COLLÈGE DE FRANCE
1530

Chaire de Physique Mésoscopique
Michel Devoret
Année 2010, 11 mai - 22 juin

INTRODUCTION AU CALCUL QUANTIQUE

INTRODUCTION TO QUANTUM COMPUTATION

Sixième Leçon / *Sixth Lecture*

VISIT THE WEBSITE OF THE CHAIR OF MESOSCOPIC PHYSICS

http://www.college-de-france.fr

then follow
Enseignement > Sciences Physiques > Physique Mésoscopique > Site web

or

http://www.physinfo.fr/lectures.html

PDF FILES OF ALL PAST LECTURES ARE POSTED

Questions, comments and corrections are welcome!

write to "phymeso@gmail.com"

CONTENT OF THIS YEAR'S LECTURES

QUANTUM COMPUTATION FROM THE PERSPECTIVE OF MESOSCOPIC CIRCUITS

1. Introduction, c-bits versus q-bits
2. The Pauli matrices and quantum computation primitives
3. Stabilizer formalism for state representation
4. Clifford calculus
5. Algorithms
6. <u>Quantum error correction</u>

10-VI-3

CALENDAR OF SEMINARS

May 11: Cristian Urbina, (Quantronics group, SPEC-CEA Saclay)
Josephson effect in atomic contacts and carbon nanotubes

May 18: Benoît Douçot (LPTHE / Université Pierre et Marie Curie)
Towards the physical realization of topologically protected qubits

June 1: Takis Kontos (LPA / Ecole Normale Supérieure)
Points quantiques et ferromagnétisme

June 8: Cristiano Ciuti (MPQ, Université Paris - Diderot)
Ultrastrong coupling circuit QED : vacuum degeneracy and quantum phase transitions

June 15: Leo DiCarlo (Yale)
Preparation and measurement of tri-partite entanglement in a superconducting quantum circuit

June 22: Vladimir Manucharian (Yale)
The fluxonium circuit: an electrical dual of the Cooper-pair box?

10-VI-4

LECTURE IV : ERROR CORRECTION

Maintaining by an active feedback process,
not simply a single state, but a manifold of quantum states.

1. Classical error correction
2. A simplified example: 3-qubit code
3. Error processes
4. The 7-qubit code

10-VI-5

OUTLINE

1. Classical error correction
2. A simplified example: 3-qubit code
3. Error processes
4. The 7-qubit code

10-VI-5a

BASICS OF CLASSICAL ERROR-CORRECTION

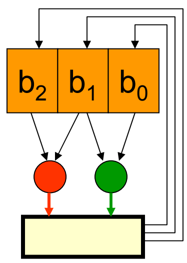
Redundancy: 0, 1 *replaced by:* 0 0 0, 1 1 1 — "repetition code"

Possible 1-bit errors:

1 0 0	0 1 0	0 0 1
0 1 1	1 0 1	1 1 0

Probability: ε per unit time (2-bit errors: ε^2)

Feedback information on parity of first 2 bits and last 2 bits:

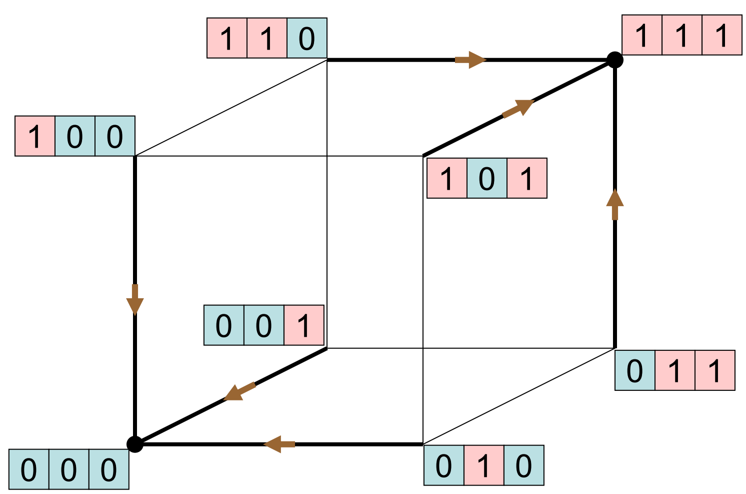


$$\begin{cases} e = b_2 \oplus b_1 \\ f = b_1 \oplus b_0 \end{cases}$$

e \ f	0	1
0	do nothing	$b_0 \to b_0 \oplus 1$
1	$b_2 \to b_2 \oplus 1$	$b_1 \to b_1 \oplus 1$

10-VI-7c

PRINCIPLE OF CLASSICAL ERROR-CORRECTION



1 error is not lethal, system is kept within basin of attraction of protected state

10-VI-8

GENERAL PARITY ERROR BIT

$N{+}1$ bits $\quad \vec{x} = (x_N, x_{N-1}, ..., x_2, x_1, x_0) \in \mathbb{B}^{N+1}$

constraint: $\quad e = \sum_{i=0}^{N} \oplus x_i \quad \longleftarrow$ Boolean sum, N free bits

↑ parity error bit, normal state is $e = 0$

If 1 or an odd number of errors occur, constraint is violated, as shown by $e = 0 \rightarrow e = 1$.
It is possible to detect that an error has occurred, but it is impossible to correct it, if nothing is added.

