COLLÈGE DE FRANCE
1530

Chaire de Physique Mésoscopique
Michel Devoret
Année 2010, 11 mai - 22 juin

INTRODUCTION AU CALCUL QUANTIQUE

INTRODUCTION TO QUANTUM COMPUTATION

Cinquième Leçon / *Fifth Lecture*

10-V-1

VISIT THE WEBSITE OF THE CHAIR OF MESOSCOPIC PHYSICS

http://www.college-de-france.fr

then follow
Enseignement > Sciences Physiques > Physique Mésoscopique > Site web

or

http://www.physinfo.fr/lectures.html

PDF FILES OF ALL PAST LECTURES ARE POSTED

Questions, comments and corrections are welcome!

write to "phymeso@gmail.com"

10-V-2

CONTENT OF THIS YEAR'S LECTURES

QUANTUM COMPUTATION FROM THE PERSPECTIVE OF MESOSCOPIC CIRCUITS

1. Introduction, c-bits versus q-bits
2. The Pauli matrices and quantum computation primitives
3. Stabilizer formalism for state representation
4. Clifford calculus
5. Algorithms
6. Error correction

10-V-3

CALENDAR OF SEMINARS

May 11: Cristian Urbina, (Quantronics group, SPEC-CEA Saclay)
Josephson effect in atomic contacts and carbon nanotubes

May 18: Benoît Douçot (LPTHE / Université Pierre et Marie Curie)
Towards the physical realization of topologically protected qubits

June 1: Takis Kontos (LPA / Ecole Normale Supérieure)
Points quantiques et ferromagnétisme

June 8: Cristiano Ciuti (MPQ, Université Paris - Diderot)
Ultrastrong coupling circuit QED : vacuum degeneracy and quantum phase transitions

June 15: Leo DiCarlo (Yale)
Preparation and measurement of tri-partite entanglement in a superconducting quantum circuit

June 22: Vladimir Manucharian (Yale)
The fluxonium circuit: an electrical dual of the Cooper-pair box?

10-V-4

LECTURE IV : ALGORITHMS

Processing information with sequences of
controlled reversible physical processes in a circuit.

1. Review of Clifford operations & logical circuits
2. The C-NOT and C-Phase gate
3. Preparing the GHZ state
4. Teleportation

10-V-5

OUTLINE

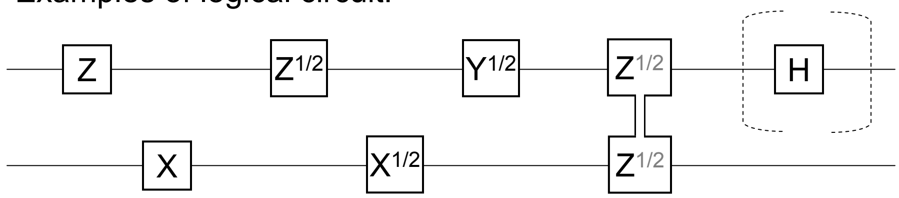
1. Review of Clifford operations & logical circuits
2. The C-NOT and C-Phase gates
3. Preparing the GHZ state
4. Teleportation

10-V-5a

CLIFFORD PRIMITIVES

They are 1- and 2-qubit gates that can simply be implemented by physical circuits and signal protocols.

Examples of logical circuit:



Corresponding super-operators for algebraic calculations:

$$[\![ZI]\!] \quad [\![IX]\!] \quad [\![ZI]\!]^{1/2} \; [\![IX]\!]^{1/2} \; [\![YI]\!]^{1/2} \quad [\![ZZ]\!]^{1/2} \quad \left([\![ZI]\!]^{1/2}[\![XI]\!]^{1/2}[\![ZI]\!]^{1/2}\right)$$

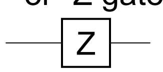
For 2-qubits these gates generate a group with 11520 elements!

n	1	2	3	4	5
C_n	24	11520	92897280	12128668876800	25410822678459187200

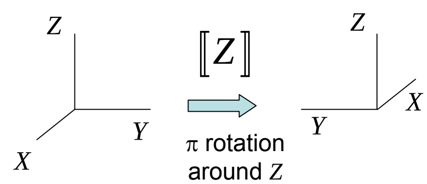
10-V-6b

1-QUBIT π-ROTATIONS

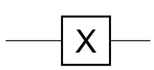
Phase flip
or "Z gate"



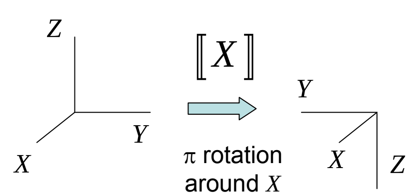
$[\![Z]\!]$	
Z	Z
X	$-X$



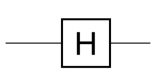
NOT, bit flip
or "X gate"



