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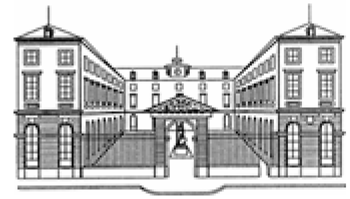


Chaire de Physique Mésoscopique

Michel Devoret

Année 2012, 15 mai - 19 juin

RÉSONATEURS NANOMÉCANIQUES DANS LE RÉGIME QUANTIQUE

NANOMECHANICAL RESONATORS IN QUANTUM REGIME

Sixième leçon / *Sixth lecture*

Last update: 120620

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction to nanomechanical systems

Lecture II: How do we model the coupling between electromagnetic modes and mechanical motion?

Lecture III: Is zero-point motion of the detector variables a limitation in the measurement of the position of a mechanical oscillator?

Lecture IV: In the active cooling of a nanomechanical resonator, is measurement-based feedback equivalent to autonomous feedback?

Lecture V: How close to the ground state can we bring a nanoresonator?

Lecture VI: What oscillator characteristics must we choose to optimally convert quantum information from the microwave domain to the optical domain?

CALENDAR OF 2012 SEMINARS

May 15: **Rob Schoelkopf (Yale University, USA)**
Quantum optics and quantum computation with superconducting circuits.

May 22: **Konrad Lehnert (JILA, Boulder, USA)**
Micro-electromechanics: a new quantum technology.

May 29: **Olivier Arcizet (Institut Néel, Grenoble)**
A single NV defect coupled to a nanomechanical oscillator: hybrid nanomechanics.

June 5: **Ivan Favero (MPQ, Université Paris Diderot)**
From micro to nano-optomechanical systems: photons interacting with mechanical resonators.

June 12: **A. Douglas Stone (Yale University, USA)**
Lasers and anti-lasers: a mesoscopic physicist's perspective on scattering from active and passive media.

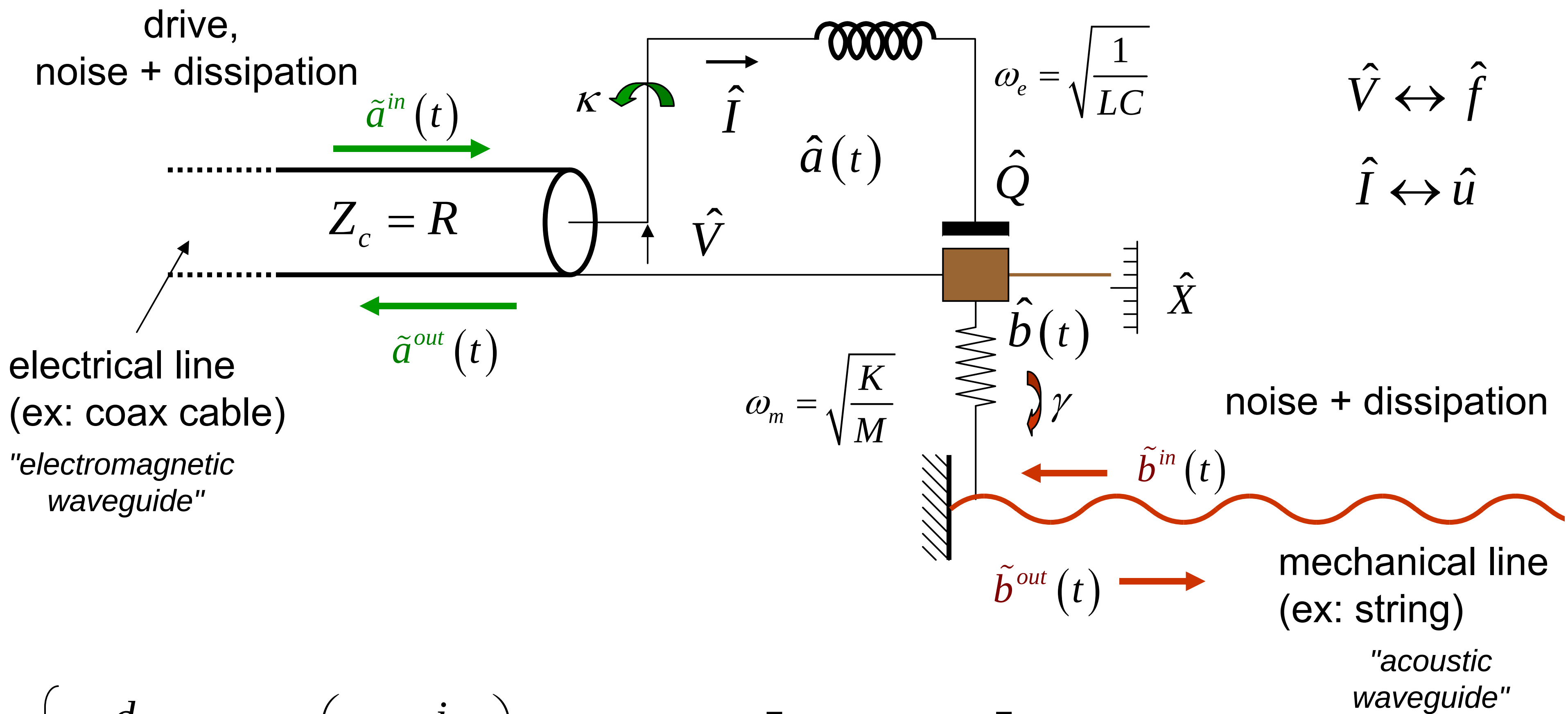
June 19: **Tobias J. Kippenberg (EPFL, Lausanne, Suisse)**
Cavity optomechanics: exploring the coupling of light and micro- and nano-mechanical oscillators.

LECTURE VI : SWAPPING PHONONS AND PHOTONS

OUTLINE

1. Langevin equations for opto- and electro-mechanical nano-resonators: linearly-coupled effective oscillators
2. Susceptibilities, spectral densities and scattering matrix in the strong coupling regime
3. Emulating phonon-photon swapping with the Josephson parametric converter

QUANTUM LANGEVIN EQUATIONS FOR ELECTRO/OPTO-MECHANICAL COUPLED SYSTEMS



$$\begin{cases} \frac{d}{dt}\hat{a}(t) = -i\left(\omega_e - \frac{i}{2}\kappa\right)\hat{a}(t) - ig_3\hat{a}(t)\left[\hat{b}(t) + \hat{b}^\dagger(t)\right] + \sqrt{\kappa}\tilde{a}^{in}(t) \\ \frac{d}{dt}\hat{b}(t) = -i\left(\omega_m - \frac{i}{2}\gamma\right)\hat{b}(t) - ig_3\hat{a}^\dagger(t)\hat{a}(t) + \sqrt{\gamma}\tilde{b}^{in}(t) \end{cases}$$

INPUT BOSON FIELD (TIME AND FREQUENCY)

Define: $$\tilde{a}^{in}(t) = \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} e^{-i\omega t} \hat{a}^{in}[\omega] d\omega$$

complex operator with only <u>positive</u> frequencies contribution

$$\tilde{a}^{in}(t)^\dagger = \tilde{a}^{in\dagger}(t) = \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} e^{i\omega t} \hat{a}^{in}[-\omega] d\omega$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-i\omega t} \hat{a}^{in}[\omega] d\omega$$

complex operator with only <u>negative</u> frequencies contribution

$$\tilde{a}^{in}[\omega] = \Theta(\omega)\hat{a}^{in}[\omega]$$
$$\tilde{a}^{in\dagger}[\omega] = \Theta(-\omega)\hat{a}^{in}[\omega] = \tilde{a}^{in}[-\omega]^\dagger$$
$$\hat{a}^{in}[-\omega] = \hat{a}^{in}[\omega]^\dagger$$
$$\hat{a}^{in\dagger}[\omega] = \hat{a}^{in}[\omega]$$

Instantaneous boson flux:

$$\left\langle \tilde{a}^{in}(t)^\dagger \tilde{a}^{in}(t) \right\rangle = \left\langle \dot{N}^{in}(t) \right\rangle = \frac{1}{2\pi} \int_0^{+\infty} S_{a^{in}a^{in}}[-\omega] d\omega$$

Boson amplitude spectral density:

$$\left\langle \hat{a}^{in}[\omega_1]\hat{a}^{in}[\omega_2] \right\rangle = S_{aa}^{in}[\omega_1]\delta(\omega_1 + \omega_2)$$

$$N_a^{in}(|\omega|) = S_{aa}^{in}\left[-|\omega|\right]$$

available photon number per unit time per unit bandwidth in *a* beam

In thermal equilibrium, with drive at Ω:

$$S_{aa}^{in}[\omega] = \frac{\mathrm{sgn}(\omega)}{2}\left[\coth\left(\frac{\hbar\omega}{2k_B T}\right) + 1\right] + 2\pi \dot{N}_d \left[\delta(\omega - \Omega) + \delta(\omega + \Omega)\right]$$