10-VI-9

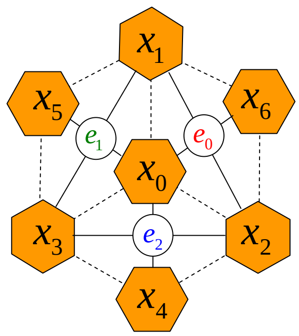
ERROR CORRECTION REQUIRES EXTRA BITS

Increase number of independent constraints on the bits until:

$$2^{\text{number of error bits}} \geq \text{number of code bits} + 1$$

10-VI-9bis

HAMMING ERROR CORRECTING CODES



Example: 7 bits protected with 3 error bits, coding for 4 free bits (2^3=7+1)

Constraints:

$$x_0 \oplus x_1 \oplus x_2 \oplus x_6 = e_0 = 0$$
$$x_0 \oplus x_1 \oplus x_3 \oplus x_5 = e_1 = 0$$
$$x_0 \oplus x_2 \oplus x_3 \oplus x_4 = e_2 = 0$$

can be written as: $\mathbf{A}\vec{x} = \vec{e} = \vec{0}$

where:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

(i ←, j ↓)

After one error:

$$\mathbf{A}\vec{x}' = \vec{e} \neq \vec{0}$$

The error syndrome matrix $\mathbf{A}$ is used to detect which error has occurred **and** to correct it.

$$x_i \rightarrow x_i \oplus \prod_{j=0}^{2} \left(e_j \oplus \bar{A}_{ji} \right)$$

Requires seven 3-way AND + linear gates

10-VI-10e

CORRECTING QUBIT ERRORS?!

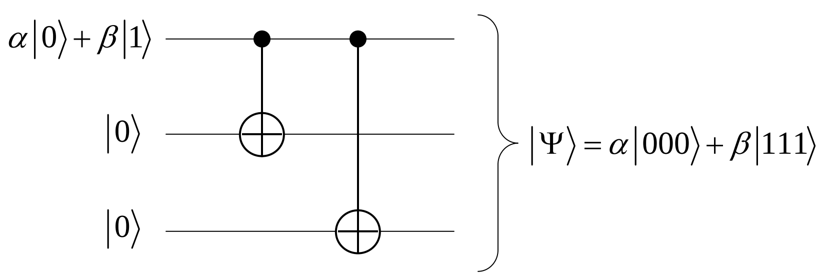
- Necessary, no internal stabilizing dynamics as in c-bits.
- Cannot check errors directly: measurement destroys state.
- Qubit errors, unlike c-bit errors, occur continuously.
- Bit flips are not the only errors. Must correct phase flips.

10-VI-11

OUTLINE

10-VI-5b

ENCODING



$\begin{Bmatrix} +IZI \\ +IIZ \end{Bmatrix}$

$\begin{Bmatrix} +ZZI \\ +ZIZ \end{Bmatrix}$

↑ stabilizer of qubit manifold

10-VI-12b

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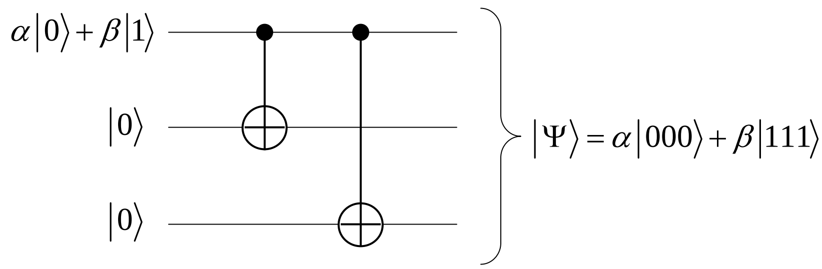
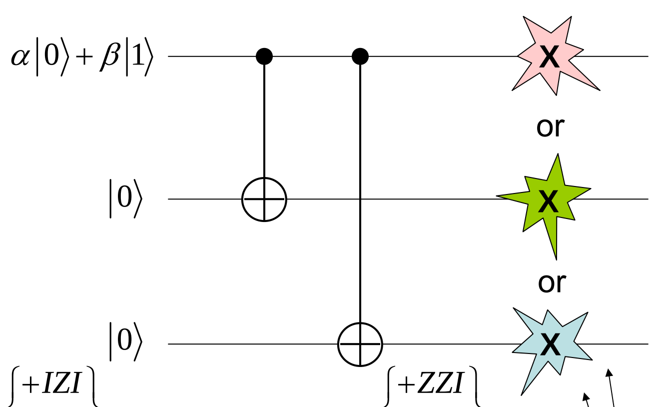