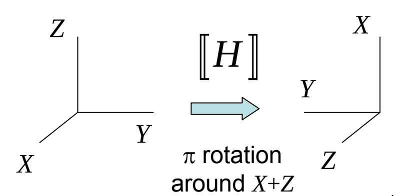
$[\![X]\!]$	
Z	$-Z$
X	X



Hadamard



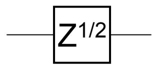
$[\![H]\!]$	
Z	X
X	Z



10-V-7

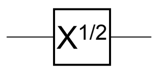
1-QUBIT $\pi/2$-ROTATIONS

"Phase"



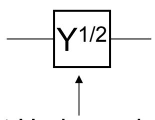
$[\![Z]\!]^{1/2}$	
Z	Z
X	Y

NOT1/2



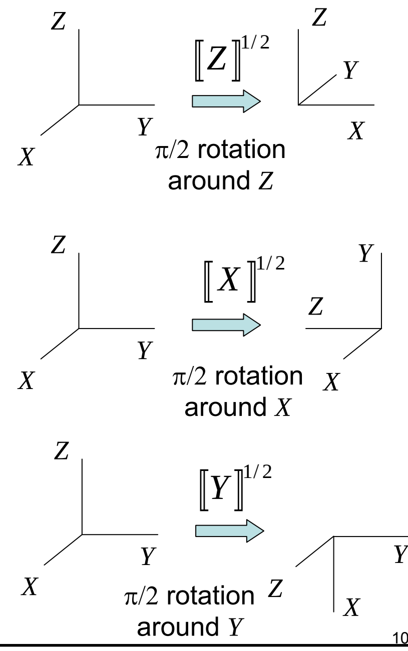
$[\![X]\!]^{1/2}$	
Z	$-Y$
X	X

i-NOT1/2

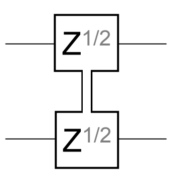


Almost Hadamard

$[\![Y]\!]^{1/2}$	
Z	X
X	$-Z$



THE EFFICIENT 2-QUBIT PRIMITIVE



"Ising gate"

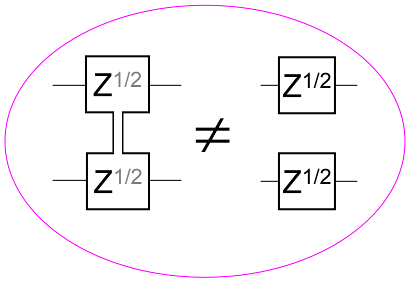
$[\![ZZ]\!]^{1/2}$	
IZ	IZ
ZI	ZI
IX	ZY
XI	YZ

Generated by secular interaction

$$\hat{H}_{\text{int}} = g_{\parallel}\sigma_z^1\sigma_z^2$$

duration $\tau_s = \dfrac{\pi\hbar}{4g_{\parallel}}$

THE EFFICIENT 2-QUBIT PRIMITIVE



!

$[\![ZZ]\!]^{1/2}$	
IZ	IZ
ZI	ZI
IX	ZY
XI	YZ

≠

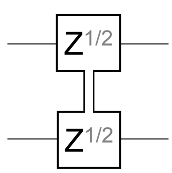
$[\![ZI]\!]^{1/2}[\![IZ]\!]^{1/2}$	
IZ	IZ
ZI	ZI
IX	IY
XI	YI

↑ can generate entanglement from product state

↑ transforms product state into product state

10-V-9a

THE EFFICIENT 2-QUBIT PRIMITIVE



"Ising gate"

$[\![ZZ]\!]^{1/2}$	
IZ	IZ
ZI	ZI
IX	ZY
XI	YZ

Generated by secular interaction

$$\hat{H}_{\text{int}} = g_{\parallel}\sigma_z^1\sigma_z^2$$

duration $\tau_s = \dfrac{\pi\hbar}{4g_{\parallel}}$

$$[\![ZZ]\!]^{1/2}\ IY = -ZX$$

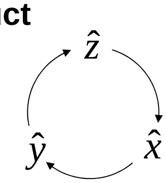
$$[\![ZZ]\!]^{1/2}\ YI = -XZ$$

$$[\![ZZ]\!]^{1/2}\ XX = XX$$

$$[\![ZZ]\!]^{1/2}\ YY = YY$$

vector product rule

$$\hat{z}\times\hat{x} = \hat{y}$$



supplemented by...

10-V-9b

RULES OF CLIFFORD CALCULUS FOR N QUBITS

They are found starting from: $[\![B]\!]^{\alpha} A = [B]^{-\alpha} A [B]^{\alpha}$

$[\![B]\!]^{1}[A] = [-A]$	if A and B anticommute
$= [A]$	if A and B commute
$[\![B]\!]^{1/2}[A] = [B][A]$	if A and B anticommute
$= [A]$	if A and B commute

Example: $[\![Z]\!]^{1/2} X = Y$

Note that : $[\![-B]\!]^{\alpha} = [\![B]\!]^{-\alpha}$

$[A] = [B] \Leftrightarrow A = B$

10-V-10a

OUTLINE

1. Review of Clifford operations & logical circuits
2. The C-NOT and C-Phase gates
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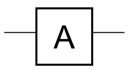
4. Teleportation

