Chaire de Physique Mésoscopique
Michel Devoret
Année 2009, 12 mai - 23 juin

CIRCUITS ET SIGNAUX QUANTIQUES (II)

QUANTUM SIGNALS AND CIRCUITS (II)

Troisième leçon / *Third Lecture*

VISIT THE WEBSITE OF THE CHAIR OF MESOSCOPIC PHYSICS

http://www.college-de-france.fr

then follow Enseignement > Sciences Physiques > Physique Mésoscopique >

PDF FILES OF ALL LECTURES WILL BE POSTED ON THIS WEBSITE

Questions, comments and corrections are welcome!

write to "phymeso@gmail.com"

CALENDAR OF SEMINARS

May 12: Daniel Esteve, (Quantronics group, SPEC-CEA Saclay)
Faithful readout of a superconducting qubit

May 19: Christian Glattli (LPA/ENS)
Statistique de Fermi dans les conducteurs balistiques : conséquences expérimentales et exploitation pour l'information quantique

June 2: Steve Girvin (Yale)
Quantum Electrodynamics of Superconducting Circuits and Qubits

June 9: Charlie Marcus (Harvard)
Electron Spin as a Holder of Quantum Information: Prospects and Challenges

June 16: Frédéric Pierre (LPN/CNRS)
Energy exchange in quantum Hall edge channels

June 23: Lev Ioffe (Rutgers)
Implementation of protected qubits in Josephson junction arrays

09-III-3

CONTENT OF THIS YEAR'S LECTURES

OUT-OF-EQUILIBRIUM NON-LINEAR QUANTUM CIRCUITS

1. Introduction and review of last year's course
2. Non-linearity of Josephson tunnel junctions
3. Readout of qubits
4. Amplifying quantum fluctuations
5. Dynamical cooling and quantum error correction
6. Defying the fine structure constant: Fluxonium qubit and observation of Bloch oscillations.

NEXT YEAR: QUANTUM COMPUTATION WITH SOLID STATE CIRCUITS

09-III-4

LECTURE III : USING THE NON-LINEARITY OF JOSEPHSON QUANTUM CIRCUITS FOR QUBIT READOUT

1. Electrodynamics of the junction in its environment (ctn'd)
2. Artificial superconducting atoms
3. Semi-classical analysis
4. Readout of superconducting qubits

09-III-5

OUTLINE

1. Electrodynamics of the junction in its environment (ctn'd)
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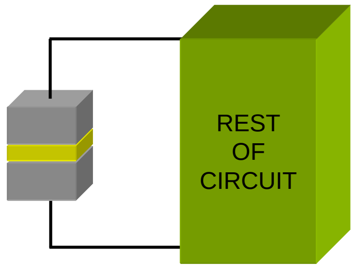
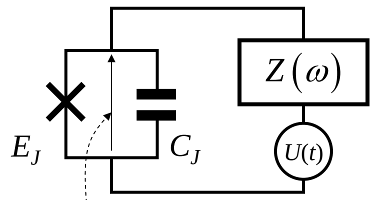
4. Readout of superconducting qubits

09-III-5a

ELECTRO^DYNAMICS OF JUNCTION IN ITS ENV^RONMENT





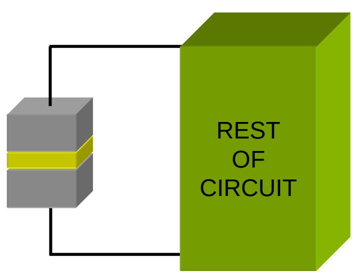
$$\phi(t) = \int_{-\infty}^{t} \int_{1}^{2} E(x,\tau)\, dx\, d\tau$$

"gauge invariant phase difference"

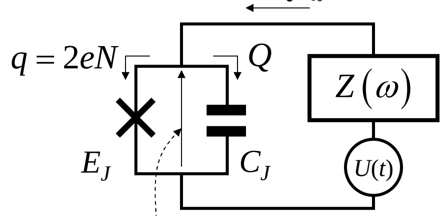
N.B. The electric field E encompasses here all contributions of the force on the electrons doing work, including those usually called chemical potential effects.

