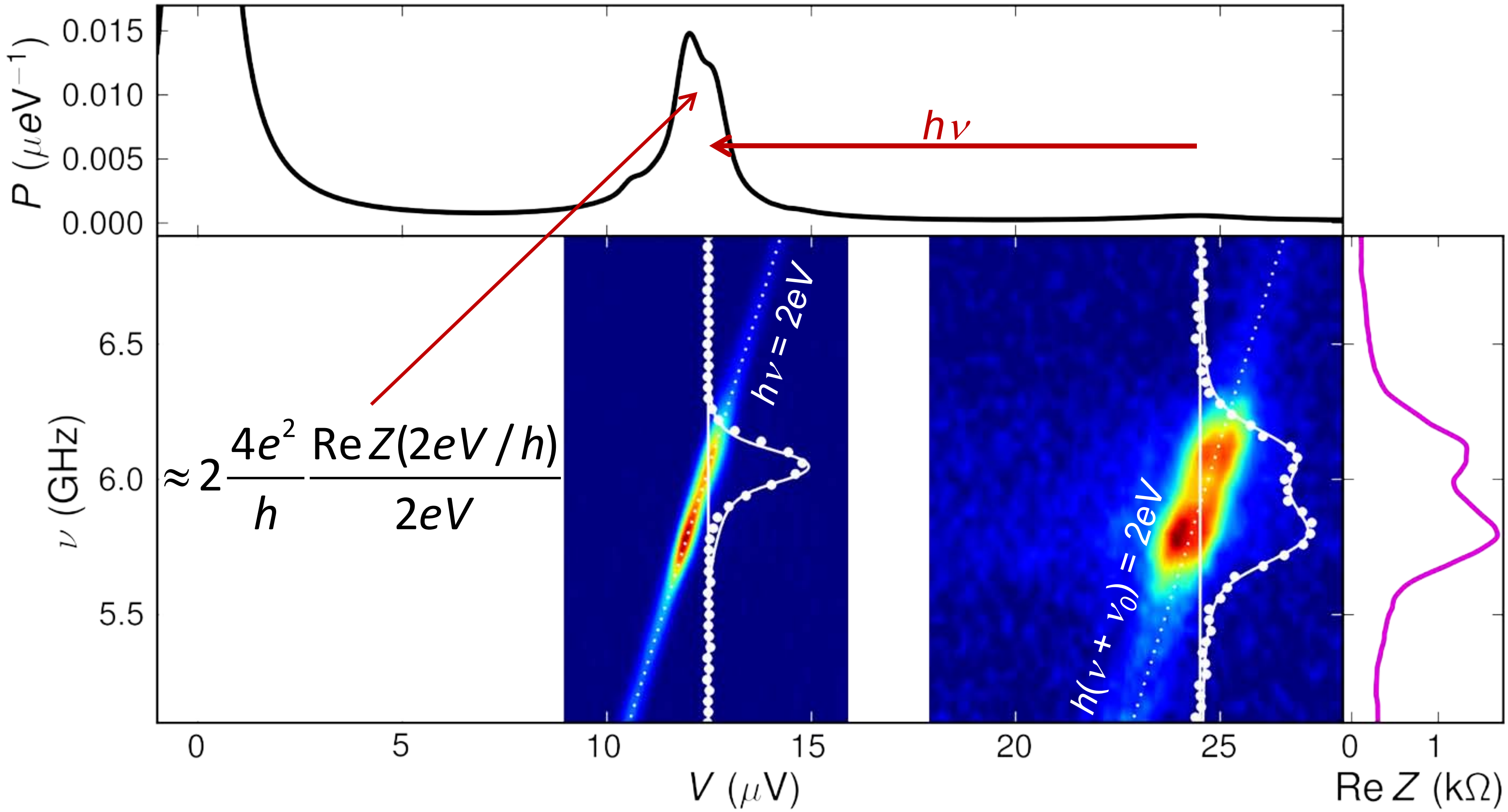
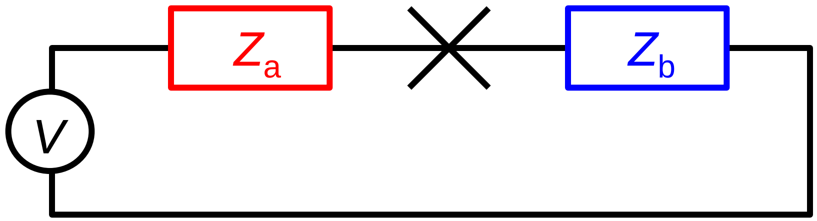
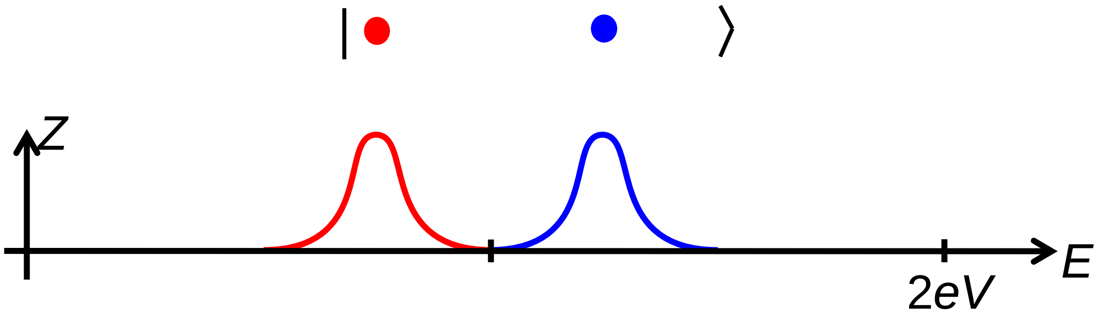
Spectral properties of emitted radiation