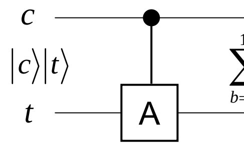
10-V-5b

CONTROLLED UNITARY

General definition:



: an arbitrary 1-qubit gate

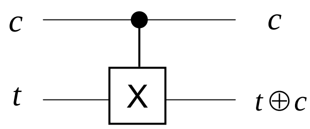


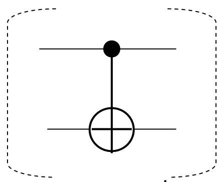
$$\sum_{b=0}^{1} \langle b|c\rangle |b\rangle [A]^b |t\rangle$$

Entangler!

Two simple examples in the Clifford group:

C-NOT



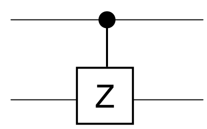


more usual notation for

C-NOT

	$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$\|11\rangle$
$\|00\rangle$	1	0	0	0
$\|01\rangle$	0	1	0	0
$\|10\rangle$	0	0	0	1
$\|11\rangle$	0	0	1	0

C-Phase



	$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$\|11\rangle$
$\|00\rangle$	+1	0	0	0
$\|01\rangle$	0	+1	0	0
$\|10\rangle$	0	0	+1	0
$\|11\rangle$	0	0	0	−1

10-V-11b

EXPRESSING CONDITIONALITY

Take $[A]^\beta = e^{-i\beta\frac{\pi}{2}A}$ where A is a multi-qubit Pauli

and extend β to a multi-qubit Pauli operator B, then

$$[A]^B = e^{-i\frac{\pi}{2}AB} = [AB] = [B]^A$$

when A and B are Pauli's and commute

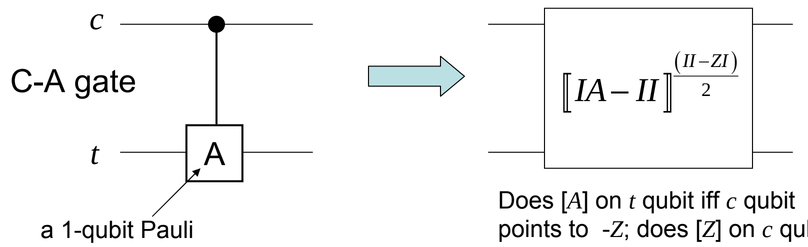
Doing B conditioned on the value of A is obtained by doing AB?

Yes, but then, A is conditioned on the value of B !

Action-reaction principle constrains information processing

10-V-12a

EXPRESSION OF CONTROLLED UNITARY

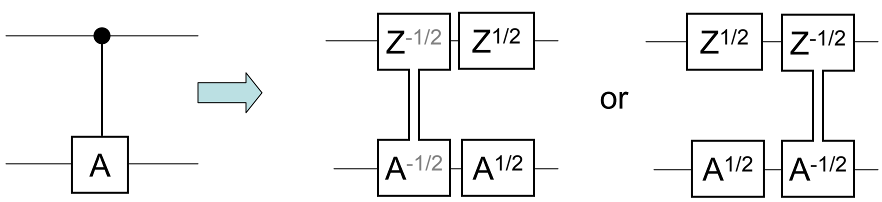


$$[\![IA - II]\!]^{\frac{(II - ZI)}{2}}$$

Does [*A*] on *t* qubit iff *c* qubit points to -*Z*; does [*Z*] on *c* qubit iff *t* qubit points to -*A*.

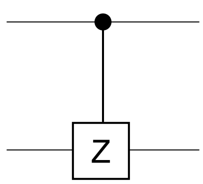
Using $[\![P]\!]^{Q} = [\![QP]\!]$ when P and Q are Paulis and commute, we get:

$$[\![\text{C-}A]\!] = [\![ZA]\!]^{-\frac{1}{2}} [\![ZI]\!]^{+\frac{1}{2}} [\![IA]\!]^{+\frac{1}{2}}$$

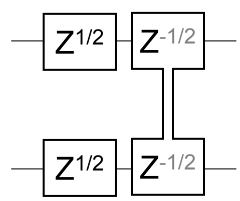


10-V-13a

THE C-PHASE GATE



$$[\![ZZ]\!]^{\frac{1}{2}} [\![ZI]\!]^{-\frac{1}{2}} [\![IZ]\!]^{-\frac{1}{2}}$$



C-Phase	
IZ	*IZ*
ZI	*ZI*
IX	*ZX*
XI	*XZ*

Check: $\hat{H}_{\text{int}} = g_{\parallel}\left(\sigma_z^1 \sigma_z^2 - \sigma_z^1 - \sigma_z^2\right)$

$$= g_{\parallel}\left(\sigma_z^1 - 1\right)\left(\sigma_z^2 - 1\right) + \text{const.}$$