09-III-6

ELECTRO^DYNAMICS OF JUNCTION IN ITS ENV^RONMENT



$$Q_{\text{ext}} = \int_{-\infty}^{t} I\, d\tau$$



$$\phi(t) = \int_{-\infty}^{t} \int_{1}^{2} E(x,\tau)\, dx\, d\tau$$

"gauge invariant phase difference"

Equation of motion:

$$C_J \ddot{\phi} + \frac{\partial}{\partial \phi}\left[-E_J \cos\left(\frac{\phi}{\phi_0}\right)\right] = I\left(\phi, \dot{\phi}, ...\right)$$

Can be obtained in general from a Lagrangian:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \qquad \mathcal{L} = \mathcal{L}_J + \mathcal{L}_{\text{ext}} = \frac{C_J}{2}\dot{\phi}^2 + E_J \cos\frac{\phi}{\phi_0} + \mathcal{L}_{\text{ext}}\left(\phi, \dot{\phi}, ...\right)$$

09-III-7

TWO CHARACTERISTIC ENERGIES OF ENVIRONMENT

Total environment admittance: $Y_{\text{tot}}(\omega) = jC_J\omega + Z^{-1}(\omega)$

Effective shunt capacitance of junction: $C_\Sigma = \lim_{\omega\to\infty} \frac{\text{Im}\left[Y_{\text{tot}}(\omega)\right]}{\omega}$

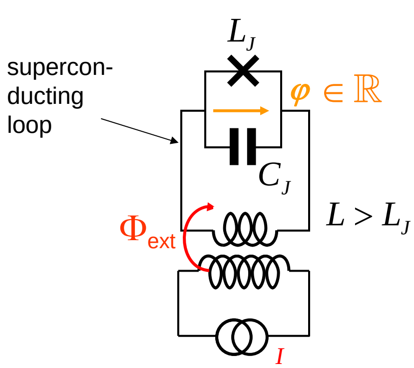
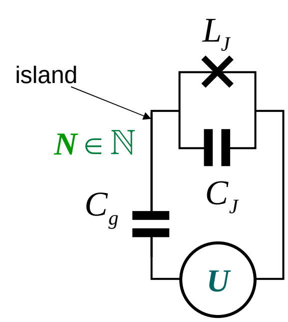
Effective shunt inductance of junction $L_{\text{eff}}^{-1} = \lim_{\omega\to 0}\left\{\text{Im}\left[\omega Y_{tot}(\omega)\right]\right\}$

Coulomb charging energy $E_C = \frac{e^2}{2C_\Sigma}$ (electron charge appear here instead of Cooper pair charge for convenience in some formulas)

Inductive energy $E_L = \frac{(\hbar/2e)^2}{L_{\text{eff}}}$ (form chosen for easy comparison with Josephson energy)

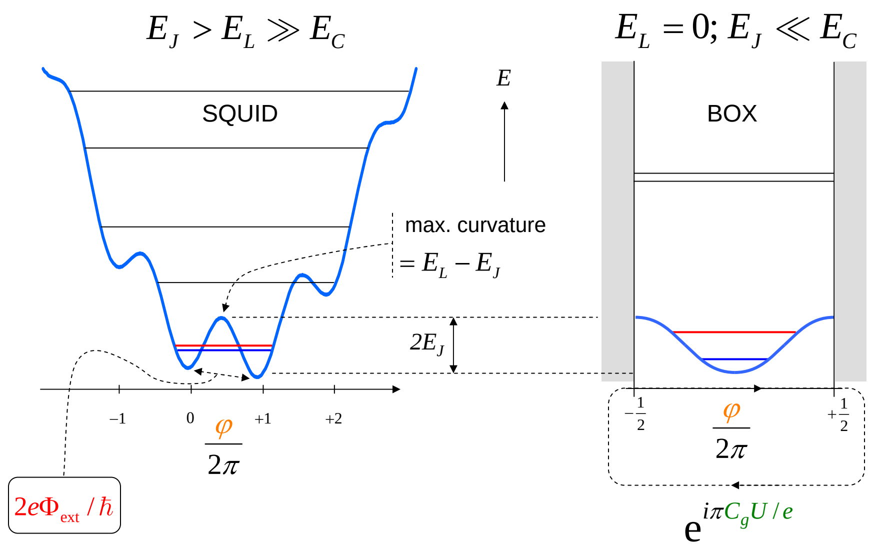
09-III-8a

TWO BASIC SUPERCONDUCTING "ATOMS"

flux	charge
	
"HYSTERETIC RF SQUID"	"COOPER PAIR BOX"
$E_J > E_L \gg E_C \qquad \frac{\Delta\varphi}{2\pi} \ll 1$	$E_L = 0;\ E_J \ll E_C \qquad \Delta N < 1$
Friedman et al. (2000)	Bouchiat et al. (1998), Nakamura, Pashkin, Tsai (1999)