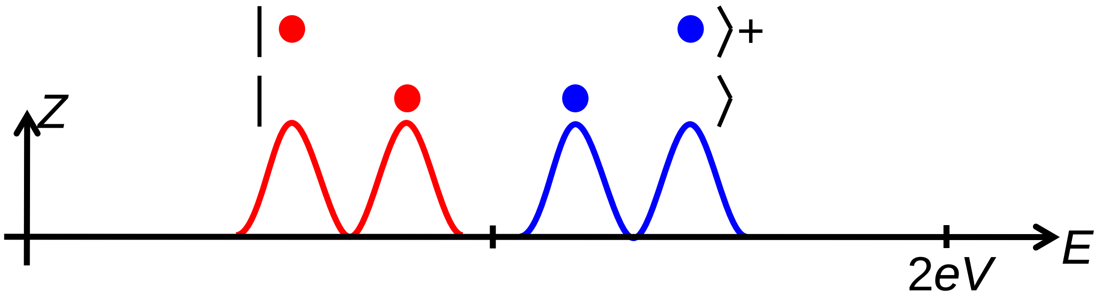
$$\frac{\delta\Gamma}{\delta\nu} = 2\frac{4e^2}{h}\frac{\mathrm{Re}\,Z(\nu)}{\nu} \times \frac{\pi}{2\hbar}E_J^2 P(2eV - h\nu)$$