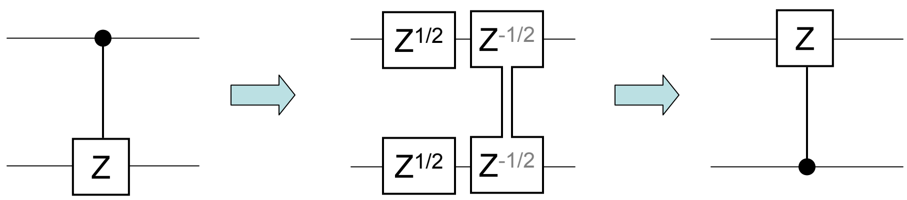
$$\hat{U}(\tau) = \exp\left(-i\hat{H}_{\text{int}}\tau / \hbar\right) \qquad \tau_s = \frac{\pi\hbar}{4g_{\parallel}}$$

$$\hat{U}(\tau_s) = \begin{bmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

up to an overall constant phase factor

10-V-14a

QUANTUM MECHANICS IMPOSES A FORM OF SYMMETRY BETWEEN CONTROL AND TARGET

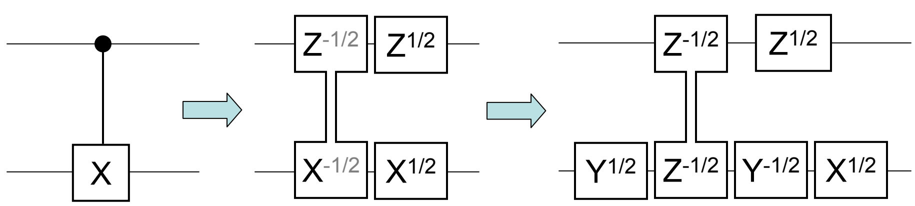


C-Phase	
IZ	*IZ*
ZI	*ZI*
IX	*ZY*
XI	*YZ*

This back-action of the target qubit on the control bit is fully general

10-V-15

THE C-NOT GATE

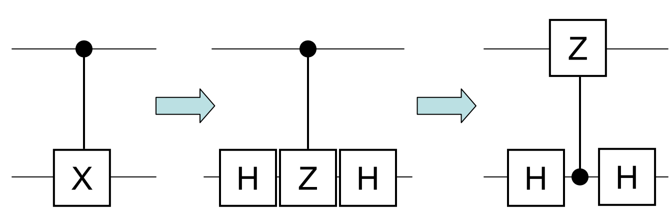


$$[ZI]^{\frac{1}{2}}[IX]^{\frac{1}{2}}[ZX]^{-\frac{1}{2}} \rightarrow [ZI]^{\frac{1}{2}}[IX]^{\frac{1}{2}}[IY]^{-\frac{1}{2}}[ZZ]^{-\frac{1}{2}}[IY]^{+\frac{1}{2}}$$

C-NOT	
IZ	*ZZ*
ZI	*ZI*
IX	*IX*
XI	*XX*

phase kick-back

other useful equivalences displaying c-t symmetry:



10-V-16a

OUTLINE

1. Review of Clifford operations & logical circuits
2. The C-NOT and C-Phase gates
3. Preparing the Greenberger-Horne-Zeilinger state
4. Teleportation

10-V-5c

THE GHZ STATE

Greenberger-Horne-Zeilinger (1989) arXiv:0712.0921

MAXIMAL THREE-PARTITE ENTANGLEMENT
NECESSARY FOR QUANTUM ERROR-CORRECTION

$$|\mathrm{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

$+IZZ$
$+ZIZ$
$+XXX$

$$\frac{|000\rangle - |111\rangle}{\sqrt{2}}$$

$+IZZ$
$+ZIZ$
$-XXX$

$$XXX\,|000\rangle = |111\rangle$$
$$XXX\,|111\rangle = |000\rangle$$

10-V-17

CHARACTERISTIC PROPERTY OF GHZ: VIOLATION OF LOCAL HIDDEN VARIABLES THY's

Full stabilizer: $\{+IZZ, +ZIZ, +ZZI, +XXX, -XYY, -YXY, -YYX\}$

MEASURE THESE OPERATORS (arrows point to $+XXX$, $-XYY$, $-YXY$, $-YYX$)

CAN BE DONE ON THE SAME STATE SINCE THEY COMMUTE!

THEY INDIVIDUALLY YIELD RESULT +1
AND THUS THEIR PRODUCT IS +1

YET, "CLASSICALLY" THEIR PRODUCT SHOULD BE -1!

Multiply the 4 "words" as if the "letters" were either I or -I

$$\begin{array}{r} (+XXX) \\ \times(-XYY) \\ \times(-YXY) \\ \times(-YYX) \\ \hline (-III) \end{array}$$

$$X^2 = Y^2 = Z^2 = I$$

Every symbol X and Y appears twice in each of the 3 columns!
A contradiction between common sense and experiment?