EXERCISE: PHOTON POPULATION OF ONE DAMPED OSCILLATOR IN THERMAL EQUILIBRIUM

Start from Langevin equation:

(valid in this form only in the very weak damping limit)

$$\frac{d}{dt}\hat{a} = -i\omega_e \hat{a} - \frac{\kappa}{2}\hat{a} + \sqrt{\kappa}\,\tilde{a}^{in}(t)$$

Go to Fourier domain:

$$-i\omega\hat{a} = -i\omega_e \hat{a} - \frac{\kappa}{2}\hat{a} + \sqrt{\kappa}\,\tilde{a}^{in}[\omega]$$

Photon amplitude susceptibility:

$$\hat{a}[\omega] = \tilde{\chi}_{aa}[\omega]\,\tilde{a}^{in}[\omega]$$

$$\tilde{\chi}_{aa}[\omega] = \frac{\sqrt{\kappa}}{\frac{\kappa}{2} - i(\omega - \omega_e)}$$

$$\left|\tilde{\chi}_{aa}[\omega]\right|^2 = 2\frac{\kappa/2}{(\kappa/2)^2 + (\omega - \omega_e)^2}$$

strongly peaked at ω_e, integral over pos. freq. = 2π

Photon number in oscillator:

$$\langle N \rangle = \langle \hat{a}^\dagger \hat{a} \rangle = \frac{1}{2\pi}\int_0^{+\infty} \langle \hat{a}[\omega]^\dagger \hat{a}[\omega] \rangle d\omega = \frac{1}{2\pi}\int_0^{+\infty} \left|\tilde{\chi}_{aa}[\omega]\right|^2 N_a^{in}(\omega)\, d\omega$$

$$\langle N \rangle_T = \frac{1}{2}\left[\coth\left(\frac{\hbar\omega_e}{2k_B T}\right) - 1\right] = \left[\exp\left(\frac{\hbar\omega_e}{k_B T}\right) - 1\right]^{-1}$$

Have recovered stat. mech. result from scattering treatment!

LINEARIZATION OF QUANTUM LANGEVIN EQUATIONS

$$\begin{cases} \dfrac{d}{dt}\hat{a}(t) = -i\left(\omega_e - \dfrac{i}{2}\kappa\right)\hat{a}(t) - ig_3\hat{a}(t)\left[\hat{b}(t)+\hat{b}^\dagger(t)\right] + \sqrt{\kappa}\,\tilde{a}^{in}(t) \\ \dfrac{d}{dt}\hat{b}(t) = -i\left(\omega_m - \dfrac{i}{2}\gamma\right)\hat{b}(t) - ig_3\hat{a}^\dagger(t)\hat{a}(t) + \sqrt{\gamma}\,\tilde{b}^{in}(t) \end{cases}$$

$$\hat{a} \rightarrow \alpha \mathrm{e}^{-i\Omega t} + \delta\hat{a}\mathrm{e}^{-i\Omega t}$$

$$\tilde{a}^{in}(t) \rightarrow \alpha^{in}\mathrm{e}^{-i(\Omega t+\theta)} + \delta\tilde{a}^{in}(t)\mathrm{e}^{-i\Omega t}$$

$$\alpha = \frac{i\sqrt{\kappa}\,\alpha^{in}e^{-i\theta}}{\Omega - \omega_e + \dfrac{i}{2}\kappa}$$

$$\Omega = \omega_e + \Delta$$

θ chosen to make α a positive real quantity

↑ complex function of t with ind. <>0 freq. comp^{nents}, describes modulation of the photon field.

$$\begin{cases} \dfrac{d}{dt}\delta\hat{a}(t) = -i\left(-\Delta - \dfrac{i}{2}\kappa\right)\delta\hat{a}(t) - ig_3\alpha\left[\hat{b}(t)+\hat{b}^\dagger(t)\right] + \sqrt{\kappa}\,\delta\tilde{a}^{in}(t) \\ \dfrac{d}{dt}\hat{b}(t) = -i\left(\omega_m - \dfrac{i}{2}\gamma\right)\hat{b}(t) - ig_3\alpha\left[\delta\hat{a}(t)+\delta\hat{a}^\dagger(t)\right] + \sqrt{\gamma}\,\tilde{b}^{in}(t) \end{cases}$$

LINEARLY-COUPLED EFFECTIVE OSCILLATORS

MHz or GHz phonons

<u>Drive: GHz or THz photons</u> $\Omega = \omega_e + \Delta$

$$\frac{\hat{H}}{\hbar} = -\Delta\,\delta\hat{a}^\dagger\delta\hat{a} + \omega_m \hat{b}^\dagger\hat{b} + g\left(\delta\hat{a} + \delta\hat{a}^\dagger\right)\left(\hat{b} + \hat{b}^\dagger\right)$$

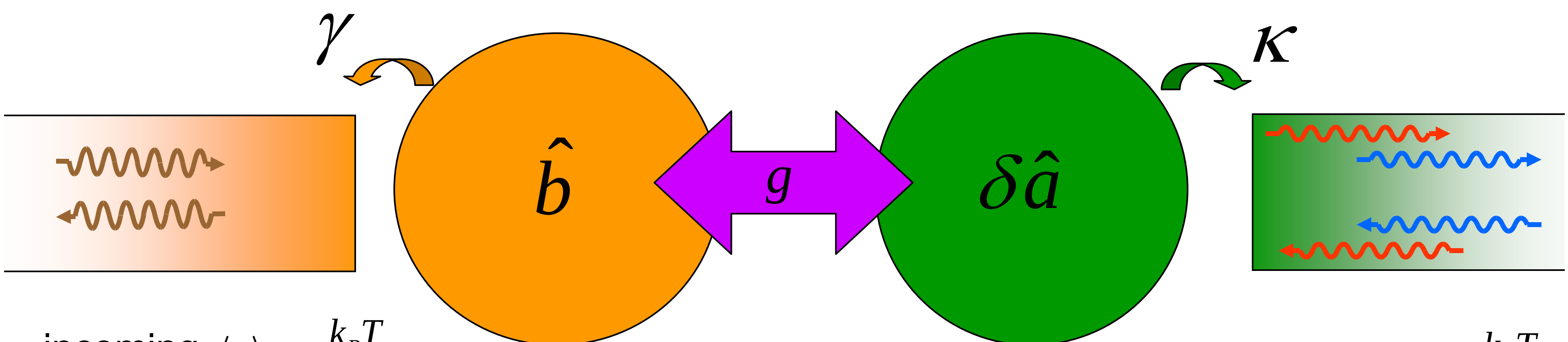
elec./opt. modulation

effective coupling rate: $g = g_3\alpha$

$= g_3\sqrt{\bar{N}_e}$ ← mean number of drive photons

MECHANICAL OSCILLATOR

ELECTRICAL/OPTICAL MODULATIONS OSCILLATOR



incoming $\langle n\rangle_T = \frac{k_B T}{\hbar\omega_m}$ phonons per unit time per unit bandwidth

ω_m

$-\Delta$

$\cong \omega_m$

incoming $\langle N\rangle_T = \frac{k_B T}{\hbar\omega_e}$ photons per unit time per unit bandwidth

FOURIER DOMAIN EXPRESSIONS

$$\begin{cases} \frac{d}{dt}\delta\hat{a}(t) = -i\left(-\Delta - \frac{i}{2}\kappa\right)\delta\hat{a}(t) - ig_3\alpha\left[\hat{b}(t) + \hat{b}^\dagger(t)\right] + \sqrt{\kappa}\,\delta\tilde{a}^{in}(t) \\ \frac{d}{dt}\hat{b}(t) = -i\left(\omega_m - \frac{i}{2}\gamma\right)\hat{b}(t) - ig_3\alpha\left[\delta\hat{a}(t) + \delta\hat{a}^\dagger(t)\right] + \sqrt{\gamma}\,\tilde{b}^{in}(t) \end{cases}$$

$$\begin{cases} -i\omega\delta\hat{a}[\omega] = -i\left(-\Delta - \frac{i}{2}\kappa\right)\delta\hat{a}[\omega] - ig_3\alpha\left[\hat{b}[\omega] + \hat{b}^\dagger[\omega]\right] + \sqrt{\kappa}\,\delta\tilde{a}^{in}[\omega] \\ -i\omega\hat{b}[\omega] = -i\left(\omega_m - \frac{i}{2}\gamma\right)\hat{b}[\omega] - ig_3\alpha\left[\delta\hat{a}[\omega] + \delta\hat{a}^\dagger[\omega]\right] + \sqrt{\gamma}\,\tilde{b}^{in}[\omega] \end{cases}$$

$$g = g_3\alpha = g_3\sqrt{\bar{N}_e}$$

$$\begin{cases} \chi_{aa}^{bare}(\omega)^{-1}\,\delta\hat{a}[\omega] = -ig\left[\hat{b}[\omega] + \hat{b}^\dagger[\omega]\right] + \sqrt{\kappa}\,\delta\tilde{a}^{in}[\omega] \\ \chi_{bb}^{bare}(\omega)^{-1}\,\hat{b}[\omega] = -ig\left[\delta\hat{a}[\omega] + \delta\hat{a}^\dagger[\omega]\right] + \sqrt{\gamma}\,\tilde{b}^{in}[\omega] \end{cases}$$