09-III-9

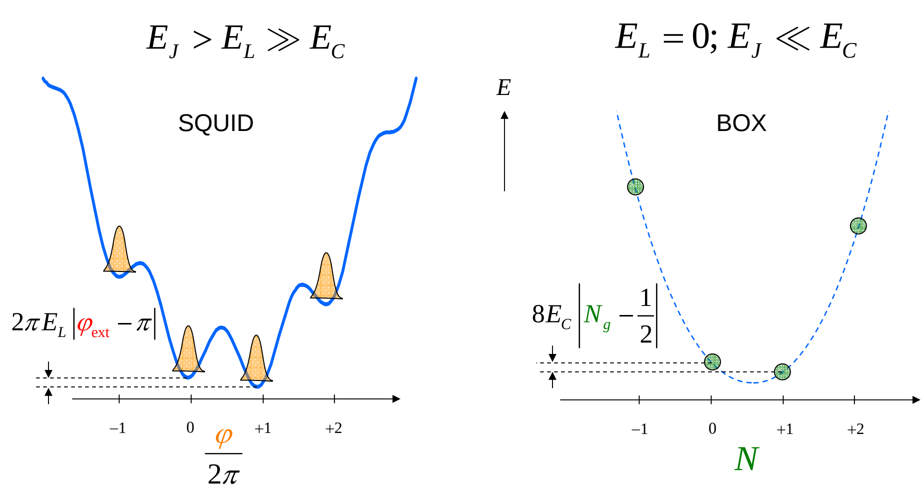
PHASE-CHARGE DUALITY



RICH LEVEL STRUCTURE, BUT EXTREME SENSITIVITY TO NOISE

09-III-10

PHASE-CHARGE DUALITY



09-III-11

OUTLINE

1. Electrodynamics of the junction in its environment (ctn'd)
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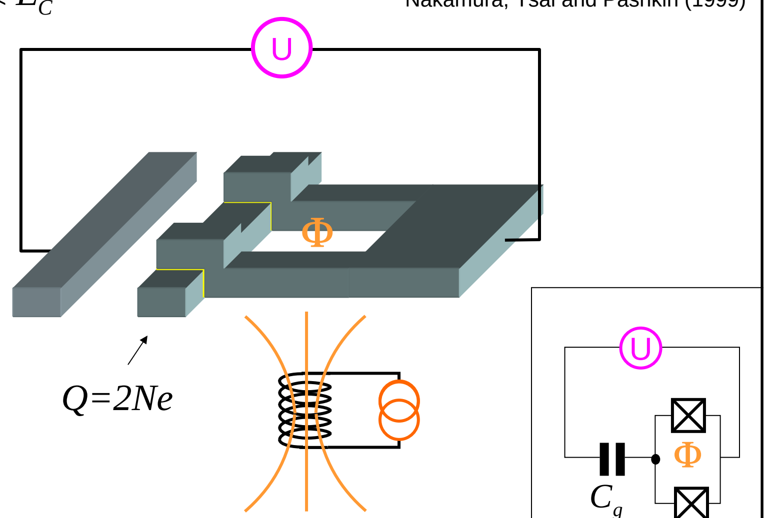
09-III-5b

THE SINGLE COOPER PAIR BOX ARTIFICIAL ATOM

$E_J < E_C$

Bouchiat et al. (1998)
Nakamura, Tsai and Pashkin (1999)