10-V-18a

GENERATING A GHZ-EQUIVALENT STATE

1) apply first rotations around x on each qubit to go to the transverse basis

$$\begin{Bmatrix} IIZ \\ IZI \\ ZII \end{Bmatrix} \xrightarrow{[\![XII]\!]^{1/2}\,[\![IXI]\!]^{1/2}\,[\![IIX]\!]^{1/2}} \begin{Bmatrix} IIY \\ IYI \\ YII \end{Bmatrix}$$

2) apply "secular" IZZ interaction during "$\pi/2$" amount of time

$$\begin{Bmatrix} IIY \\ IYI \\ YII \end{Bmatrix} \xrightarrow{[\![IZZ]\!]^{1/2}} \begin{Bmatrix} IZX \\ IXZ \\ YII \end{Bmatrix}$$

3) apply "secular" ZZI interaction during "$\pi/2$" amount of time

$$\begin{Bmatrix} IZX \\ IXZ \\ YII \end{Bmatrix} \xrightarrow{[\![ZZI]\!]^{1/2}} \begin{Bmatrix} IZX \\ ZYZ \\ XZI \end{Bmatrix}$$

10-V-19b

GHZ-EQUIVALENCE

$$\begin{Bmatrix} IZX \\ ZYZ \\ XZI \end{Bmatrix} \xrightarrow{\text{cyclic permutation X→Z→Y on qubits 1\&3}} \begin{Bmatrix} IZZ \\ YYY \\ ZZI \end{Bmatrix}$$

$$\begin{Bmatrix} IZZ \\ YYY \\ ZZI \end{Bmatrix} \xrightarrow{\text{transposition X-Y on all qubits}} \begin{Bmatrix} IZZ \\ XXX \\ ZZI \end{Bmatrix}$$

↑ ORIGINAL GHZ STABILIZER

10-V-20

GRAPH STATES

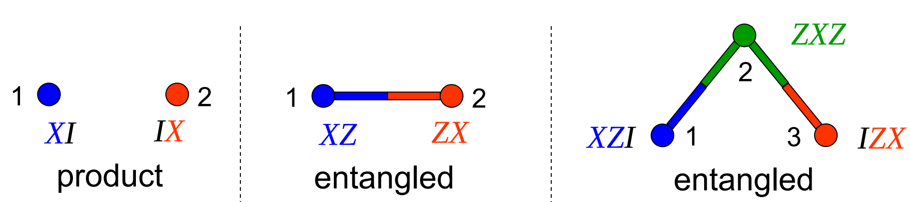
Transform stabilizer generators to “standard form” through 1-qubit (local) Clifford group operations. Resulting stabilizer has an associated graph.

Examples:

$$\{ZZ, XX\} \xrightarrow{H \text{ on 2nd qubit}} \{ZX, XZ\}$$

$$\{IZZ, ZZI, XXX\} \xrightarrow{H \text{ on 1st and 3rd qubit}} \{IZX, XZI, ZXZ\}$$

$\boldsymbol{X}$ symbol in operator (unique by construction) drawn as corresponding vertex.
$\boldsymbol{Z}$ symbol in operator drawn as half-edge in direction of corresponding vertex.
$\boldsymbol{I}$ symbol of the operator not drawn at all.



10-V-21

OUTLINE

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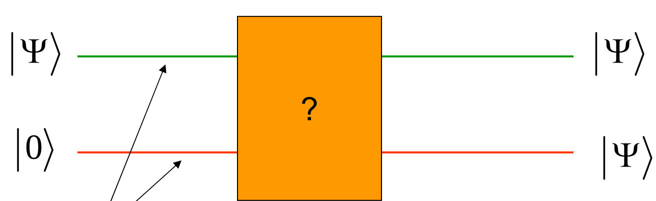
10-V-5d

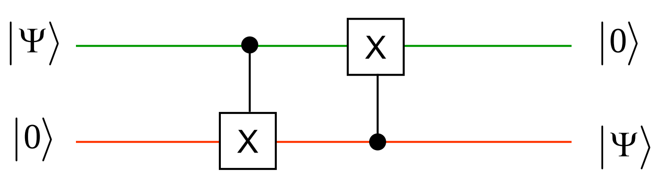
NO-CLONING THEOREM



Two different types of qubits
(ex.: nuclear spin and photon)

Impossible,
if $|\Psi\rangle$ is unknown

The only possible transfer is:



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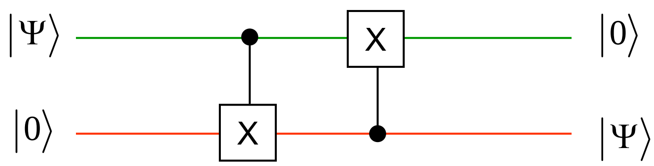
THE TELEPORTATION PROBLEM