Emitting photon pairs





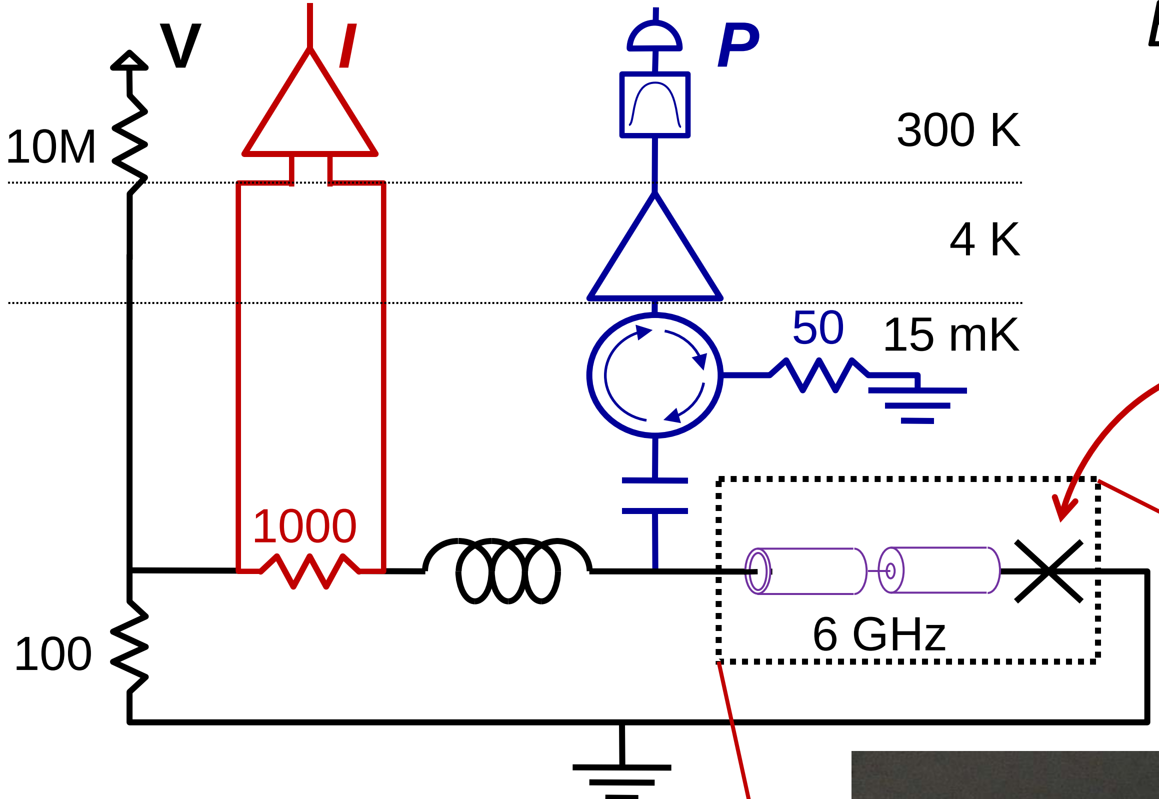
Emission of photon pairs

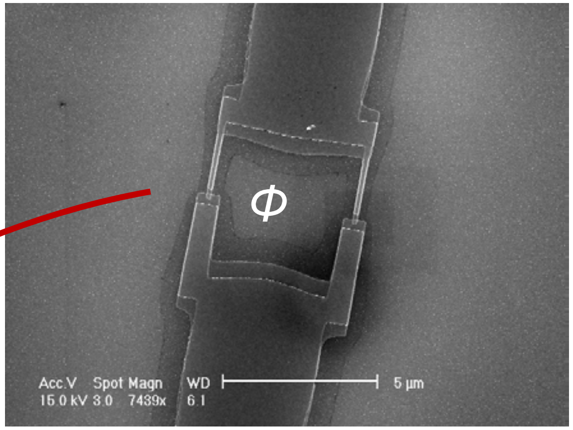


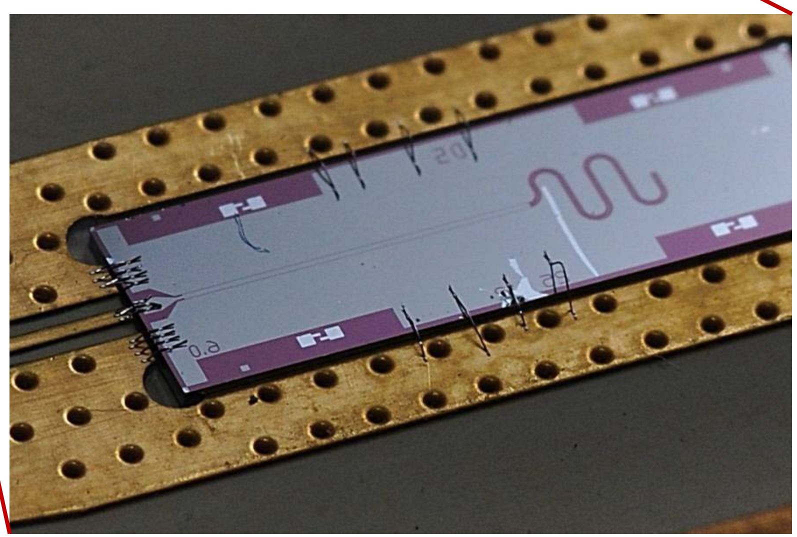
Bell-like state