$$\delta\hat{a}^\dagger[\omega] = \delta\hat{a}[-\omega]^\dagger$$

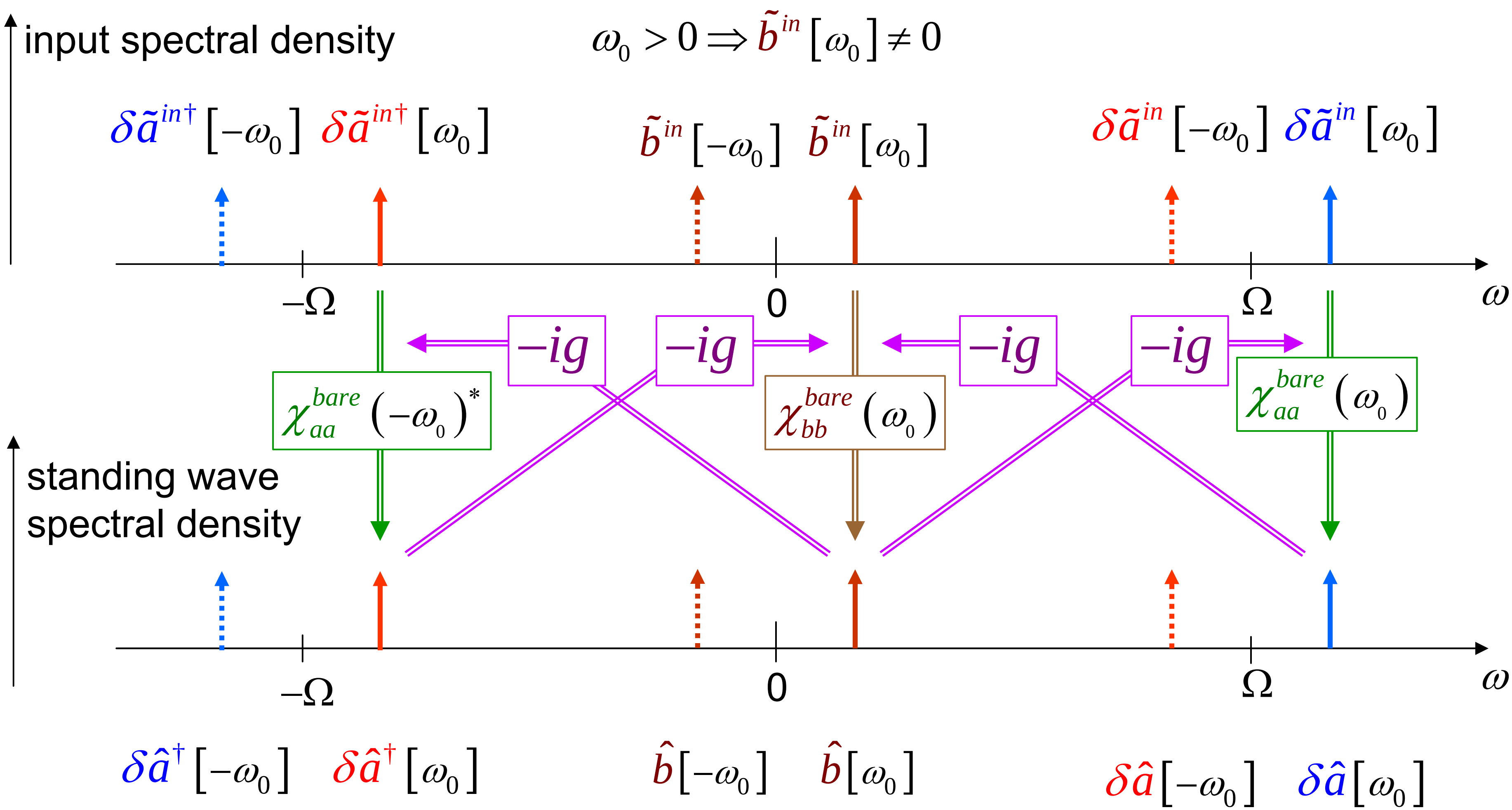
$$\delta\hat{a}[-\omega]^\dagger \neq \delta\hat{a}[\omega]$$

$$\tilde{b}^{in}[\omega] = \Theta(\omega)\,\hat{b}^{in}[\omega]$$

$$\tilde{b}^{in\dagger}[\omega] = \Theta(-\omega)\,\hat{b}^{in}[\omega]$$

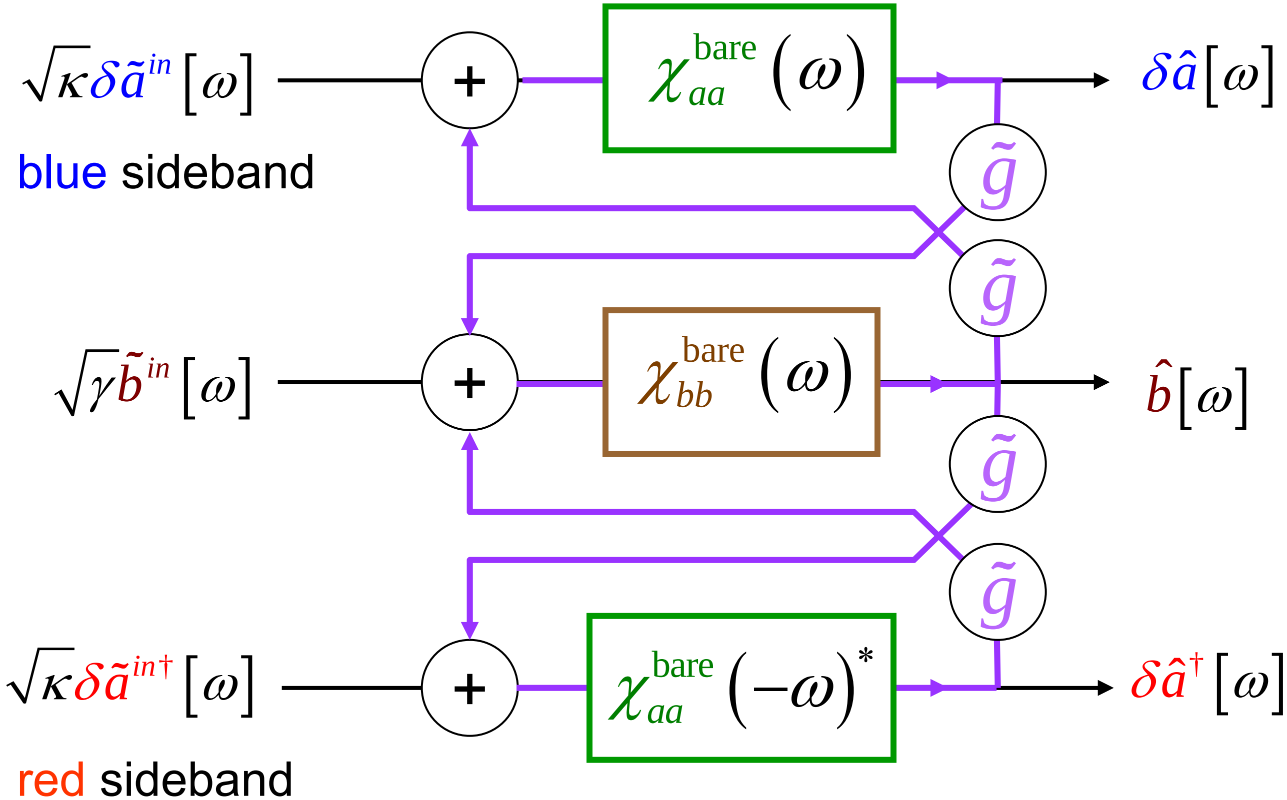
$$\chi_{aa}^{bare}(\omega) = \frac{1}{\frac{\kappa}{2} - i(\omega + \Delta)} \qquad \chi_{bb}^{bare}(\omega) = \frac{1}{\frac{\gamma}{2} - i(\omega - \omega_m)}$$

STRUCTURE OF COUPLED EQUATIONS

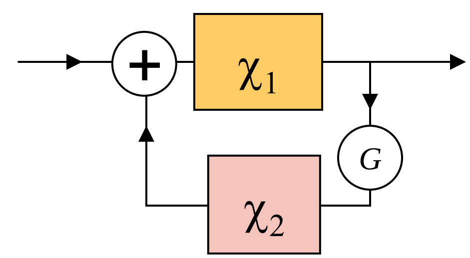


$$\chi_{aa}^{bare}(\omega) = \frac{1}{\frac{\kappa}{2} - i(\omega + \Delta)} \qquad \chi_{bb}^{bare}(\omega) = \frac{1}{\frac{\gamma}{2} - i(\omega - \omega_m)}$$