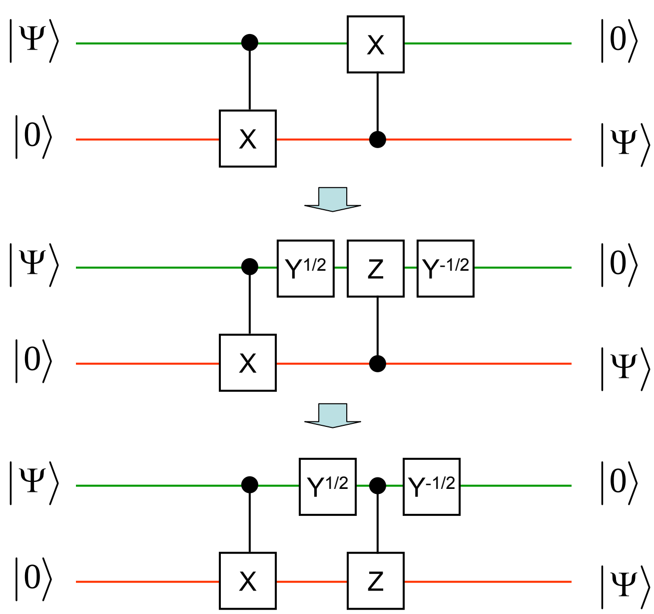
Consider the case where the basic solution to
the quantum information transfer,



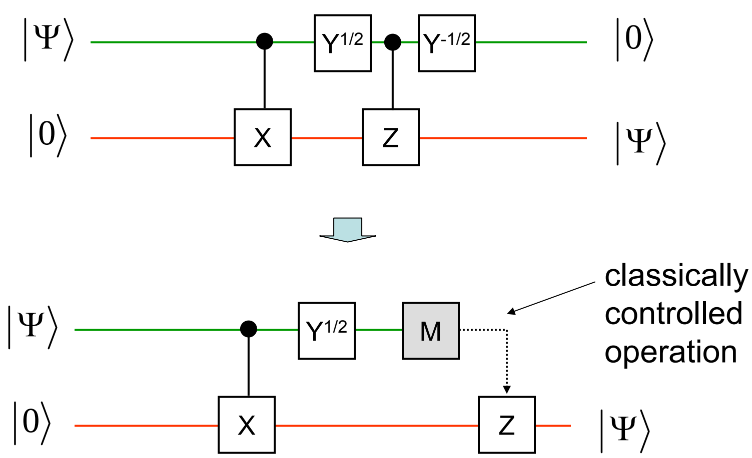
is not possible because there is no
interaction possible between the green and red qubits.
Also, the red qubit is not available for
any two-bit quantum gate at the time of the transfer.

Can we still do the work?