$\begin{Bmatrix} +IZI \\ +IIZ \end{Bmatrix}$ ← stabilizer of qubit manifold

$\begin{Bmatrix} +ZZI \\ +IZZ \end{Bmatrix}$

10-VI-12c

BIT FLIP ERRORS



$\begin{Bmatrix} +IZI \\ +IIZ \end{Bmatrix}$ $\begin{Bmatrix} +ZZI \\ +IZZ \end{Bmatrix}$

random flip of at most 1 qubit

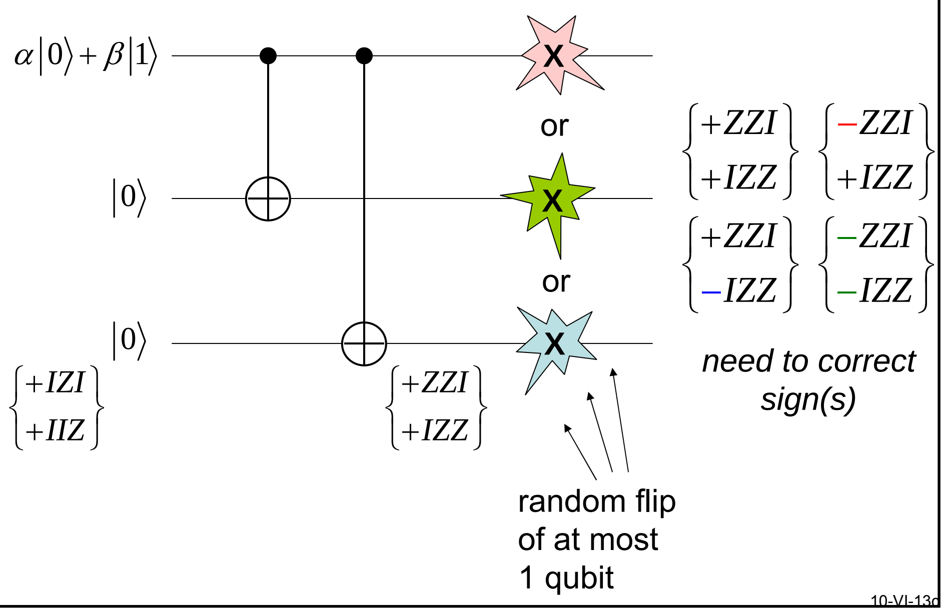
$\alpha|000\rangle + \beta|111\rangle$
or
$\alpha|100\rangle + \beta|011\rangle$
or
$\alpha|010\rangle + \beta|101\rangle$
or
$\alpha|001\rangle + \beta|110\rangle$

$\begin{Bmatrix} +ZZI \\ +IZZ \end{Bmatrix}$ $\begin{Bmatrix} -ZZI \\ +IZZ \end{Bmatrix}$

$\begin{Bmatrix} +ZZI \\ -IZZ \end{Bmatrix}$ $\begin{Bmatrix} -ZZI \\ -IZZ \end{Bmatrix}$