DRESSED SUSCEPTIBILITIES



$$\tilde{g} = -ig_3\alpha$$



$$\chi = \frac{\chi_1}{1 - G\chi_1\chi_2}$$

$$\chi^{-1} = \chi_1^{-1} - G\chi_2$$

$$\hat{b}[\omega] = \sqrt{\gamma}\chi_{bb}(\omega)\tilde{b}^{in}[\omega] + \sqrt{\kappa}\left(\chi_{ba}^{+}(\omega)\delta\tilde{a}^{in}[\omega] + \chi_{ba}^{-}(\omega)\delta\tilde{a}^{in\dagger}[\omega]\right)$$

$$\chi_{bb}^{-1}(\omega) = \chi_{bb}^{bare}(\omega)^{-1} + i\Sigma(\omega) \qquad i\Sigma(\omega) = -\tilde{g}^2\left[\chi_{aa}^{bare}(\omega) + \chi_{aa}^{bare}(-\omega)^*\right]$$

$$\chi_{ba}^{+}(\omega) = \tilde{g}\chi_{aa}^{bare}(\omega)\chi_{bb}(\omega) \qquad \chi_{ba}^{-}(\omega) = \tilde{g}\chi_{aa}^{bare}(-\omega)^*\chi_{bb}(\omega)$$

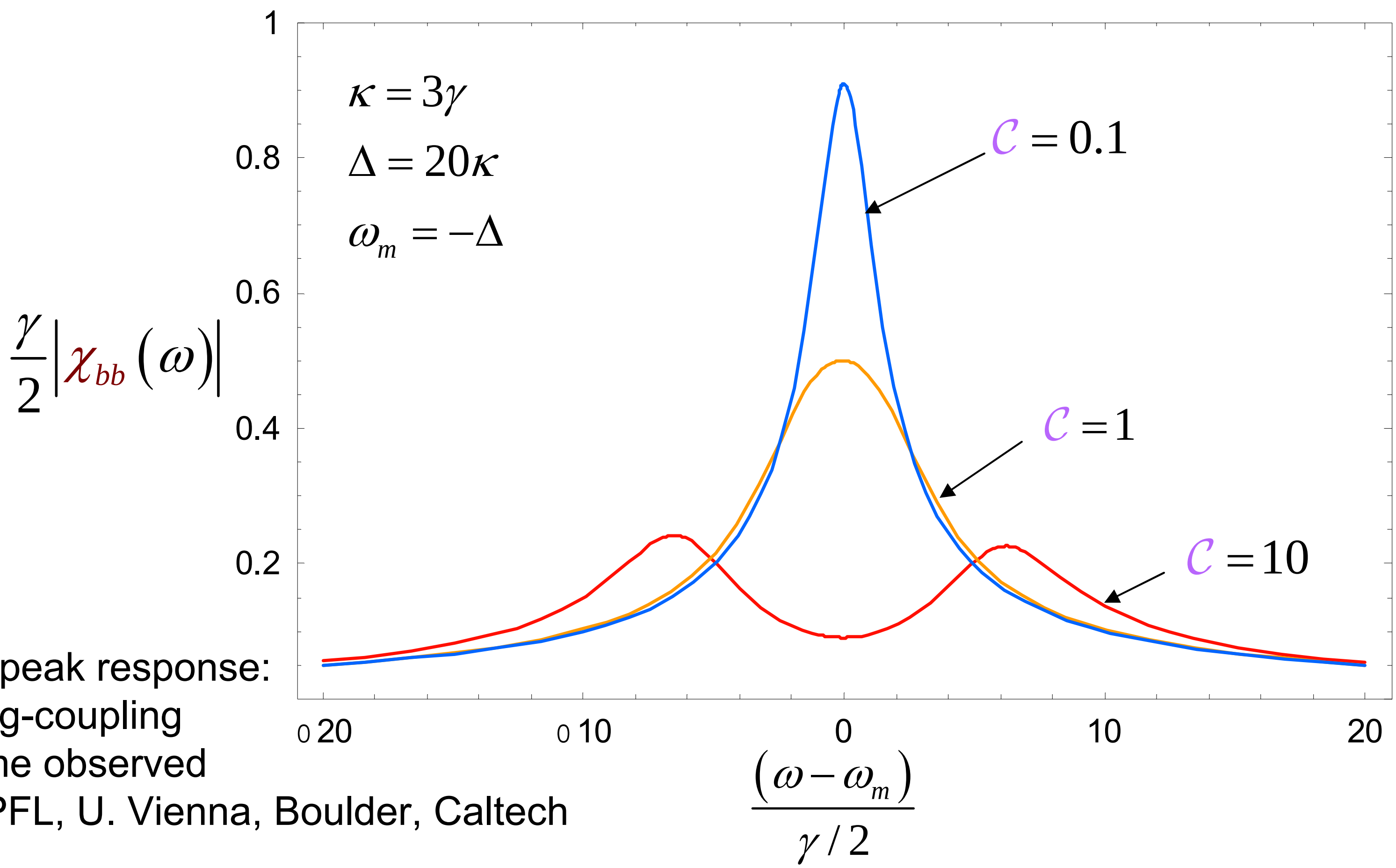
EXPRESSION OF DRESSED SUSCEPTIBILITY

$$\chi_{bb}(\omega) = \frac{2/\gamma}{1 - i\left(\frac{\omega-\omega_m}{\gamma/2}\right) + \frac{\mathcal{C}}{1 - i\left(\frac{\omega+\Delta}{\kappa/2}\right)} + \frac{\mathcal{C}}{1 - i\left(\frac{\omega-\Delta}{\kappa/2}\right)}}$$

$$\mathcal{C} = \frac{4 g_3^2 \bar{N}_e}{\gamma\kappa}$$

dim[less] coupling strength

(AKA "cooperativity")



Two-peak response:
strong-coupling
regime observed
at EPFL, U. Vienna, Boulder, Caltech

POLES OF SUSCEPTIBILITY

$$\mathcal{C} = \frac{4 g_3^2 |\alpha|^2}{\gamma\kappa}$$

$$\chi_{bb}(\omega) = \frac{2/\gamma}{1 - i\left(\frac{\omega-\omega_m}{\gamma/2}\right) + \frac{\mathcal{C}}{1 - i\left(\frac{\omega-\omega_m}{\kappa/2}\right)} + \ldots}$$

can drop non-resonant term in denominator ($2\omega_m \gg \kappa$)

$$i(\omega-\omega_m) \to z \qquad \frac{\gamma}{2} \to \Gamma_b \qquad \frac{\kappa}{2} \to \Gamma_a \qquad \Gamma_a \gg \Gamma_b$$

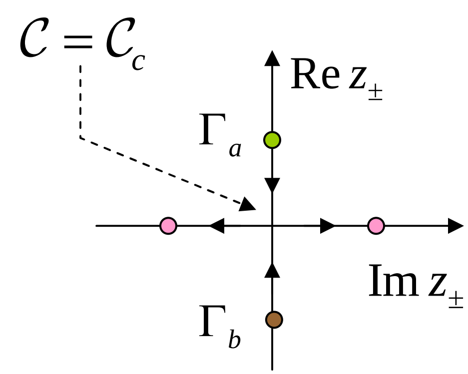
$$\chi_{bb}(z) = \frac{\Gamma_a - z}{(\Gamma_a - z)(\Gamma_b - z) + \Gamma_a \Gamma_b \mathcal{C}}$$

poles and residues

$$\chi_{bb}(z) = \frac{r_+}{z - z_+} + \frac{r_-}{z - z_-}$$

at critical cooperativity: $\mathcal{C}_c = \frac{(\Gamma_a - \Gamma_b)^2}{4\Gamma_a \Gamma_b}$

poles coincide



$$z_\pm = \frac{\Gamma_a + \Gamma_b}{2} \pm \sqrt{\frac{(\Gamma_a - \Gamma_b)^2}{4} - \Gamma_a \Gamma_b \mathcal{C}}$$

$$r_\pm = \mp\left(\frac{1/2}{\sqrt{1 - \frac{\mathcal{C}}{\mathcal{C}_c}}}\right) - \frac{1}{2}$$

when poles are well-separated, the two effective oscillators are fully hybridized

SPECTRAL DENSITY OF MECHANICAL FLUCTUATIONS

$$\left\langle \tilde{b}^{in}[\omega']^{\dagger}\,\tilde{b}^{in}[\omega]\right\rangle = N_{bb}^{in}(\omega)\,\delta(\omega-\omega'); \quad \left\langle \tilde{b}^{in}[\omega]\,\tilde{b}^{in}[\omega']^{\dagger}\right\rangle = \left[N_{bb}^{in}(\omega)+1\right]\delta(\omega-\omega')$$

$$\left\langle \delta\tilde{a}^{in}[\omega']^{\dagger}\,\delta\tilde{a}^{in}[\omega]\right\rangle = N_{aa}^{in}(\omega)\,\delta(\omega-\omega'); \quad \left\langle \delta\tilde{a}^{in}[\omega]\,\delta\tilde{a}^{in}[\omega']^{\dagger}\right\rangle = \left[N_{aa}^{in}(\omega)+1\right]\delta(\omega-\omega')$$

$$\Omega = \omega_e + \Delta;\ \Delta = -\omega_m$$
$$\delta\omega = \omega - \omega_m$$

$$N_{bb}(\omega) = \left|\chi_{bb}(\delta\omega)\right|^2 \Big[N_{bb}^{in} + \kappa\left|\chi_{aa}^{bare}(\delta\omega)\right|^2 N_{aa}^{in} + \kappa\left|\chi_{aa}^{bare}(\delta\omega+2\omega_m)\right|^2 \left(N_{aa}^{in}+1\right)\Big]$$

N_{bb}^{in}: incoming noise phonons

$N_{bb}(\omega)$: total noise phonons in standing mech. d.o.f.