Setup

$$E_J(\Phi) = E_J^0 \left|\cos(\pi\ \Phi/\Phi_0)\right|$$

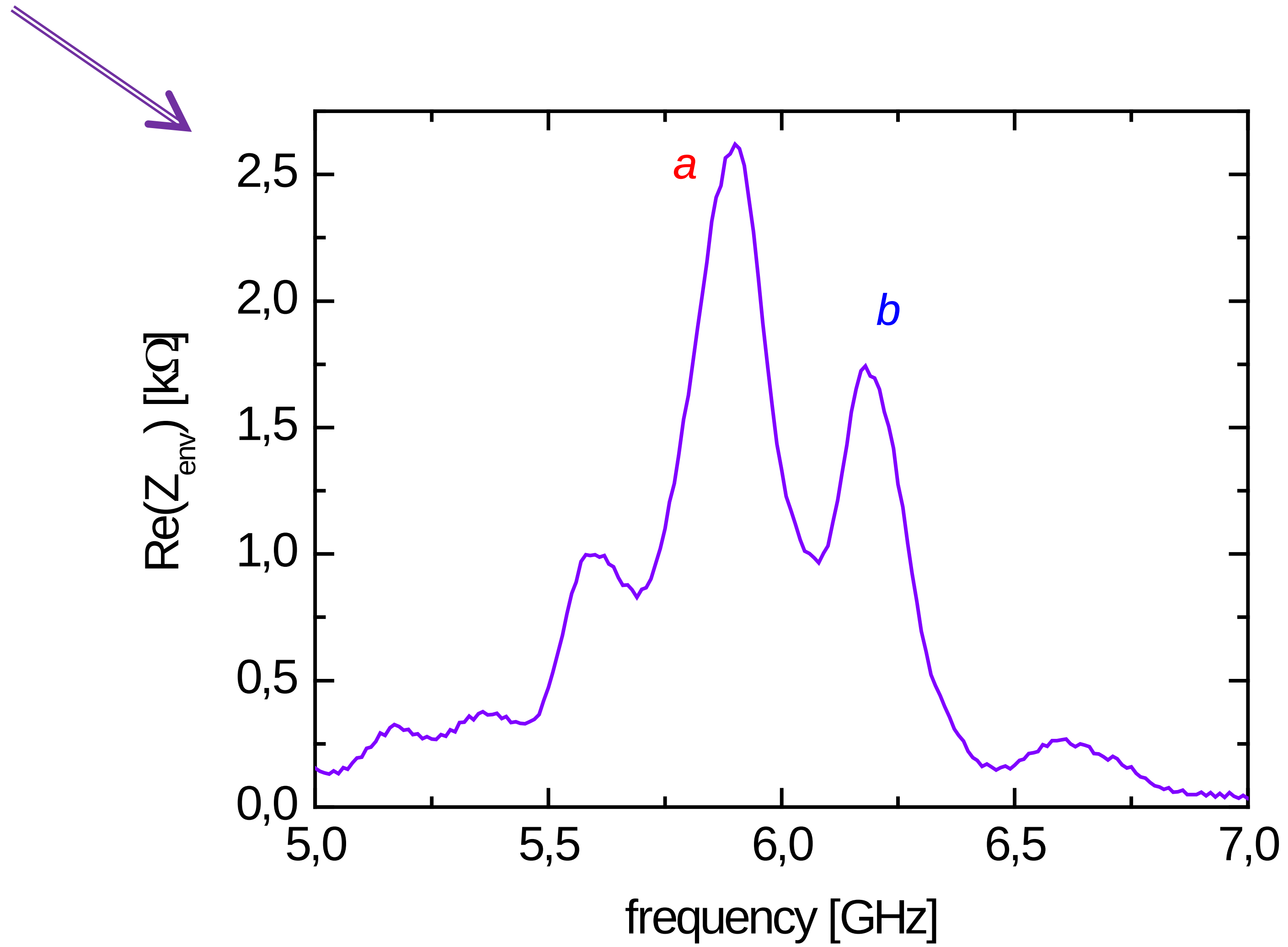




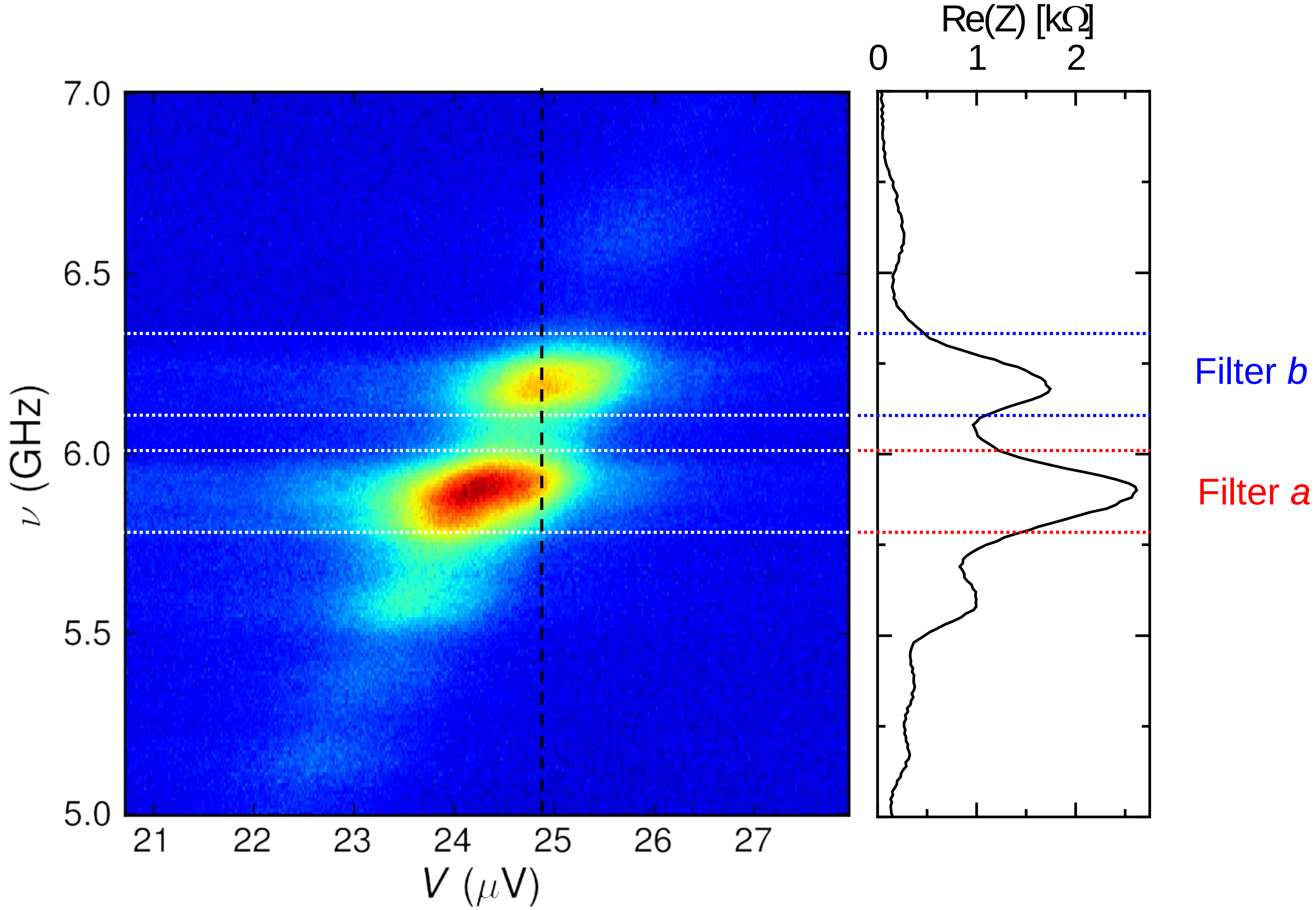


Emitting photon pairs

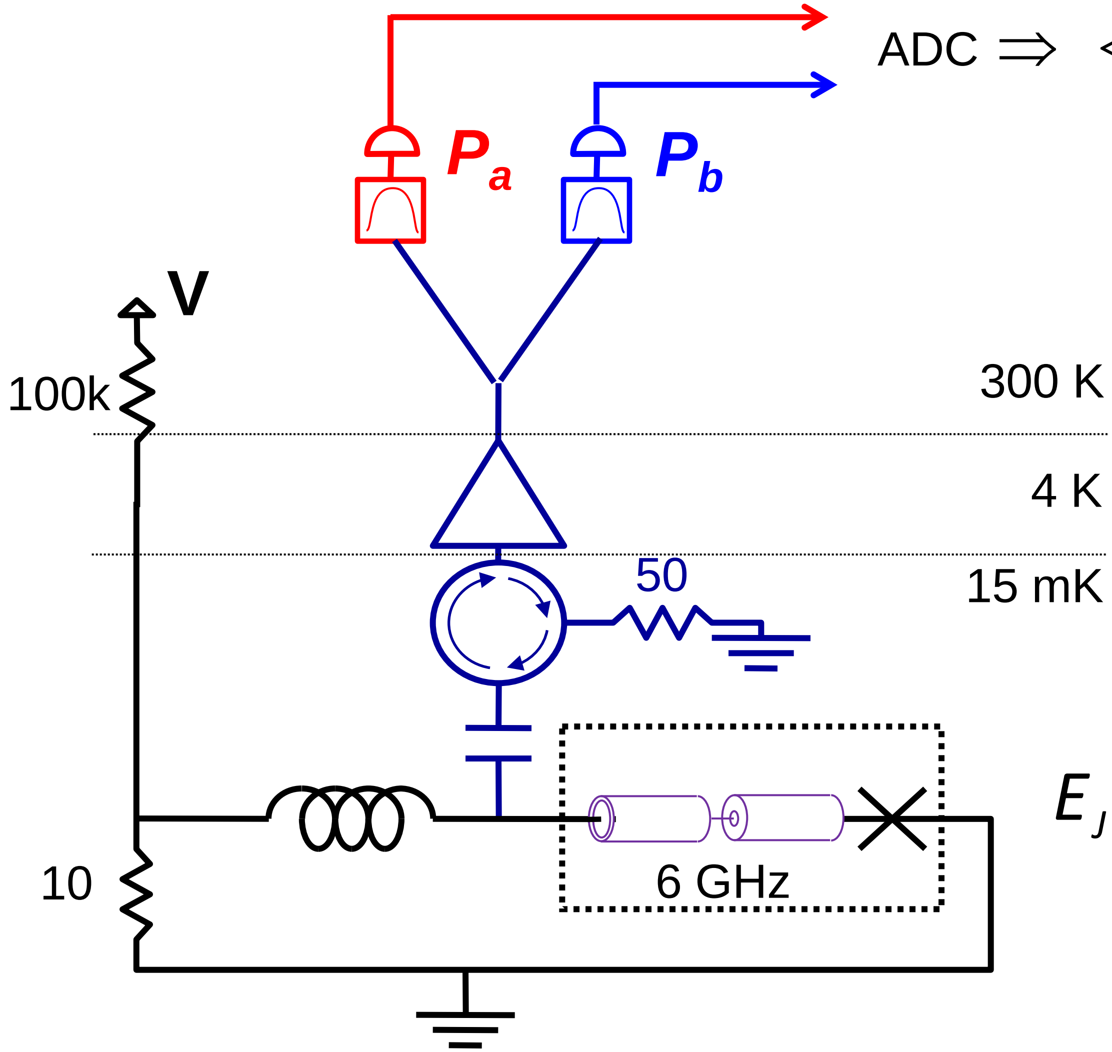