10-VI-13b

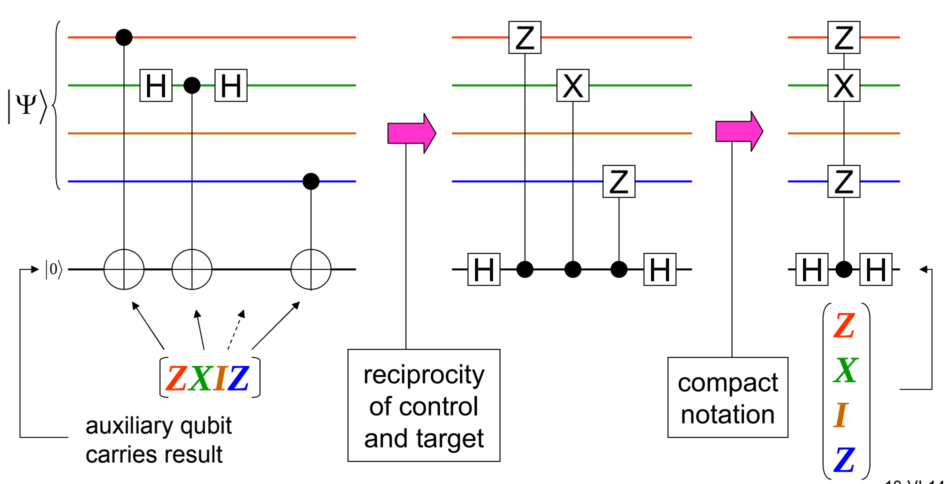
BIT FLIP ERRORS

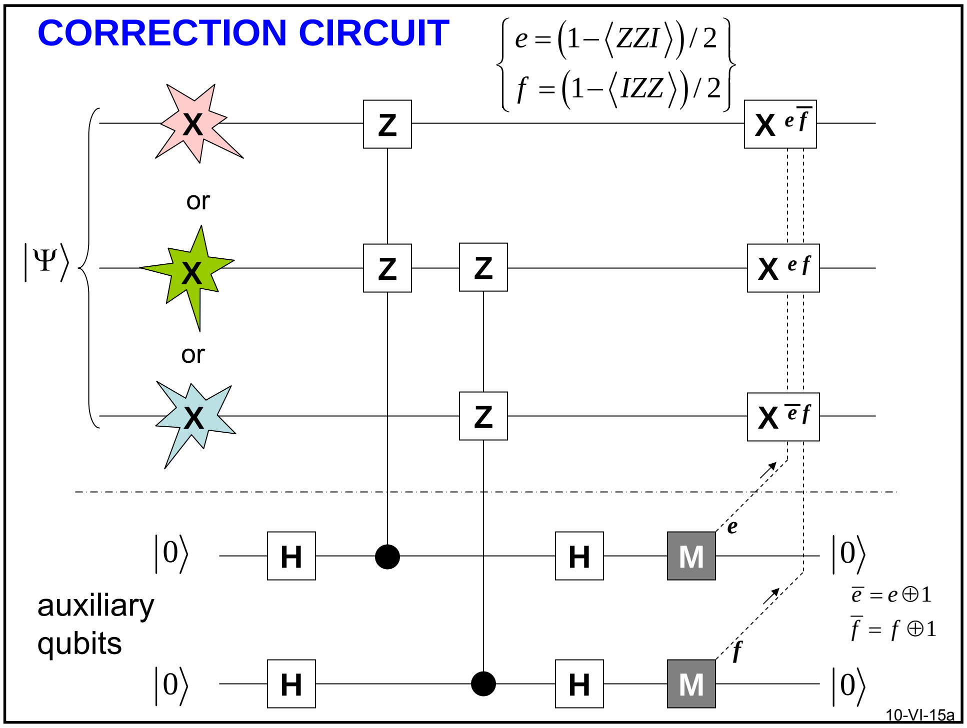


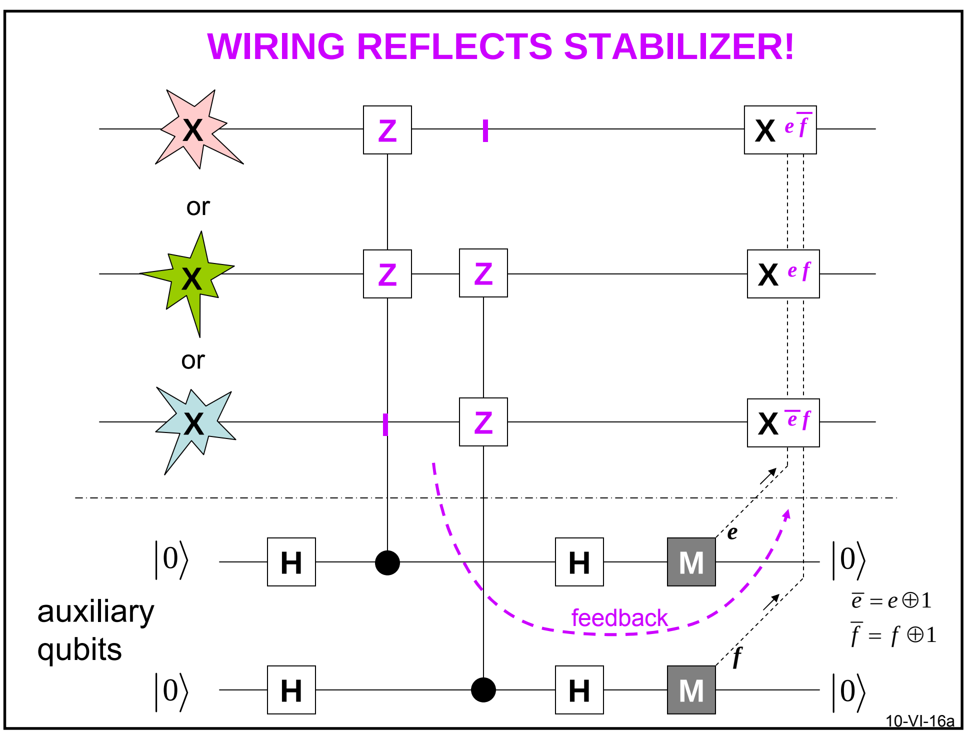
JOINT MEASUREMENTS

How do we measure joint qubit operators such as ZZI ?