TELEPORTATION SOLUTION: 2nd C-NOT



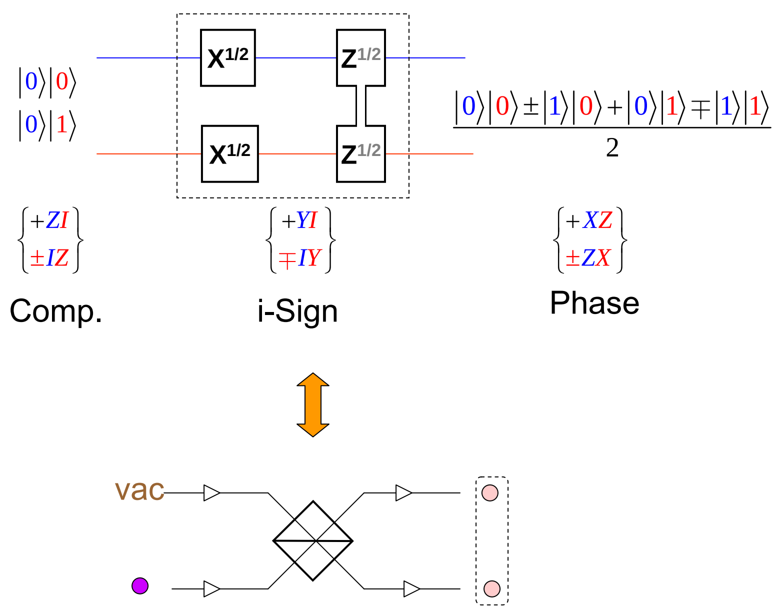
TELEPORTATION SOLUTION: 2nd C-NOT



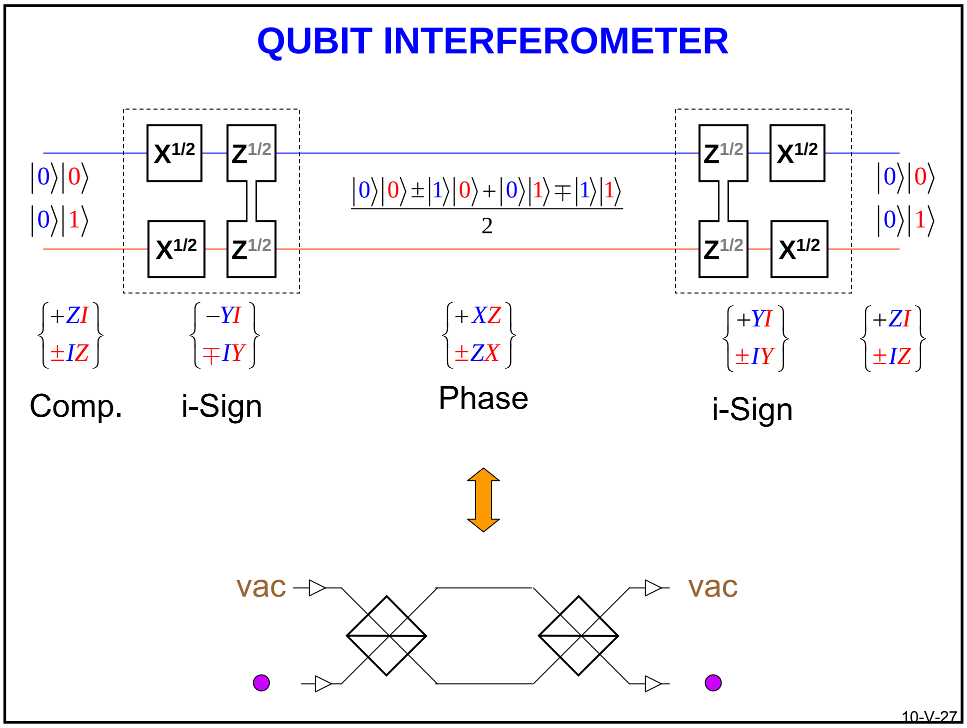
Need now to transform the first C-NOT!