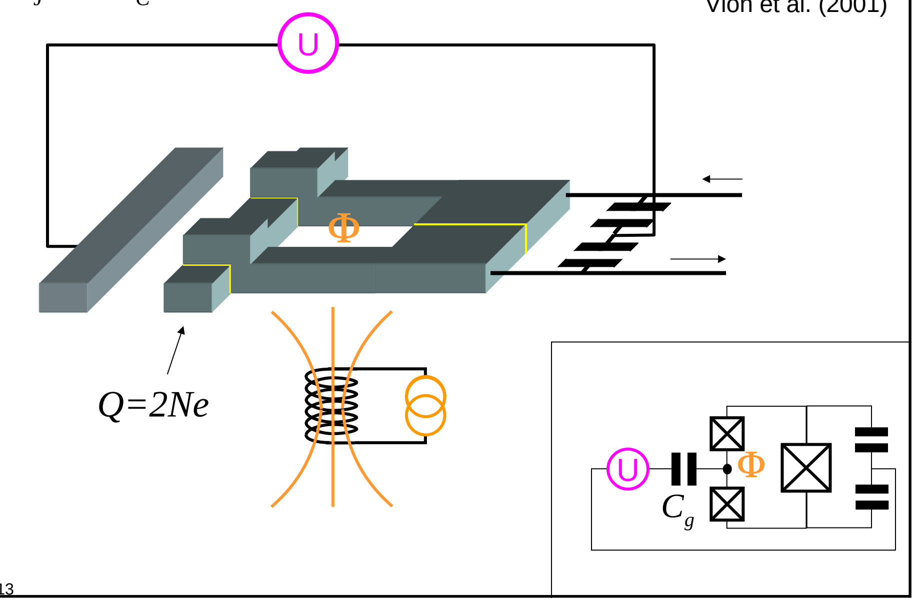


09-III-12

QUANTRONIUM

$E_J \simeq 4E_C$

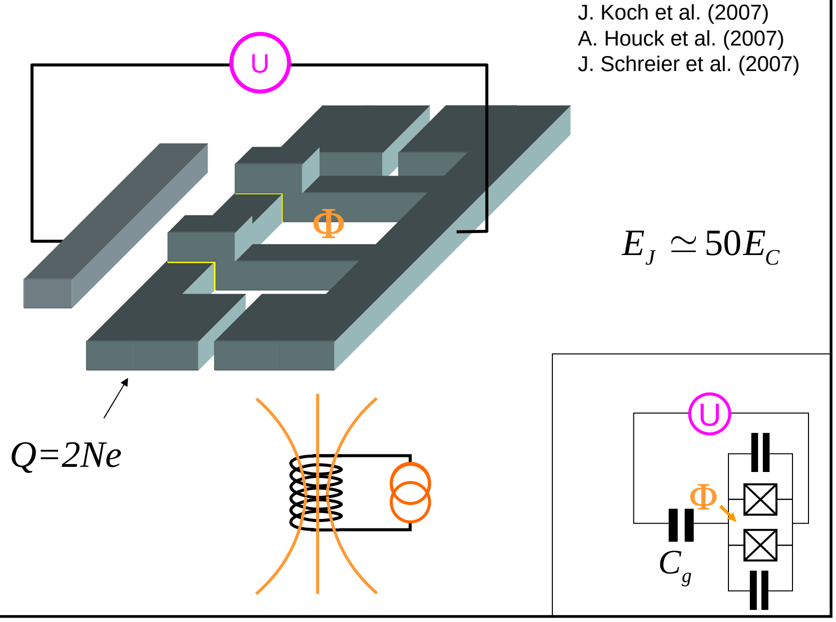
Vion et al. (2001)



09-III-13

TRANSMON COOPER PAIR BOX

J. Koch et al. (2007)
A. Houck et al. (2007)
J. Schreier et al. (2007)

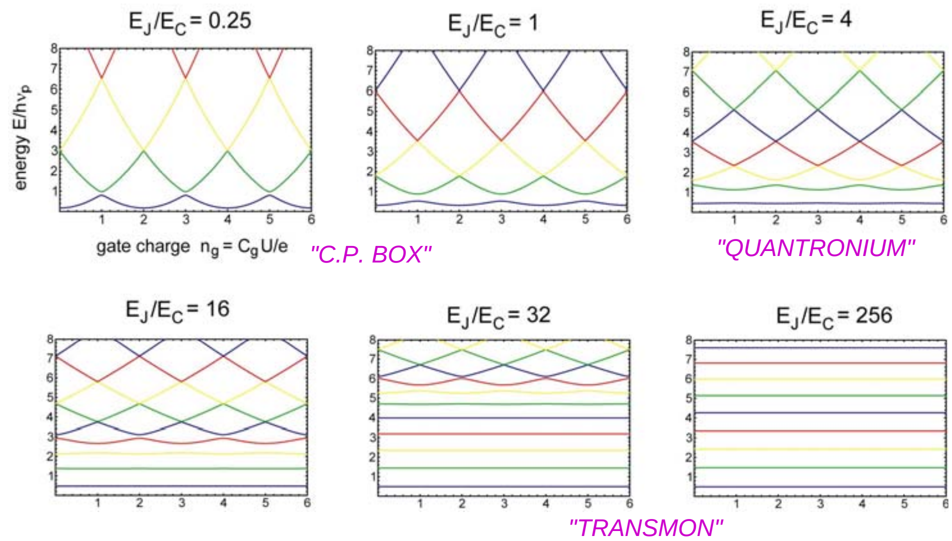


$E_J \simeq 50E_C$

09-III-14

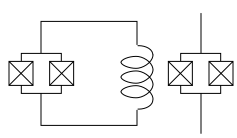
Cottet et al. (2002) Koch et al. (2007)

ANHARMONICITY vs OFFSET CHARGE INSENSITIVITY IN COOPER PAIR BOX

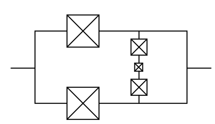


09-III-15

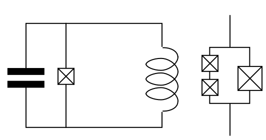
EXAMPLES OF QUANTUM CIRCUITS BELONGING TO THE RF-SQUID TYPE



Friedman, Patel, Chen, Tolpygo and J. E. Lukens, *Nature* **406**, 43 (2000).