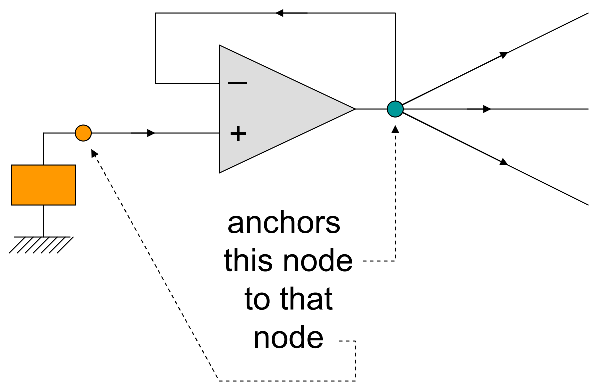
Let us take a more representative case, for instance $ZXIZ$:




CORRECTION CIRCUIT
$e = (1 - \langle ZZI \rangle)/2$
$f = (1 - \langle IZZ \rangle)/2$
$|\Psi\rangle$
X
or
X
or
X
Z
Z
Z
Z
$X^{e\bar{f}}$
X^{ef}
$X^{\bar{e}f}$
$|0\rangle$
$|0\rangle$
auxiliary qubits
H
H
H
H
M
M
e
f
$\bar{e} = e \oplus 1$
$\bar{f} = f \oplus 1$
10-VI-15a


WIRING REFLECTS STABILIZER!
X
or
X
or
X
Z
I
Z
Z
I
Z
$X^{e\bar{f}}$
X^{ef}
$X^{\bar{e}f}$
$|0\rangle$
$|0\rangle$
auxiliary qubits
H
H
H
H
M
M
e
f
feedback
$\bar{e} = e \oplus 1$
$\bar{f} = f \oplus 1$
10-VI-16a

QUBIT CORRECTION CIRCUIT WORKS LIKE ORDINARY FEEDBACK



10-VI-16bis

POINTS FOR DISCUSSION

1) Correction protocol is discrete but error process is continuous.
   How are "partial errors" dealt with?

2) Feedback goes through external circuitry.
   Can feedback be purely internal to the system?

3) Error correction removes entropy from qubit.
   Where is the entropy going?

4) Quantum error correction : feedback to a manifold, not a state.
   What symmetry allows this manifold-preserving attractive dynamics?

10-VI-17

OUTLINE

1. Classical error correction
2. A simplified example: 3-qubit code
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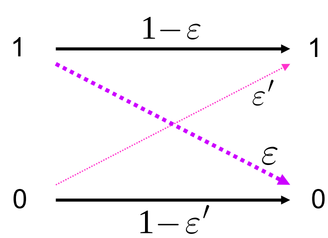
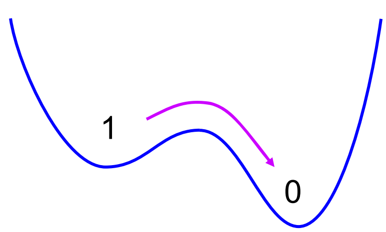
4. The 7-qubit code