quasi-particle shot noise



Emitting photon pairs



Setup



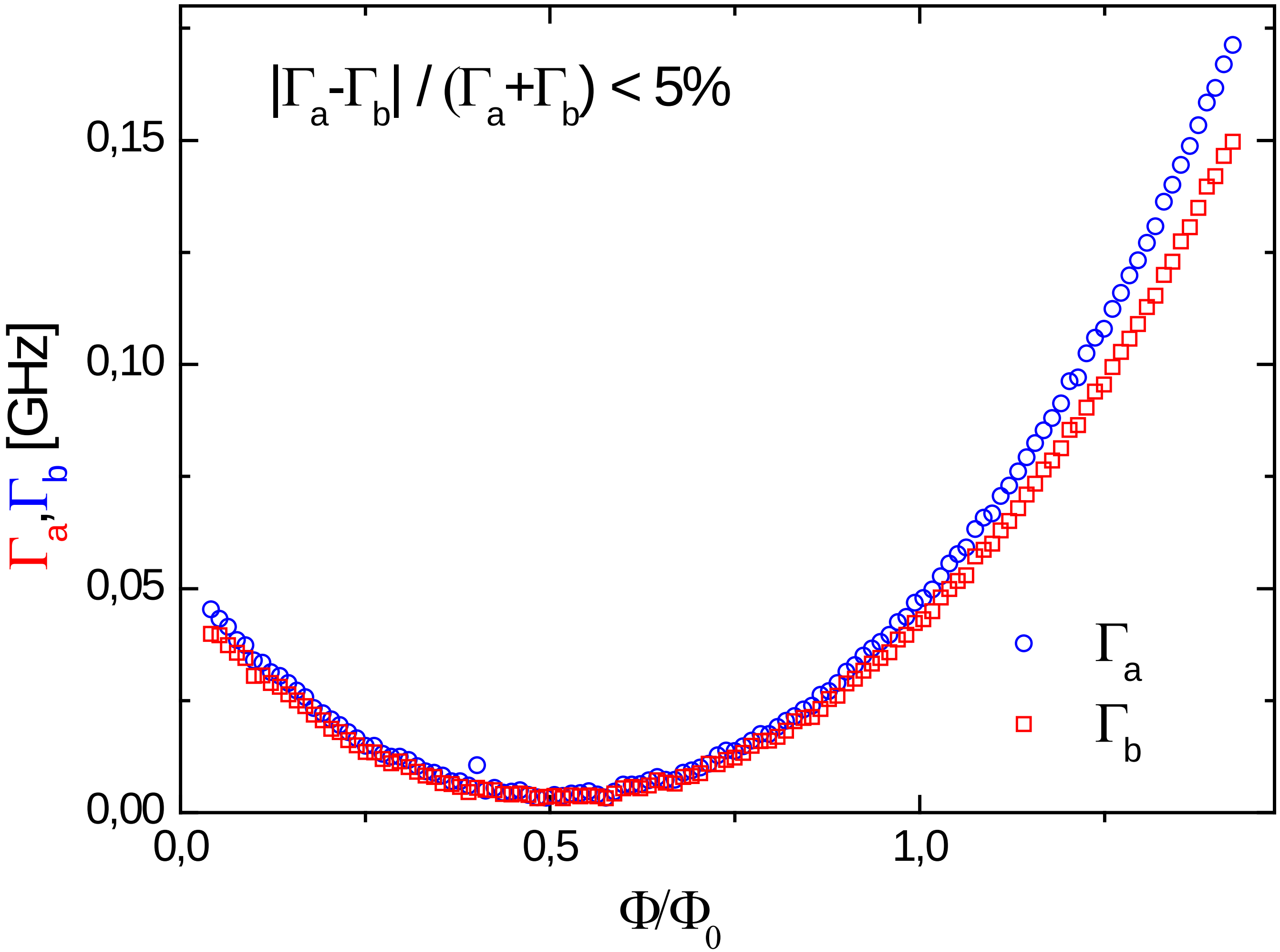
ADC $\Rightarrow$ $<\delta P_a^2>, <\delta P_b^2>, <\delta P_a \delta P_b>$