Chiorescu, Nakamura, Harmans & Mooij, *Science* **299**, 1869 (2003).



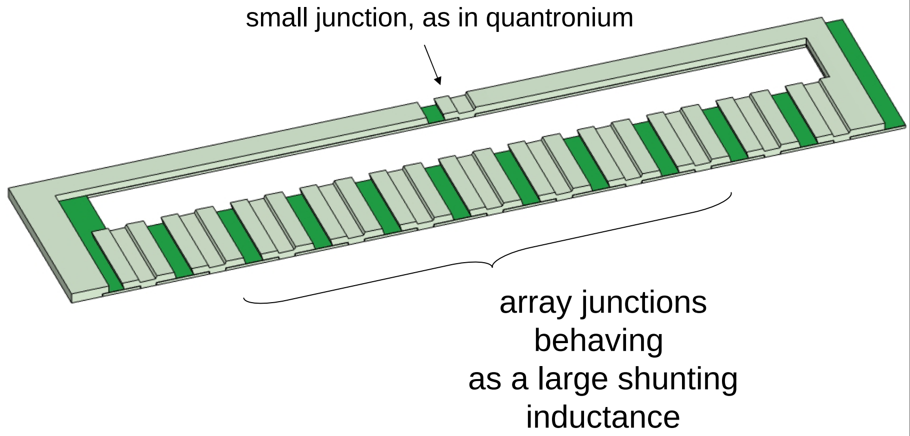
"phase" qubit

Steffen et al., *Phys. Rev. Lett.* **97**, 050502 (2006)

09-III-16

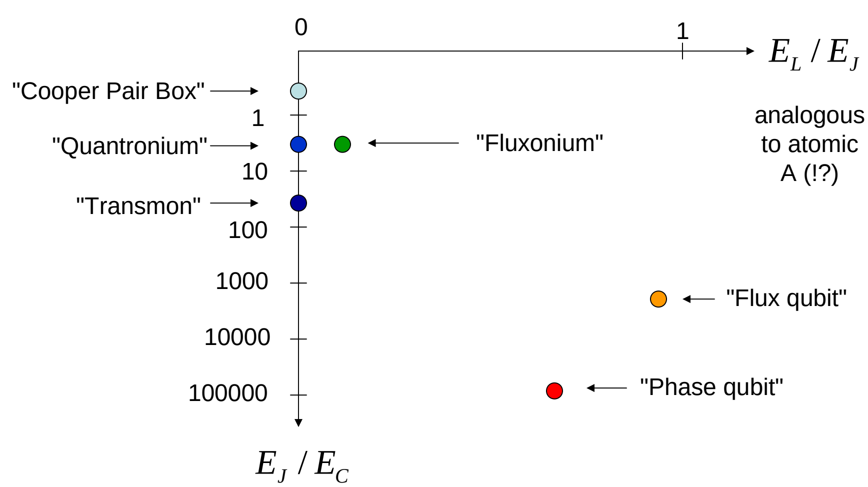
"FLUXONIUM" QUBIT

V. Manucharyan et al. (2009)



09-III-17

THE LANDSCAPE OF SUPERCONDUCTING ARTIFICIAL ATOMS



analogous to atomic Z (!?)

09-III-18

HARMONIC APPROXIMATION

$$\hat{H}_J = 8E_C \frac{\left(\hat{N} - N_{ext}\right)^2}{2} - E_J \cos\hat{\varphi} + E_L \frac{\left(\hat{\varphi} - \varphi_{\text{ext}}\right)^2}{2}$$

$\hat{\varphi} - \varphi_{\text{offset}} \to \hat{\varphi}$

$$\hat{H}_{J,h} = 8E_C \frac{\left(\hat{N} - N_{ext}\right)^2}{2} + E'_J \frac{\hat{\varphi}^2}{2} \qquad E'_J \gg E_C$$

09-III-19

HARMONIC APPROXIMATION

$$\hat{H}_J = 8E_C \frac{\left(\hat{N} - N_{ext}\right)^2}{2} - E_J \cos\hat{\varphi} + E_L \frac{\left(\hat{\varphi} - \varphi_{\text{ext}}\right)^2}{2}$$