10-VI-5c

EVEN CLASSICALLY, THERE IS MORE TO ERRORS THAN JUST RANDOM BIT FLIPS



The two error transitions might not have the same probability



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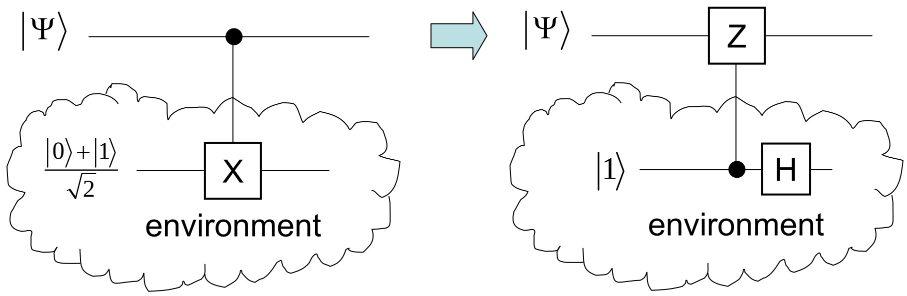
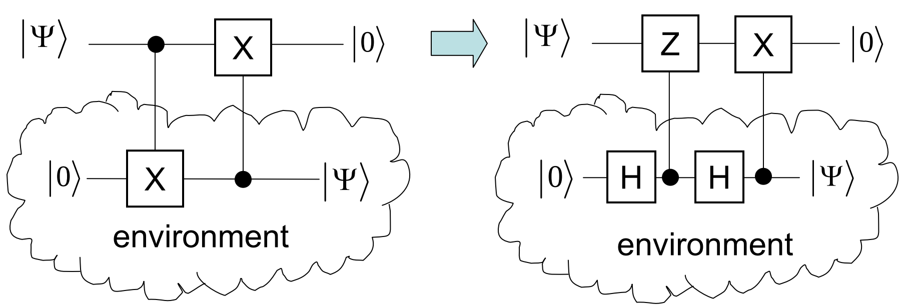
QUANTUM-MECHANICALLY, ERRORS CAN BE BIT FLIPS, PHASE FLIPS AND BIT-PHASE FLIPS.



Example of phase flip, leading to dephasing

10-VI-19a

QUANTUM-MECHANICALLY, ERRORS CAN BE BIT FLIPS, PHASE FLIPS AND BIT-PHASE FLIPS (2)



Relaxation can be seen as a combination of phase and bit flips performed by a "cold" environment

10-VI-20a

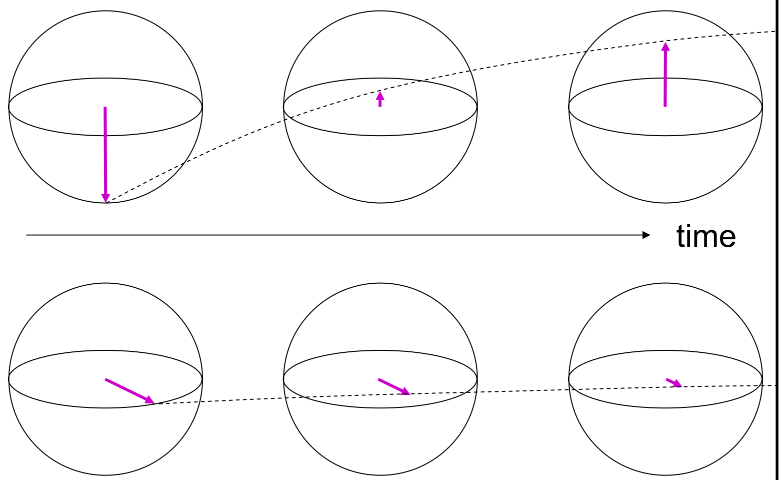
LOSSES OF QUANTUM MEMORY

T_1 PROCESS

random fields in X,Y plane uniformly distributed

T_ϕ PROCESS

random field along Z



$$T_2 = \frac{1}{\frac{1}{2T_1} + \frac{1}{T_\phi}}$$ DECOHERENCE TIME

$\omega_{01} T_2$: DECOHERENCE QUALITY FACTOR

10-VI-21a

JUST BY CORRECTING BIT FLIPS, PHASE FLIPS AND THE COMBINATION OF THESE TWO FLIPS, ANY TYPE OF ERROR CAN BE CORRECTED!

Shor (1995), Steane (1996)

10-VI-22

OUTLINE

1. Classical error correction
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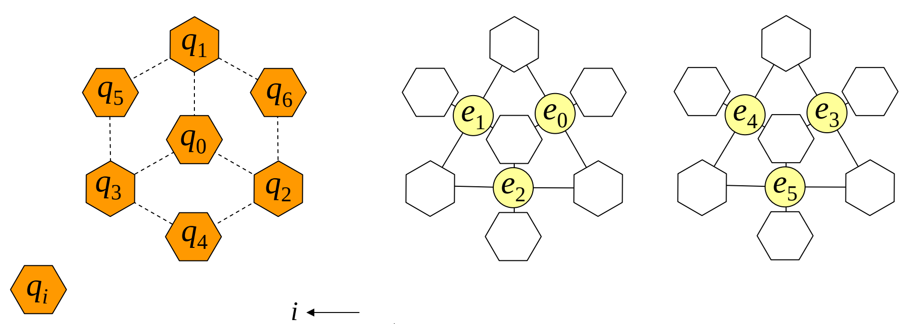
10-VI-5d

STABILIZER OF THE 7-QUBIT CODE

Steane (1996)