$$E_J(\Phi) = E_J^0 \left|\cos(\pi\ \Phi/\Phi_0)\right|$$

Tuning the photon emission rate

$$\Gamma \propto E_J^2$$

$$E_J(\Phi) = E_J^0 \left|\cos(\pi \, \Phi / \Phi_0)\right|$$



Power fluctuations cross correlation

•Poissonian source of electrons electronic shot noise due the charge granularity

$$S_{II} = 2eI = 2e^2\Gamma$$

•Poissonian source of photons

$$S_{PaPa} = 2h\nu_a P_a = 2(h\nu_a)^2\Gamma$$

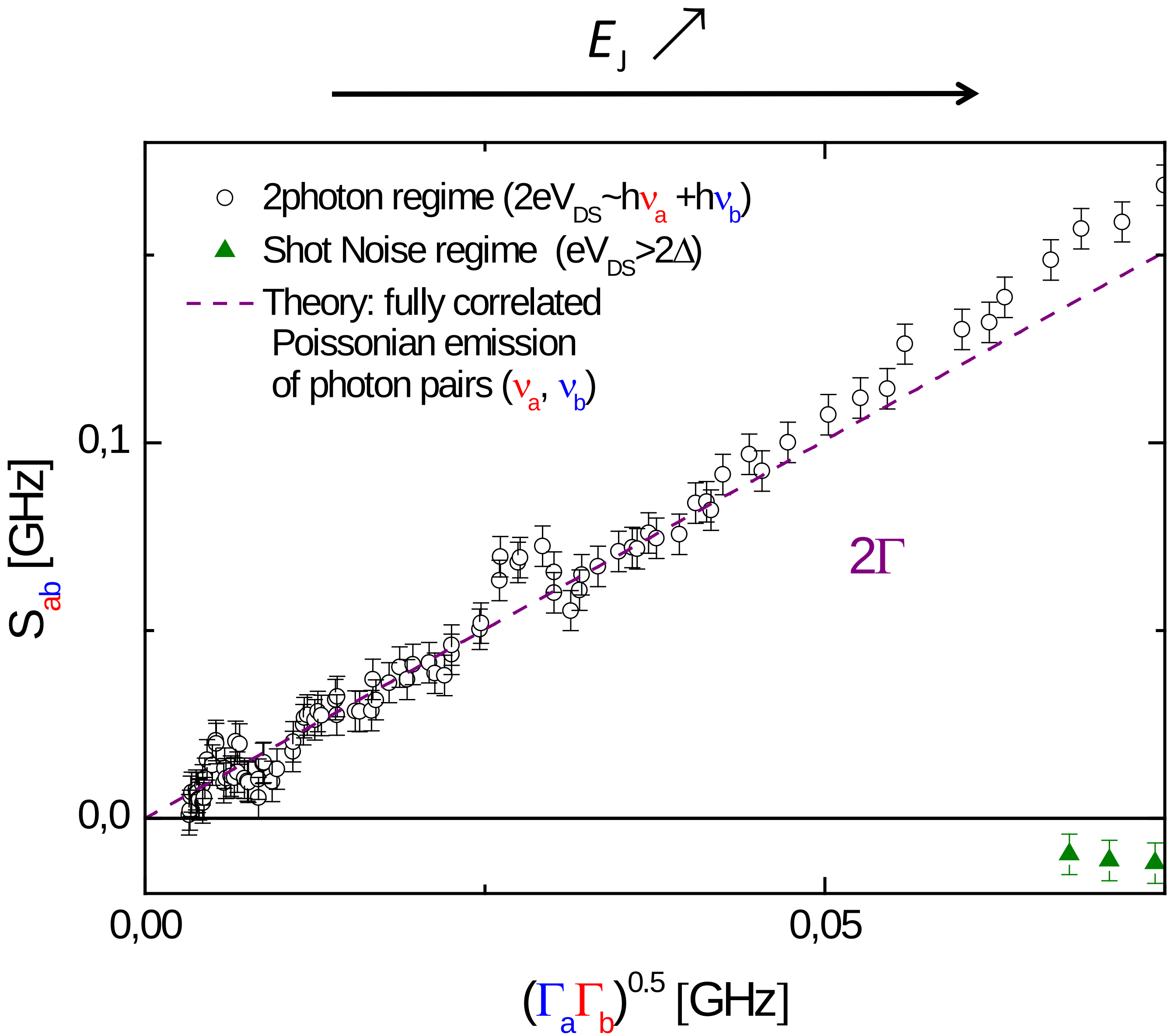
$$S_{PbPb} = 2h\nu_b P_b = 2(h\nu_b)^2\Gamma$$