N_{aa}^{in}: incoming noise of blue sideband photons (anti-stokes line)

$\left(N_{aa}^{in}+1\right)$: incoming noise of red sideband photons (stokes line)

$$\left|\chi_{aa}^{bare}(\delta\omega)\right|^2 = \frac{4}{\kappa^2 + 4\delta\omega^2}$$

$$\langle n\rangle = \left\langle \hat{b}^{\dagger}\hat{b}\right\rangle = \frac{1}{2\pi}\int_0^{+\infty}\left\langle b[\omega]^{\dagger}\,b[\omega]\right\rangle d\omega = \frac{1}{2\pi}\int_0^{+\infty} N_{bb}(\omega)\,d\omega$$

If we could neglect flux of noise phonons, integral shows, at opt. drive: $\langle n\rangle_{opt} \cong \left(\dfrac{\kappa}{4\omega_m}\right)^2$

MINIMAL EFFECTIVE PHONON TEMPERATURE

Assume oscillator δa is perfectly cold $\quad \hbar\omega_e \gg k_B T \quad N_{aa}^{in}(\omega) = 0$

while oscillator b is thermally excited: $\quad \hbar\omega_m \ll k_B T \quad N_{bb}^{in}(\omega) = \dfrac{k_B T}{\hbar\omega_m}$

For $\quad \mathcal{C} \ll \mathcal{C}_c = \dfrac{(\kappa-\gamma)^2}{4\gamma\kappa} \quad \langle n_b \rangle = N_{bb}^{in}$ OK, FDT!

(obtained from integrating one lorentzian with weight 1 and width γ)

For $\quad \mathcal{C} \gg \mathcal{C}_c = \dfrac{(\kappa-\gamma)^2}{4\gamma\kappa} \quad \langle n_b \rangle = \dfrac{1}{2} N_{bb}^{in} \dfrac{\gamma}{(\gamma+\kappa)/2}$

(obtained from integrating 2 lorentzians with weight 1/4 and width $(\gamma + \kappa)/2$)

when $\quad \kappa \gg \gamma \quad \langle n_b \rangle \to \dfrac{k_B T}{\hbar\omega_m} \dfrac{\gamma}{\kappa}$

COMPLETE CONVERSION OF MECHANICAL MODE INTO CAVITY MODULATION MODE

If there is no δa input

$$\tilde{b}^{out} = \tilde{b}^{in} - \sqrt{\gamma} b$$ from input-output relations

and if $\chi_{bb}^{-1}(\omega) = \chi_{bb}^{bare}(\omega)^{-1} + i\Sigma(\omega) = \gamma$ ⇨ $\tilde{b}^{out} = 0$

At this point, mechanical signal & noise is entirely converted into electrical signal & noise!

Full conversion drive:

$$\chi_{bb}(\omega) = \gamma^{-1} \Rightarrow 1 - \frac{i(\omega - \omega_m)}{\gamma/2} + \frac{2g^2}{\gamma}\left\{\left[\frac{\kappa}{2} - i(\omega+\Delta)\right]^{-1} + \left[\frac{\kappa}{2} - i(\omega-\Delta)\right]^{-1}\right\} = 2$$

Solution at: $\omega \cong \omega_m$
(when $\omega_m = -\Delta \gg \kappa$)

$$\mathcal{C} = \frac{4g_3^2 \bar{N}_e}{\kappa\gamma} \cong 1$$

Not to be confused with $\mathcal{C} = \mathcal{C}_c$

greater than 1 when $\kappa \gg \gamma$

PHONON-PHOTON SCATTERING MATRIX

$$\begin{cases} \chi_{aa}^{bare}(\omega)^{-1}\,\delta\hat{a}[\omega] + ig\hat{b}[\omega] = \sqrt{\kappa}\,\delta\tilde{a}^{in}[\omega] \\ \chi_{bb}^{bare}(\omega)^{-1}\,\hat{b}[\omega] + ig\,\delta\hat{a}[\omega] = \sqrt{\gamma}\,\tilde{b}^{in}[\omega] \end{cases} \qquad \begin{cases} -\chi_{aa}^{bare}(\omega)^{-1*}\,\delta\hat{a}[\omega] + ig\hat{b}[\omega] = \sqrt{\kappa}\,\delta\tilde{a}^{out}[\omega] \\ -\chi_{bb}^{bare}(\omega)^{-1*}\,\hat{b}[\omega] + ig\,\delta\hat{a}[\omega] = \sqrt{\gamma}\,\tilde{b}^{out}[\omega] \end{cases}$$

$$\begin{bmatrix} \delta\tilde{a}^{out}[\omega] \\ \tilde{b}^{out}[\omega] \end{bmatrix} = \begin{bmatrix} r_{aa} & t_{ab} \\ t_{ba} & r_{bb} \end{bmatrix} \begin{bmatrix} \delta\tilde{a}^{in}[\omega] \\ \tilde{b}^{in}[\omega] \end{bmatrix}$$

$$\begin{bmatrix} r_{aa} & t_{ab} \\ t_{ba} & r_{bb} \end{bmatrix} = \begin{bmatrix} \dfrac{-\chi_e^{-1*}\chi_m^{-1} + \mathcal{C}}{\chi_e^{-1}\chi_m^{-1} + \mathcal{C}} & \dfrac{2ie^{i\phi}\mathcal{C}^{1/2}}{\chi_e^{-1}\chi_m^{-1} + \mathcal{C}} \\ \dfrac{2ie^{-i\phi}\mathcal{C}^{1/2}}{\chi_e^{-1}\chi_m^{-1} + \mathcal{C}} & \dfrac{-\chi_m^{-1*}\chi_e^{-1} + \mathcal{C}}{\chi_e^{-1}\chi_m^{-1} + \mathcal{C}} \end{bmatrix}$$

$$\mathcal{C} = \frac{4|g_3\alpha|^2}{\gamma\kappa}$$

$$\chi_e^{-1} = 1 - i\frac{\delta\omega}{\kappa}$$

$$\chi_m^{-1} = 1 - i\frac{\delta\omega}{\gamma}$$

$$\delta\omega = \omega - \omega_m$$

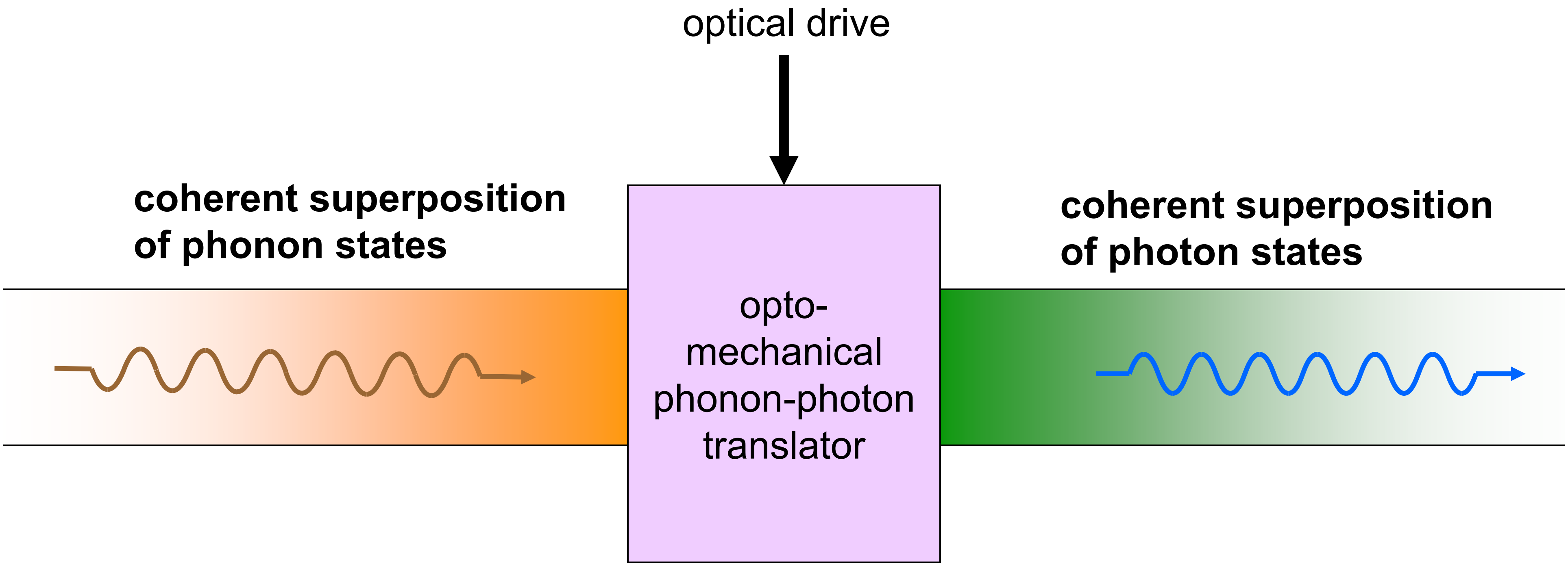
$e^{i\phi}$: pump phase factor

unitary, conserves boson number

full conversion when $\mathcal{C} = 1$, 50/50 beam splitter when $\mathcal{C} = \sqrt{2} - 1$