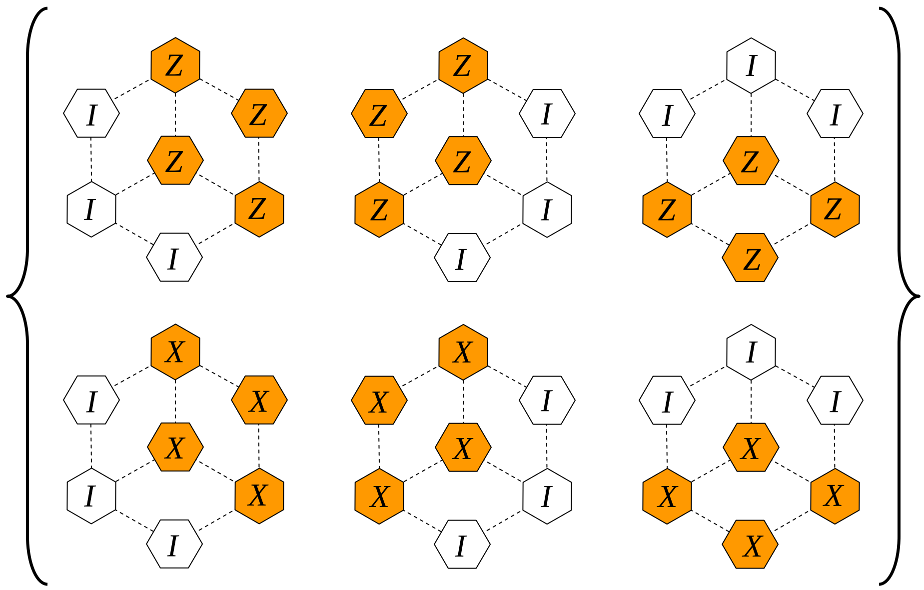
$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$i \leftarrow$, $j \downarrow$, e_j

$$\Rightarrow \left\{ \begin{matrix} +Z & I & I & I & Z & Z & Z \\ +I & Z & I & Z & I & Z & Z \\ +I & I & Z & Z & Z & I & Z \\ \hline +X & I & I & I & X & X & X \\ +I & X & I & X & I & X & X \\ +I & I & X & X & X & I & X \end{matrix} \right\}$$

10-VI-23b

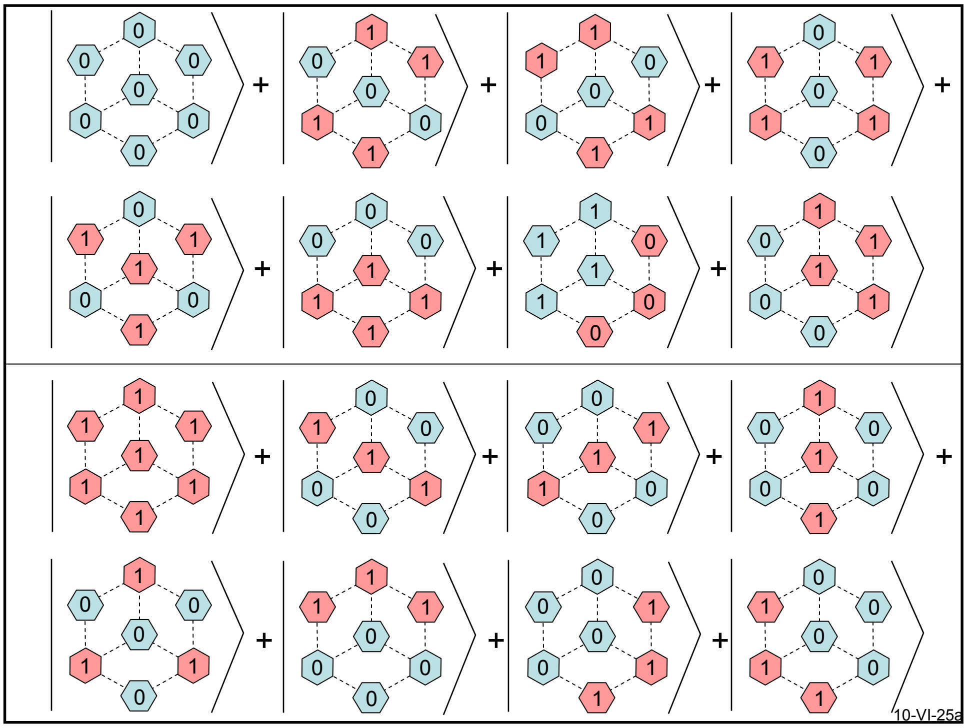
OTHER REPRESENTATION OF STEANE STABILIZER