•Poissonian source of photon pairs (ν_b, ν_a)

$$S_{PaPb} = 2h\nu_a h\nu_b\Gamma$$

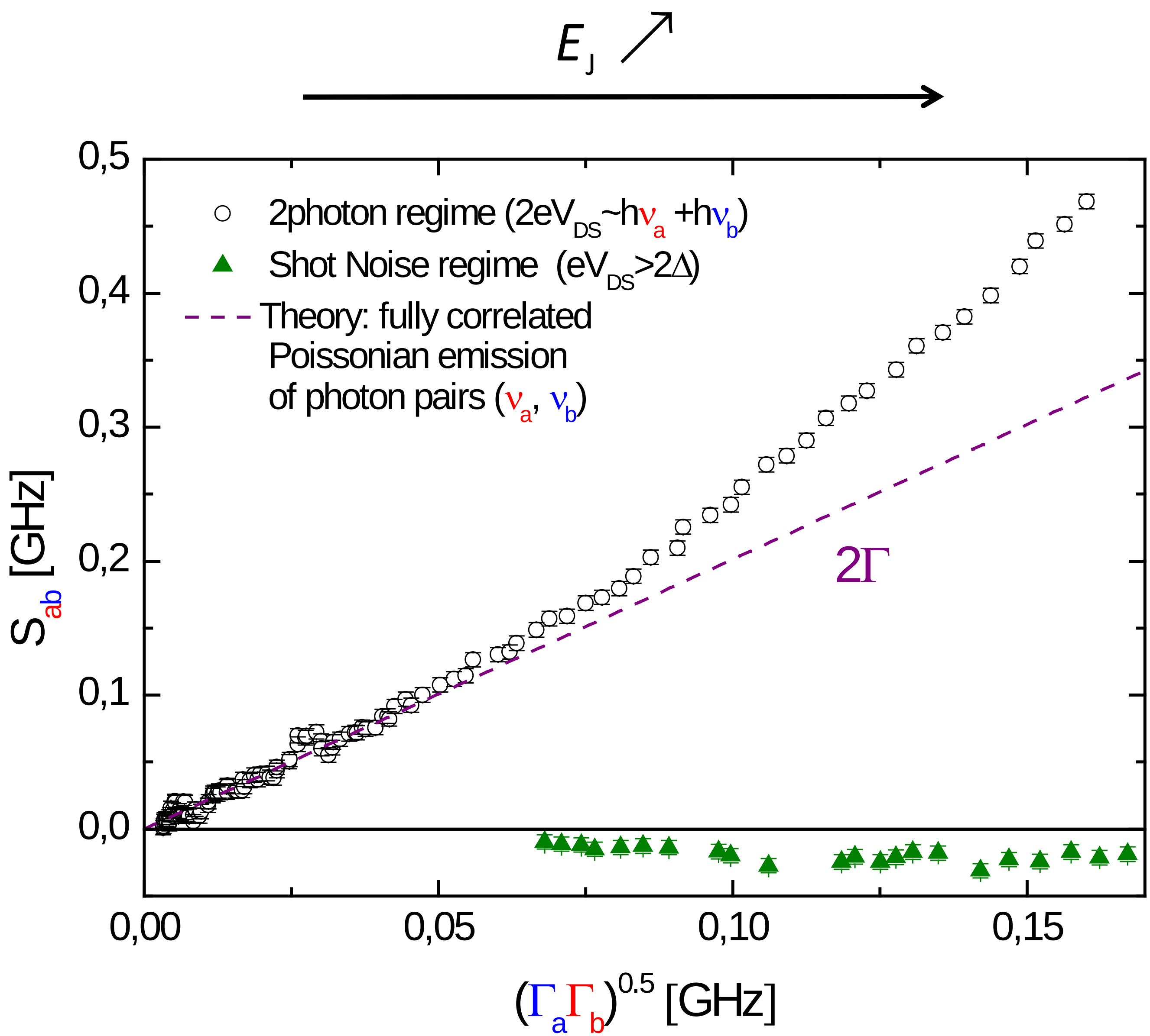
$$\Leftrightarrow S_{ab} = S_{PaPb}/(h\nu_a h\nu_b) = 2\Gamma$$

Correlated photon pairs



Evidence of Poissonian emission of photon pairs

Correlated photon pairs



Deviations due to stimulated emission?