PHONON-PHOTON TRANSLATOR

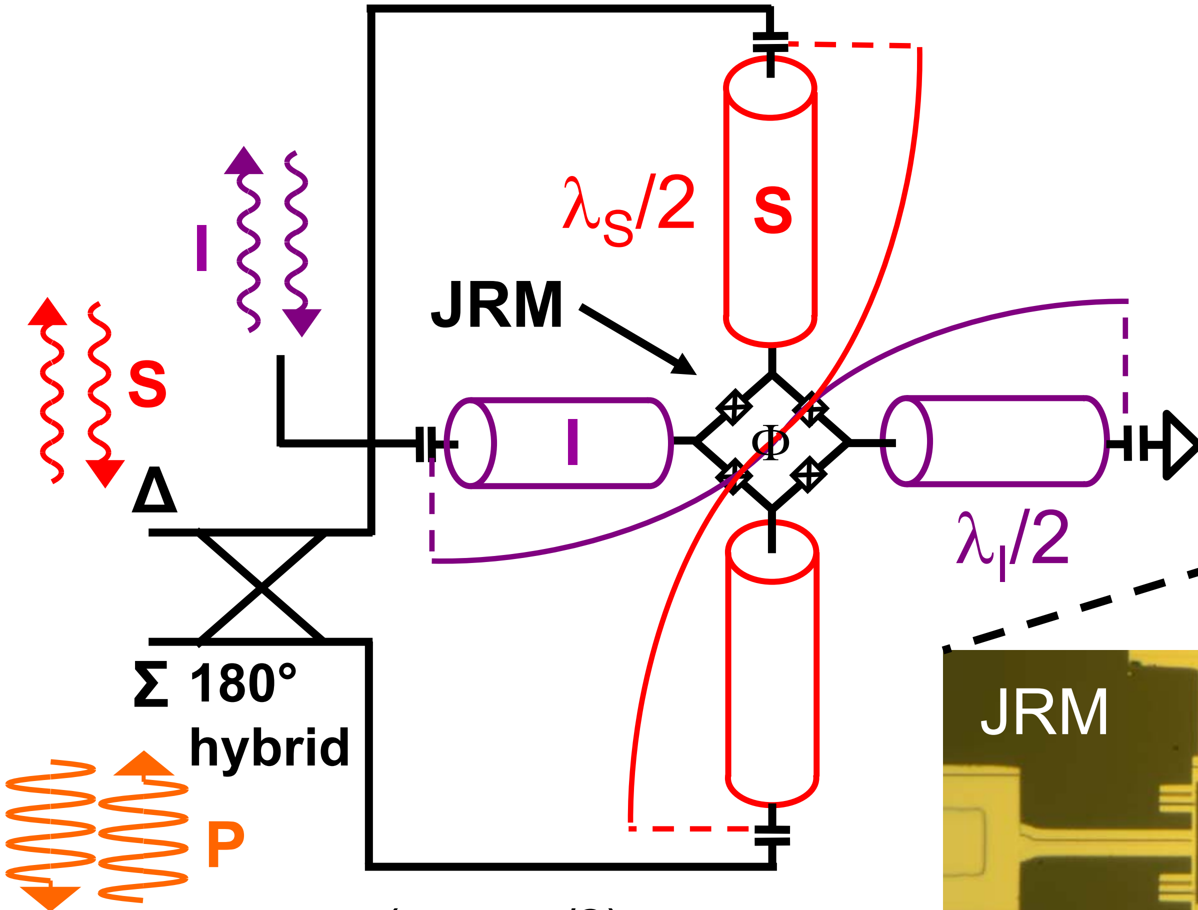
Safavi-Naeini & Painter, *New Journal of Physics* **13** (2011) 013017

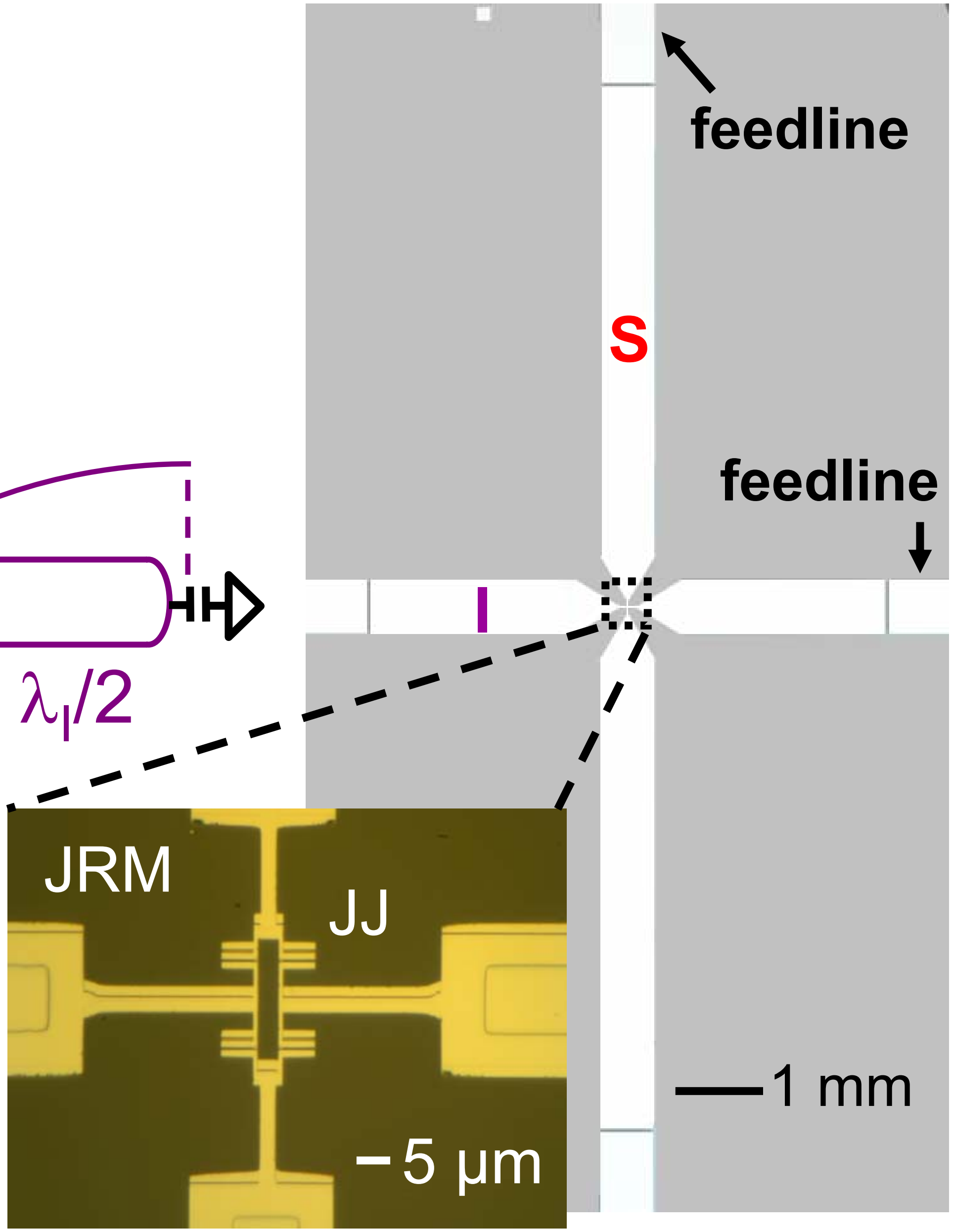


$\mathcal{C} = 1$: no reflection, noiseless conversion

JOSEPHSON PARAMETRIC CONVERTER (JPC)

(B. Abdo et al., 2012)