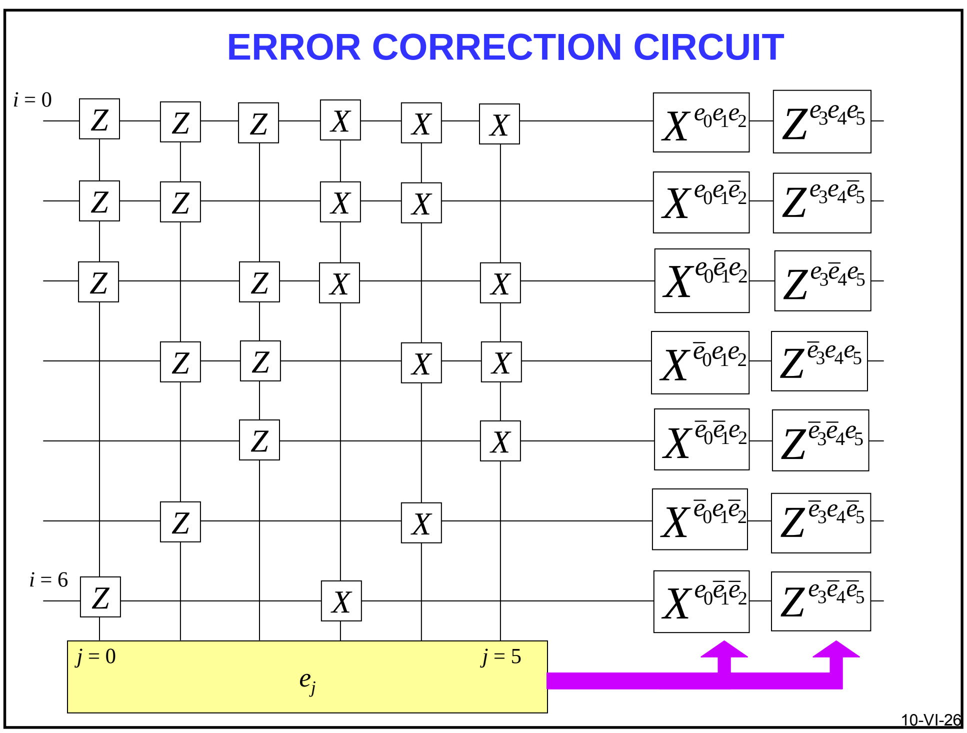


10-VI-24

NEXT SLIDE SHOWS THE TWO WAVEFUNCTIONS OF STEANE CODE

10-VI-25


10-VI-25a


ERROR CORRECTION CIRCUIT
i = 0
i = 6
j = 0
j = 5
e_j
10-VI-26

WHY DOES IT WORK?

Steane's 7-qubit code:
6 generators in stabilizer (7 physical qubits – 1 logical qubit)
2^6 =64 error syndromes > 7 qubits x (3 errors/qubit) + 1 = 22

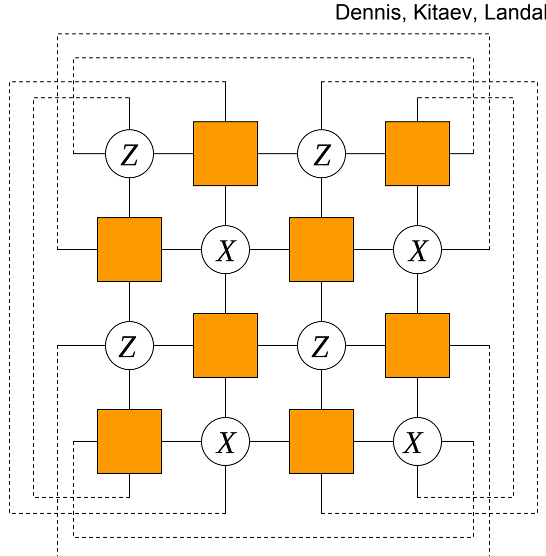
Gottesman's 5-qubit code:
4 generators in stabilizer (5 physical qubits – 1 logical qubit)
2^4 =16 error syndromes = 5 qubits x (3 errors/qubit) + 1 = 16

minimal but impractical

10-VI-27a

TORIC CODE

Dennis, Kitaev, Landahl and Preskill (2001)



see B. Douçot's seminar this year

10-VI-28

ERRORS CAN ALSO BE CORRECTED BY CONTINUOUS MONITORING AND FEEDBACK

Ahn, Doherty & Landahl, Phys. Rev. A65, 042301 (2002)

NEXT YEAR: AMPLIFICATION AND FEEDBACK
OF ENGINEERED QUANTUM SYSTEMS

10-VI-29

END OF 2010 COURSE

ACKNOWLEDGEMENTS

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