$\hat{\varphi} - \varphi_{\text{offset}} \to \hat{\varphi}$

$$\hat{H}_{J,h} = 8E_C \frac{\left(\hat{N} - N_{ext}\right)^2}{2} + E'_J \frac{\hat{\varphi}^2}{2} \qquad E'_J \gg E_C$$

$$\hat{n} = c^\dagger c; \quad \left[c, c^\dagger\right] = 1$$

photon representation

$$\hat{H}_{J,h} = \hbar\omega_P \left(\hat{n} + \frac{1}{2}\right)$$

Josephson plasma frequency

$$\omega_P = \frac{\sqrt{8E_C E'_J}}{\hbar}$$

09-III-19a

HARMONIC APPROXIMATION

$$\hat{H}_J = 8E_C \frac{\left(\hat{N} - N_{ext}\right)^2}{2} - E_J \cos\hat{\varphi} + E_L \frac{\left(\hat{\varphi} - \varphi_{\text{ext}}\right)^2}{2}$$

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$$\hat{n} = c^\dagger c; \quad \left[c, c^\dagger\right] = 1$$

photon representation

$$\hat{H}_{J,h} = \hbar\omega_P \left(\hat{n} + \frac{1}{2}\right)$$

$$c = \sqrt[4]{\frac{E_J}{16E_C}}\hat{\varphi} + i\sqrt[4]{\frac{4E_C}{E_J}}\hat{N}$$

$$c^\dagger = \sqrt[4]{\frac{E_J}{16E_C}}\hat{\varphi} - i\sqrt[4]{\frac{4E_C}{E_J}}\hat{N}$$

Josephson plasma frequency $\omega_P = \frac{\sqrt{8E_C E'_J}}{\hbar}$

Spectrum independent of DC value of N_{ext}

09-III-19b

CAN WE GO 1 STEP BEYOND
THE HARMONIC APPROXIMATION
AND OBTAIN MEANINGFUL RESULTS?

09-III-20

YES, BUT WE NEED TO CONSIDER REGIMES WHICH LEND THEMSELVES TO A SEMI-CLASSICAL APPROXIMATION

09-III-21

OUTLINE

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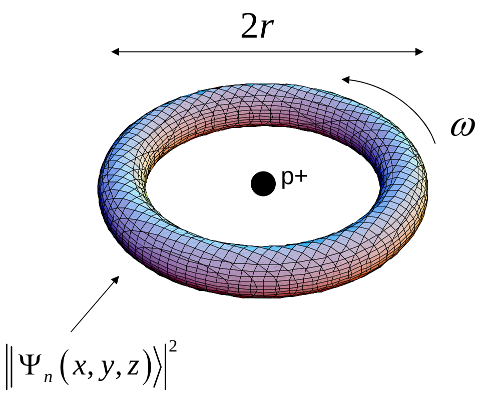
09-III-5c

ATOM IN SEMI-CLASSICAL REGIME: CIRCULAR RYDBERG ATOM



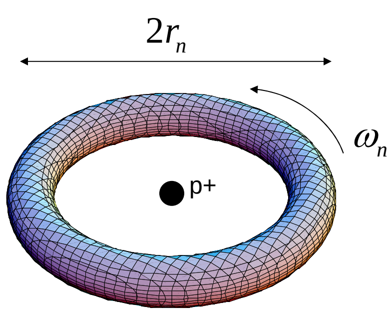
old Bohr theory essentially "exact"!

$$m_e\omega r^2 = n\hbar$$

constraint

09-III-22

ATOM IN SEMI-CLASSICAL REGIME: CIRCULAR RYDBERG ATOM



old Bohr theory essentially "exact"!

$$m_e\omega r^2 = n\hbar$$

constraint

$$r_n = a_0 n^2$$

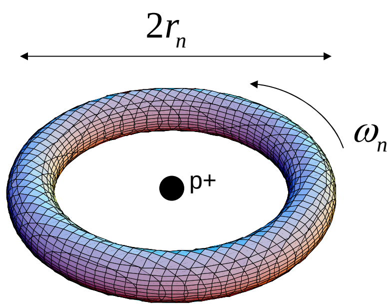
$$E_n = -\frac{\mathrm{Ry}}{n^2}$$

$$\omega_{n\to n\pm 1} = 2\frac{\mathrm{Ry}}{\hbar n^3}$$

$$d_{n\to n\pm 1} = \frac{e r_n}{\sqrt{2}}$$

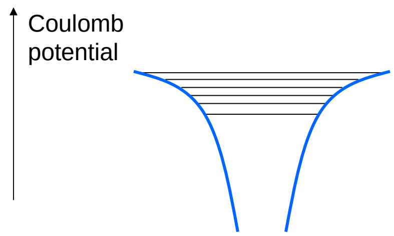
09-III-22a

ATOM IN SEMI-CLASSICAL REGIME: CIRCULAR RYDBERG ATOM



old Bohr theory essentially “exact”!