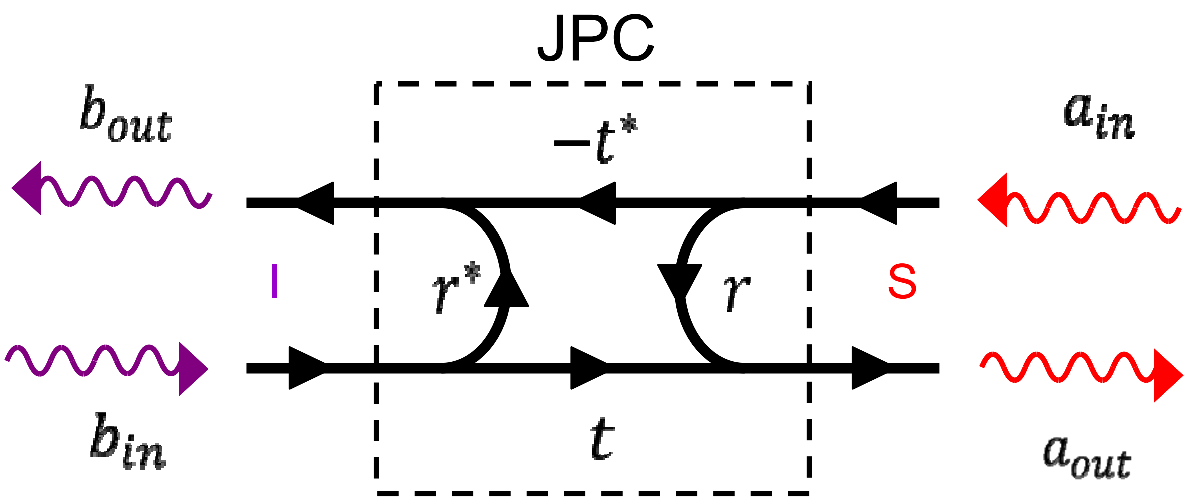
$(\Phi = \Phi_0/2)$

f_S = 8.402 GHz, f_I = 14.687 GHz,
bw_s = 100 MHz, bw_I ≈ 300 MHz,
f_P = 6.285 GHz, I_0 = 3 μA

REFLECTION - CONVERSION

Bergeal et al., Nature Physics 6, 296 (2012)

$f_P = f_I - f_S$



$|r|^2 + |t|^2 = 1$

At resonance:

$$r = \frac{1-|\rho|^2}{1+|\rho|^2}$$

$$t = \frac{2\rho}{1+|\rho|^2}$$

$$\rho = \sqrt{\frac{P_P}{P_{P0}}} e^{-i\varphi}$$

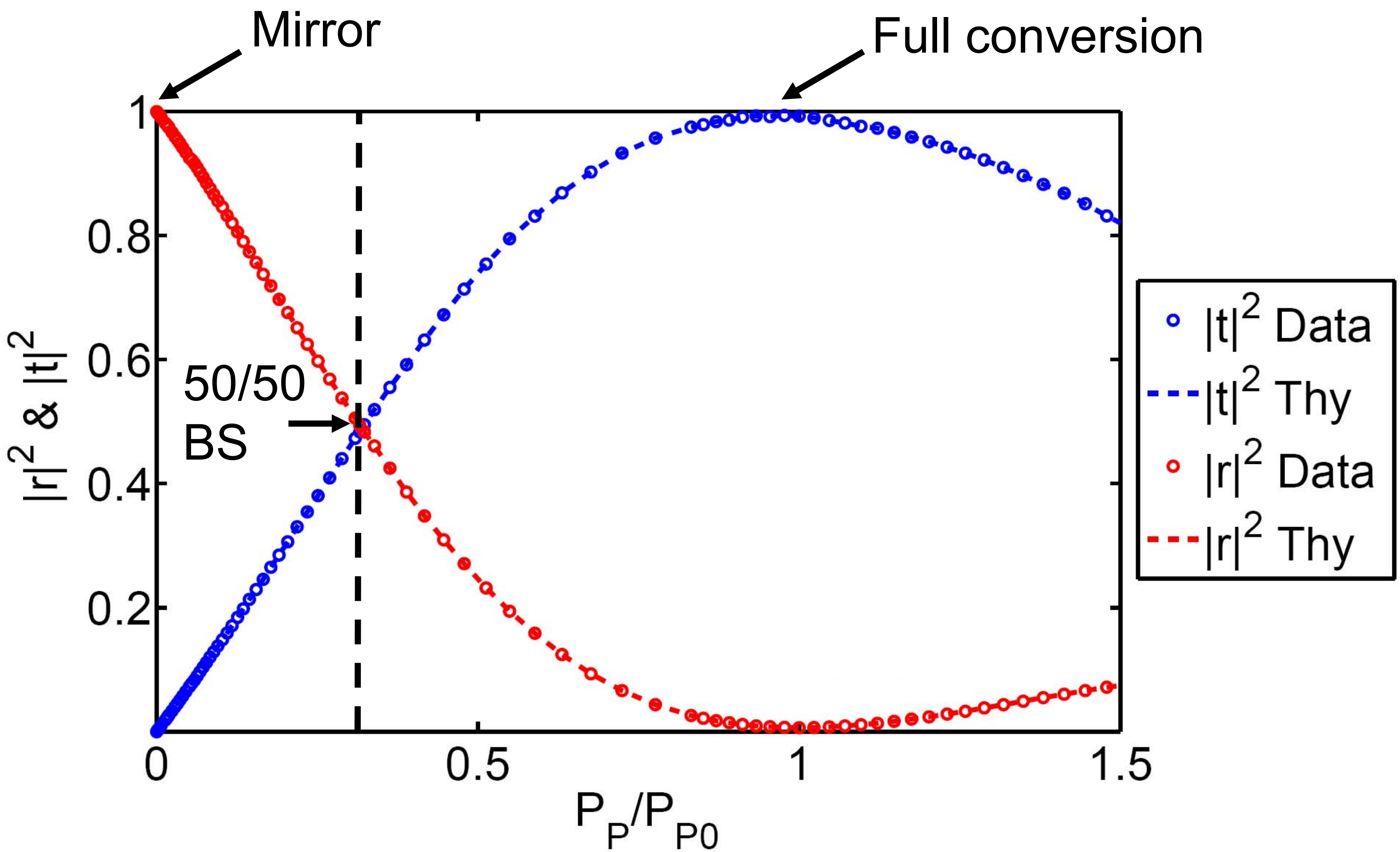
Points of interest:

Perfect mirror: $|r|^2 = 1,\ |t|^2 = 0$

50/50 beam-splitter: $|r|^2 = |t|^2 = 0.5$

Full conversion: $|r|^2 = 0,\ |t|^2 = 1$

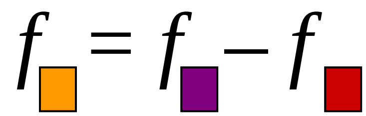
SCATTERING PARAMETER MEASUREMENT



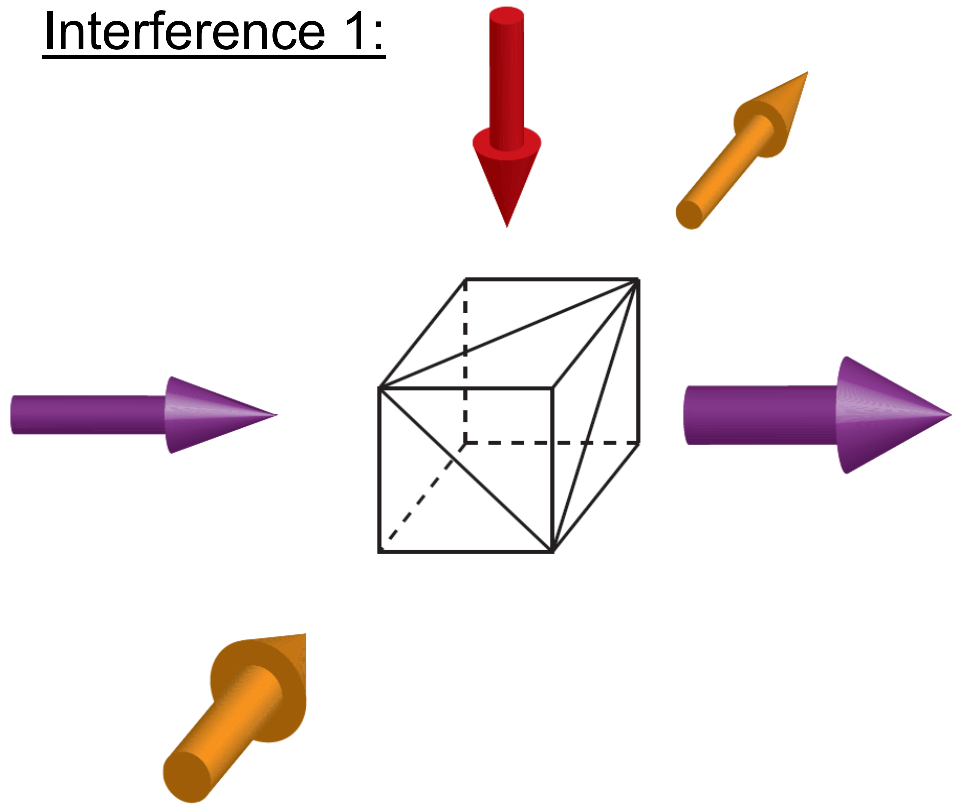
3-WAVE COHERENT SCATTERING

All 3 waves can be rapidly modulated (MHz)