Limits of Coulomb blockade theory

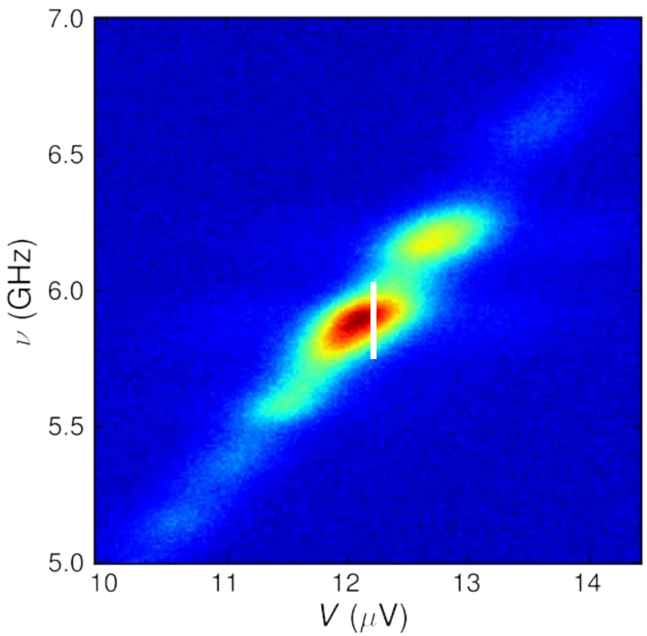
- So far good agreement with $P(E)$ theory
- But need very low E_J to fulfill assumptions
  - environment at equilibrium

$$\Gamma = \frac{\pi}{2\hbar} E_J^2 \sum_{n_0, n_1, \ldots} \left| \langle n_0, n_1, \ldots | e^{i\varphi} | 0 \rangle \right|^2 \delta(2eV - \sum_i n_i h\nu_i)$$

  - single Cooper pair regime: $E_J P(2eV) << 1$

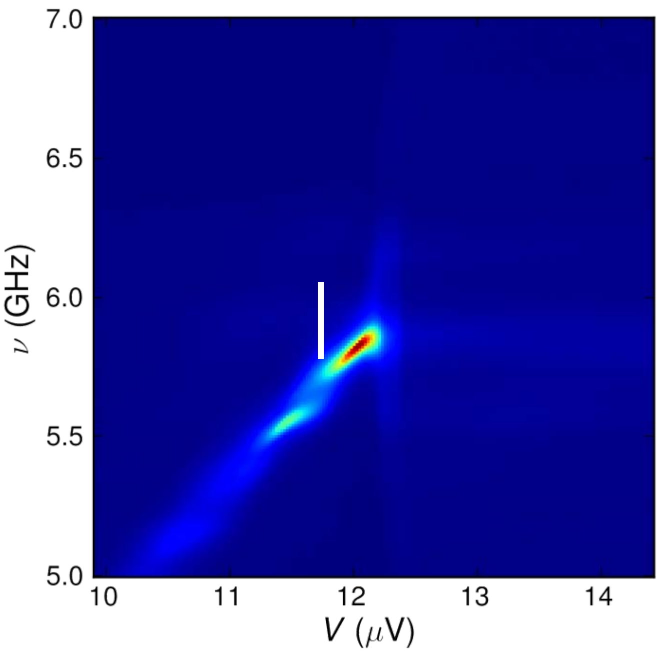
- What happens if assumptions are violated?

Out of equilibrium environment



increase E_J

Lasing-like transition ?

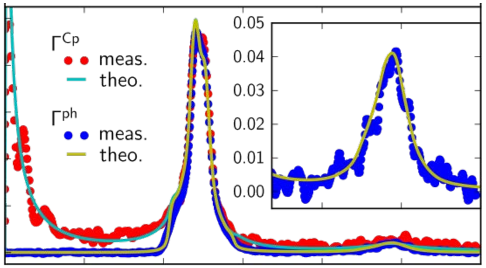


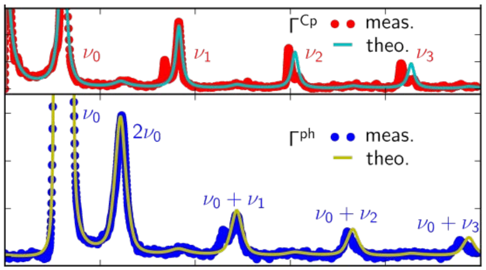
Incoherent pair tunneling

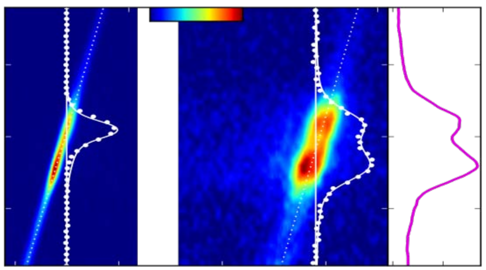
Transition ?

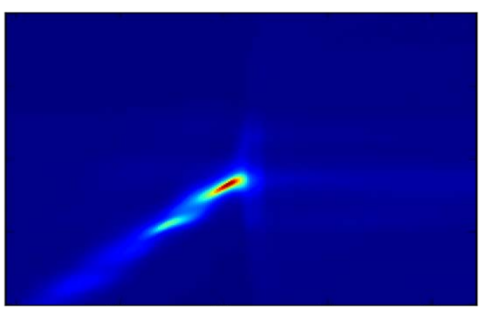
classical AC Josephson effect ?

Conclusions









- Photon side of Coulomb blockade:
  - Cooper pair vs. photon rate
  - multi photon processes
  - spectral properties

  arXiv:1102.0131, to be published in Phys. Rev. Lett.

- Perspectives:
  - interesting for quantum optics with microwave photons, need for a deeper characterization of the emitted radiation
  - out of equilibrium environment (no theory yet)