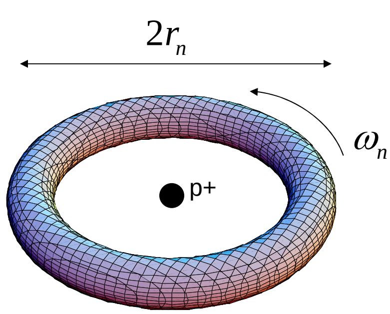
$$m_e \omega r^2 = n\hbar$$



valid when: $\left|\frac{\partial \omega(E)}{\partial E}\right| \ll \frac{1}{\hbar}$

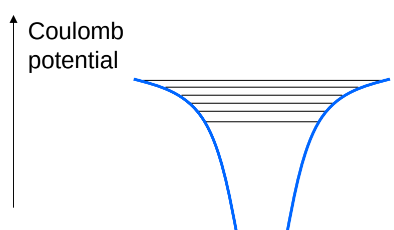
09-III-22b

ATOM IN SEMI-CLASSICAL REGIME: CIRCULAR RYDBERG ATOM



old Bohr theory essentially “exact”!

$$m_e \omega r^2 = n\hbar$$



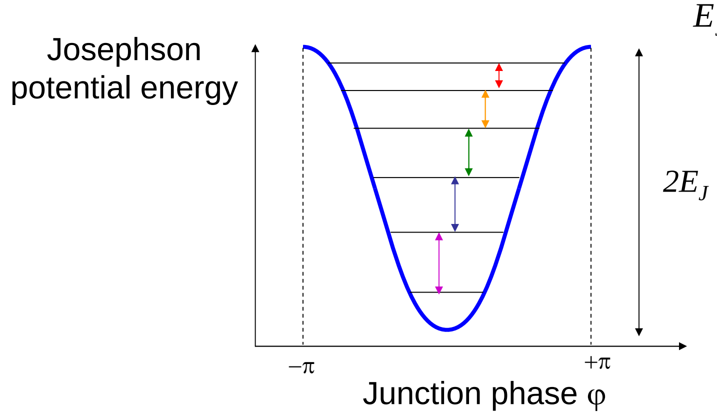
valid when: $\left|\frac{\partial \omega(E)}{\partial E}\right| \ll \frac{1}{\hbar}$

$$\omega_{n,n+1} - \omega_{n+1,n+2} \ll \omega_{n,n+1}$$

09-III-22c

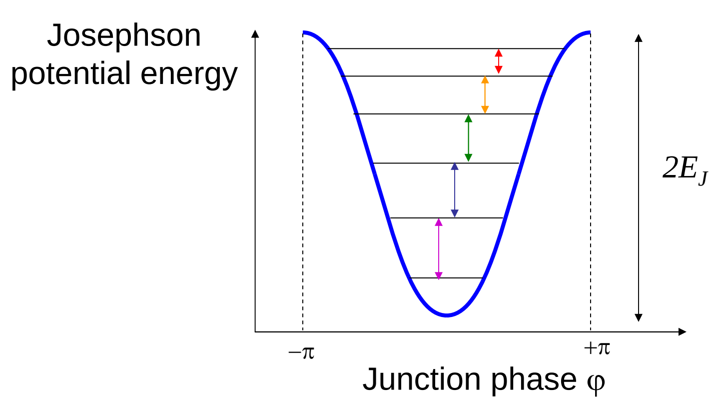
TRANSMON AS ANALOG OF CIRCULAR RYDBERG ATOMS