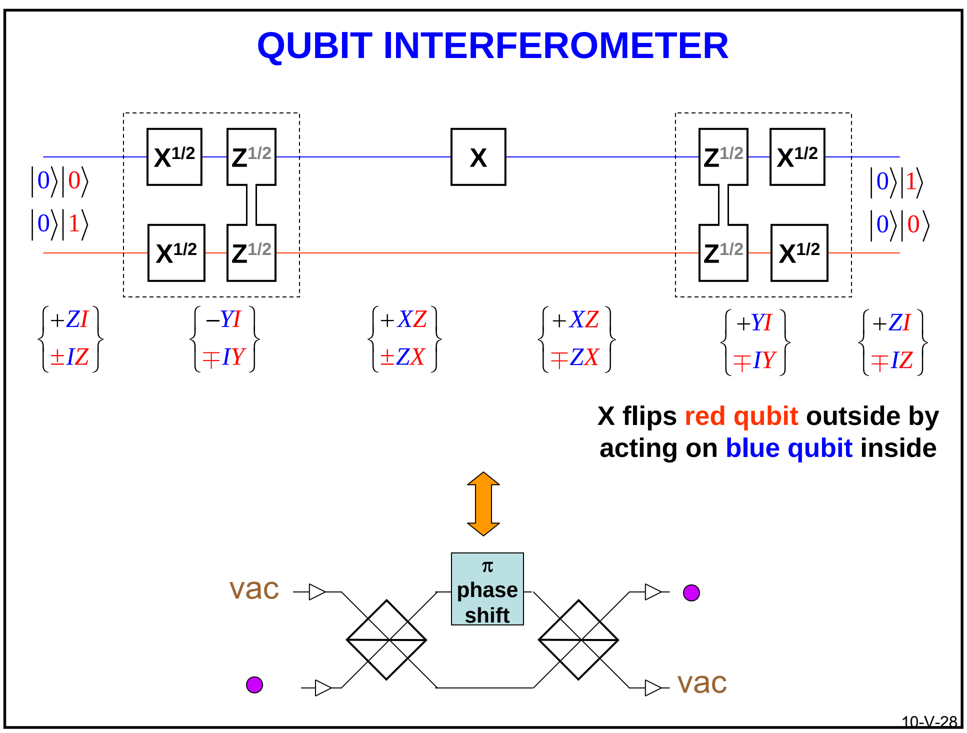
10-V-25

THE "BEAM SPLITTER" GATE

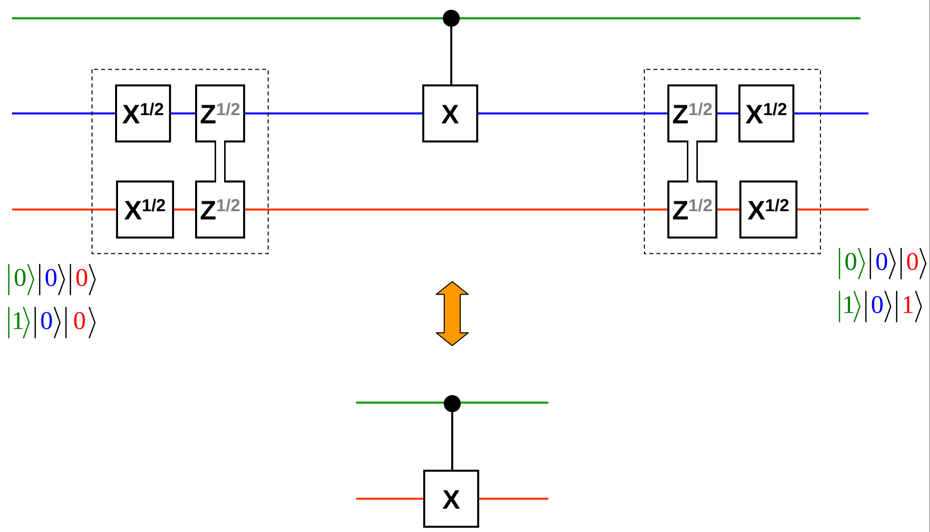


10-V-26


QUBIT INTERFEROMETER
X1/2
Z1/2
|0⟩|0⟩
|0⟩|1⟩
(|0⟩|0⟩ ± |1⟩|0⟩ + |0⟩|1⟩ ∓ |1⟩|1⟩) / 2
{+ZI, ±IZ}
{−YI, ∓IY}
{+XZ, ±ZX}
{+YI, ±IY}
{+ZI, ±IZ}
Comp.
i-Sign
Phase
i-Sign
vac
vac
10-V-27

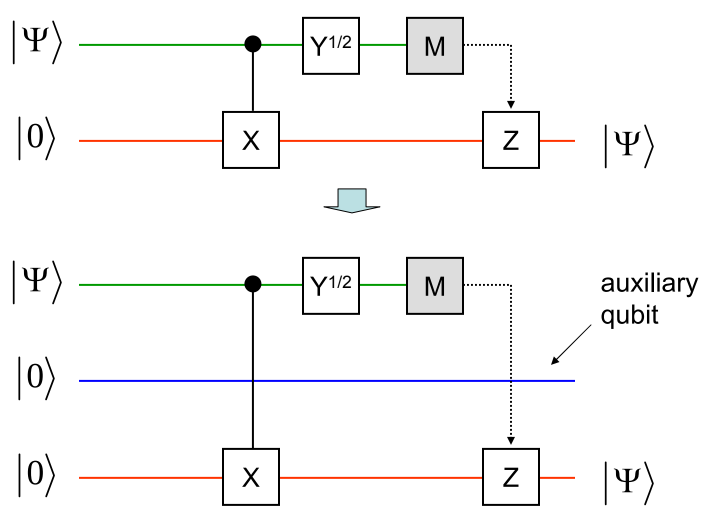

QUBIT INTERFEROMETER
X1/2
Z1/2
X
|0⟩|0⟩
|0⟩|1⟩
|0⟩|1⟩
|0⟩|0⟩
{+ZI, ±IZ}
{−YI, ∓IY}
{+XZ, ±ZX}
{+XZ, ∓ZX}
{+YI, ∓IY}
{+ZI, ∓IZ}
X flips red qubit outside by
acting on blue qubit inside
vac
π phase shift
vac
10-V-28

REMOTE CONTROLLED NOT



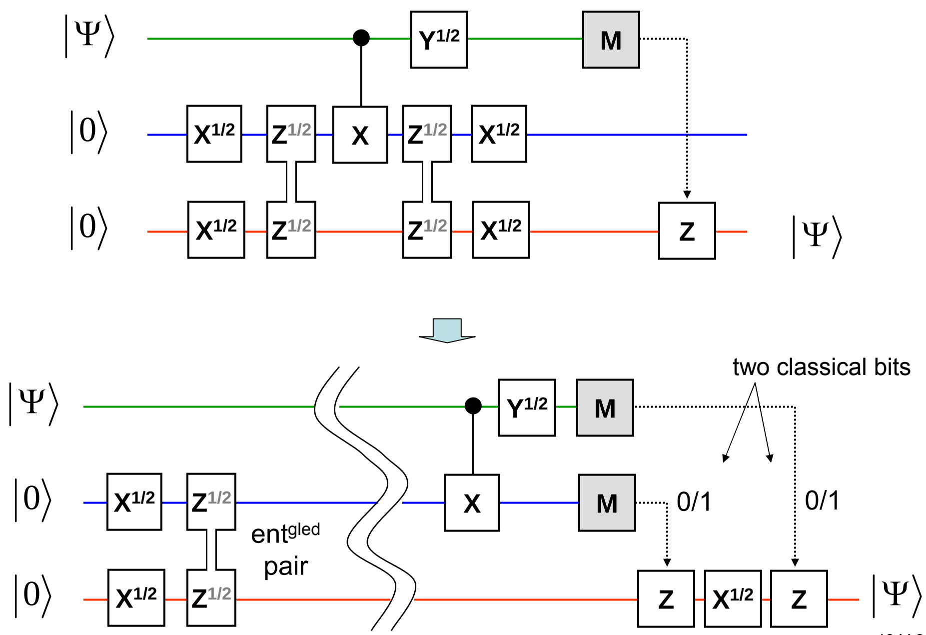
10-V-29

TELEPORTATION SOLUTION: 1nd C-NOT



10-V-30

TELEPORTATION SOLUTION: 1nd C-NOT



10-V-31

END OF LECTURE