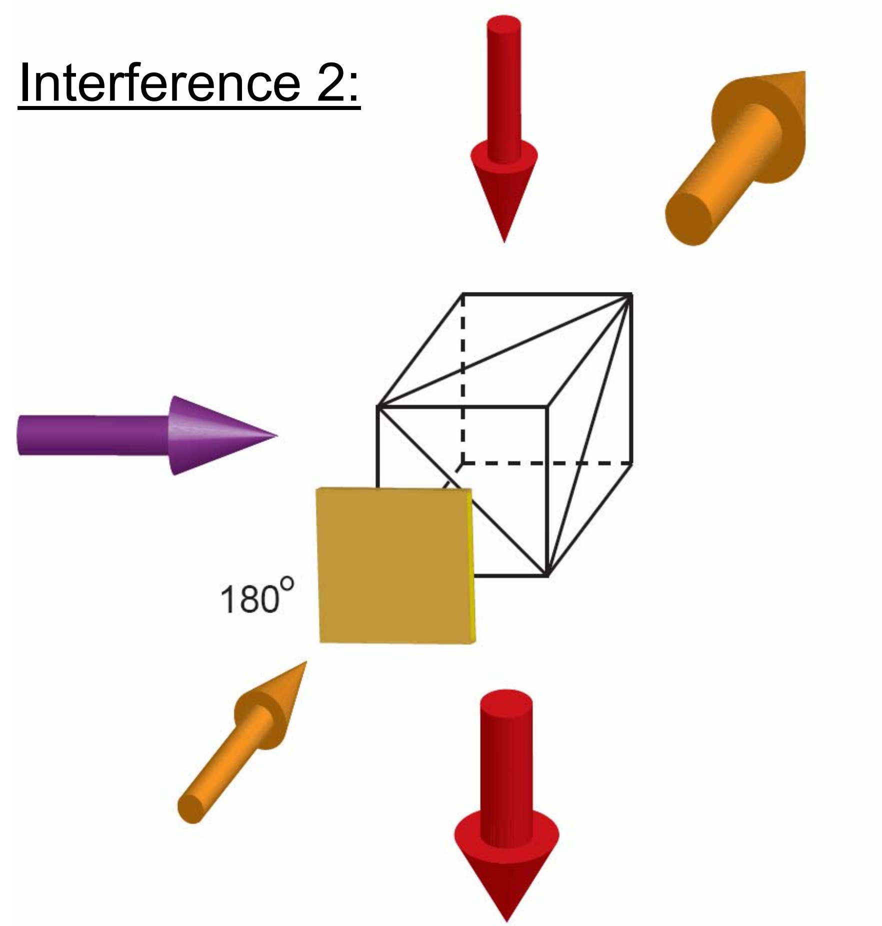
frequency locking



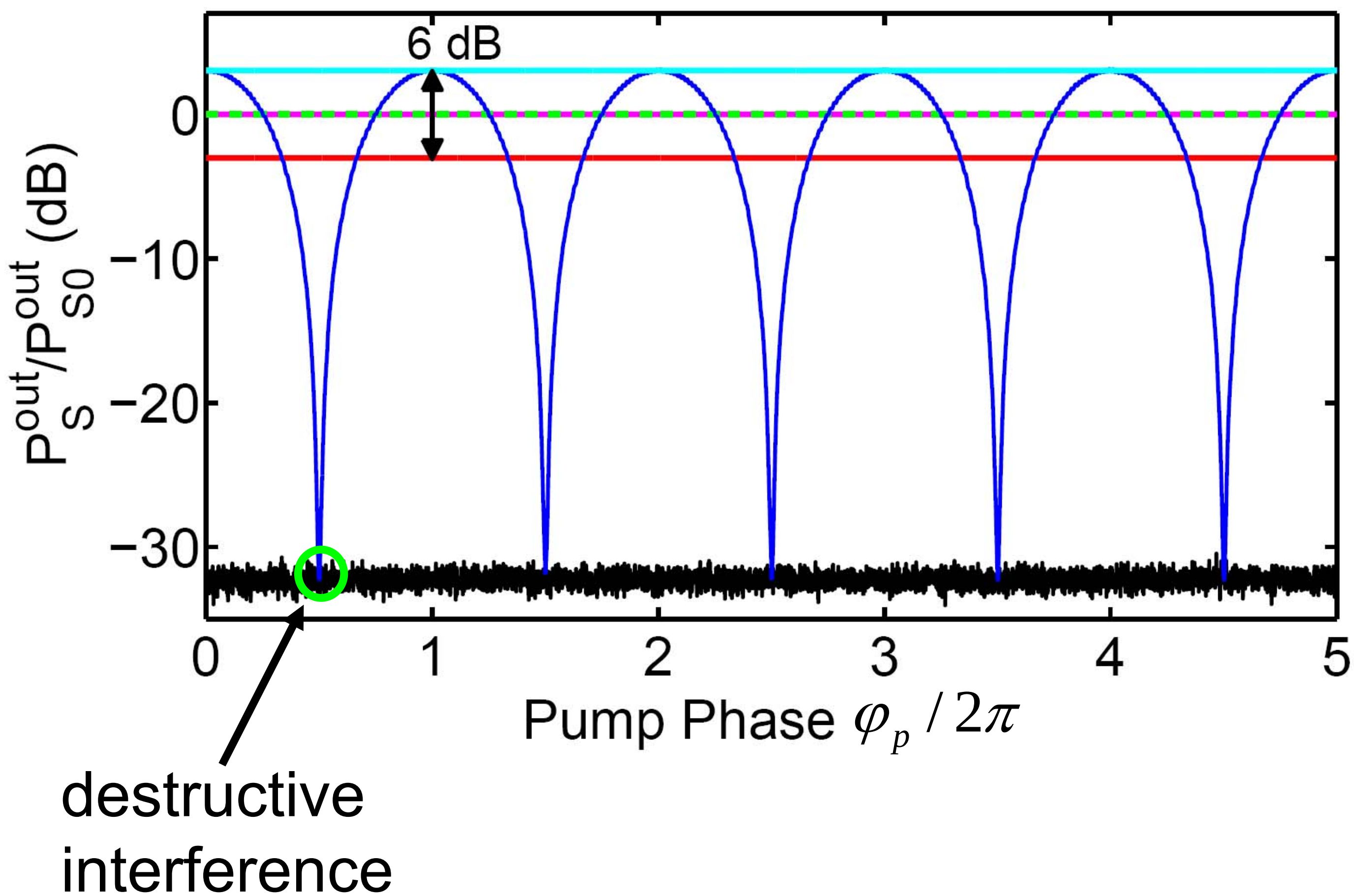
Interference 1:

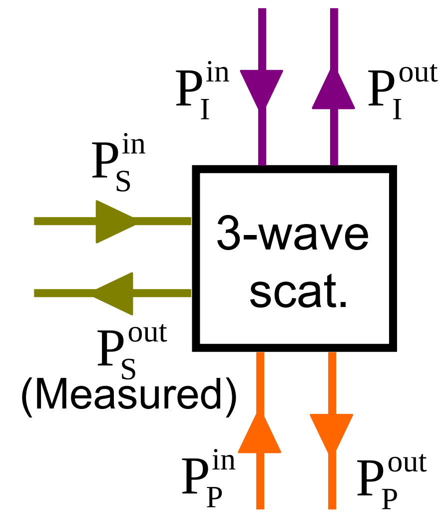


Interference 2:



INTERFEROMETRY WITH THE JPC AT THE 50/50 BEAM-SPLITTER WORKING POINT





line	P_S^{in}	P_I^{in}	P_P^{in}
pink solid	ON	OFF	OFF
green dotted	ON	ON	OFF
red solid	ON	OFF	ON
black solid	OFF	OFF	OFF
blue solid	ON	ON	ON
cyan solid	ON	ON	ON

END OF 2012 COURSE ON NANOMECHANICAL RESONATORS.

THERE WILL BE NO COURSE IN 2013.
TOPICS OF INTEREST AFTER 2013: SINGLE SPIN DETECTION, AUTONOMOUS FEEDBACK CONTROL OF QUANTUM STATES AND MANIFOLDS.

ACKNOWLEDGEMENTS:

B. Abdo, N. Bergeal, P. Campagne, A. Clerk, D. Esteve, E. Flurin, L. Frunzio, K. Geerlings, S. Girvin, L. Glazman, J. Harris, M. Hatridge, B. Huard, A. Kamal, J. Koch, K. Lehnert, M. Mirrahimi, F. Mallet, V. Manucharyan, N. Masluk, D. Prober, N. Roch, F. Schackert, R. Schoelkopf, S. Shankar, K. Sliwa and A.-M. Tremblay.