$E_J \gg E_C$



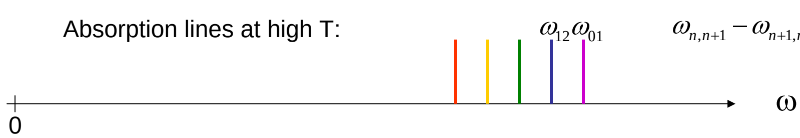
09-III-23

TRANSMON AS ANALOG OF CIRCULAR RYDBERG ATOMS



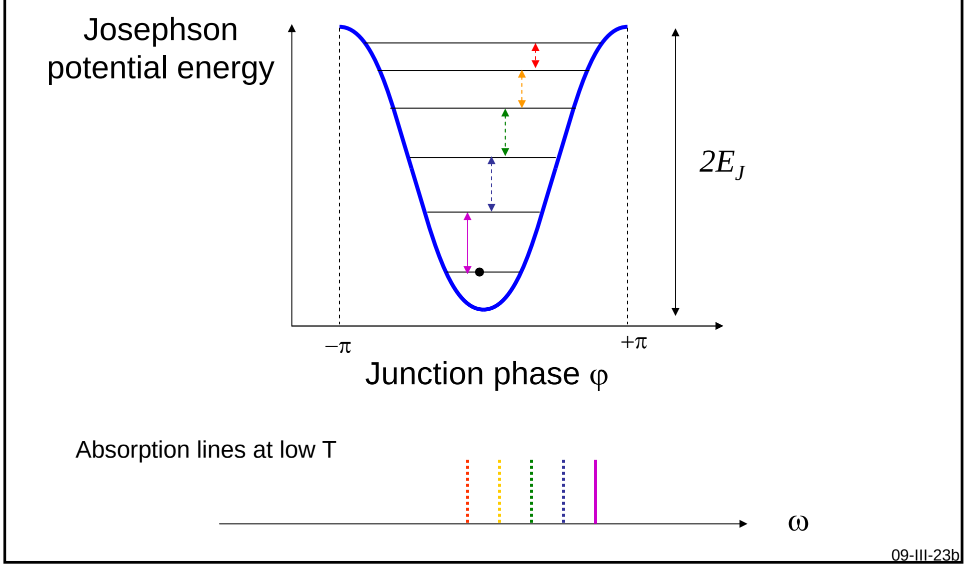
Absorption lines at high T:

$\omega_{n,n+1} - \omega_{n+1,n+2} \ll \omega_{n,n+1}$

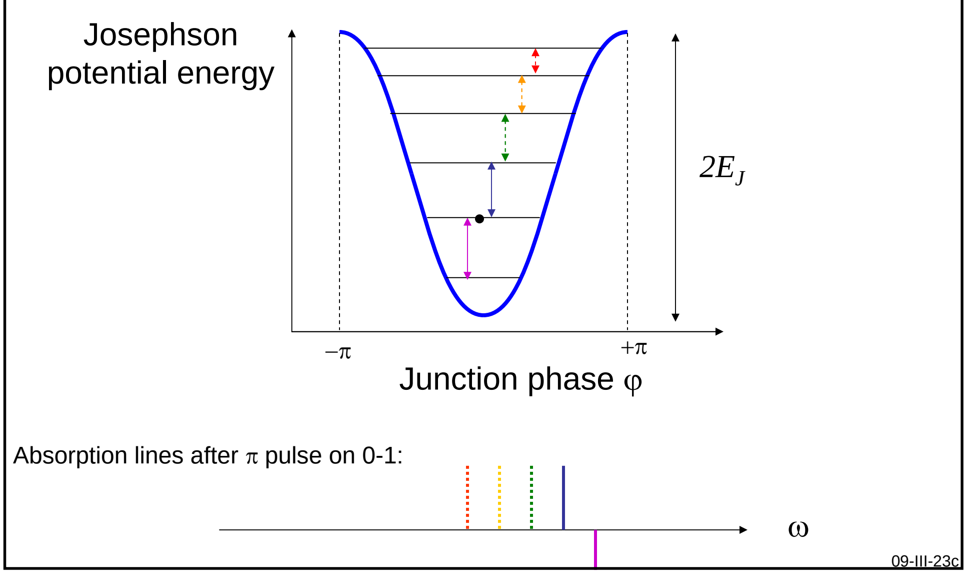


09-III-23a

TRANSMON AS ANALOG OF CIRCULAR RYDBERG ATOMS

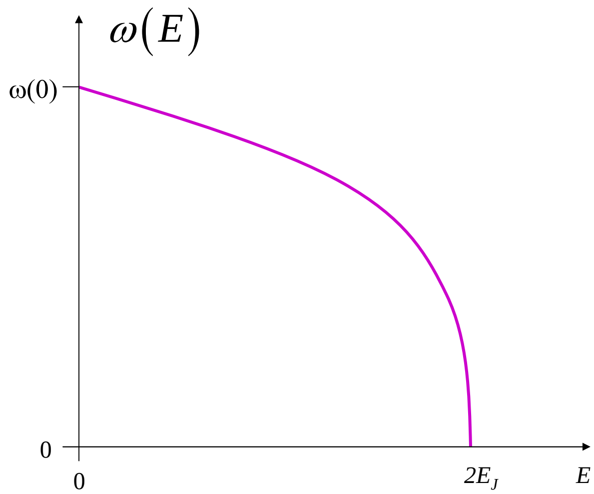


TRANSMON AS ANALOG OF CIRCULAR RYDBERG ATOMS



JUNCTION NON-LINEAR INDUCTANCE

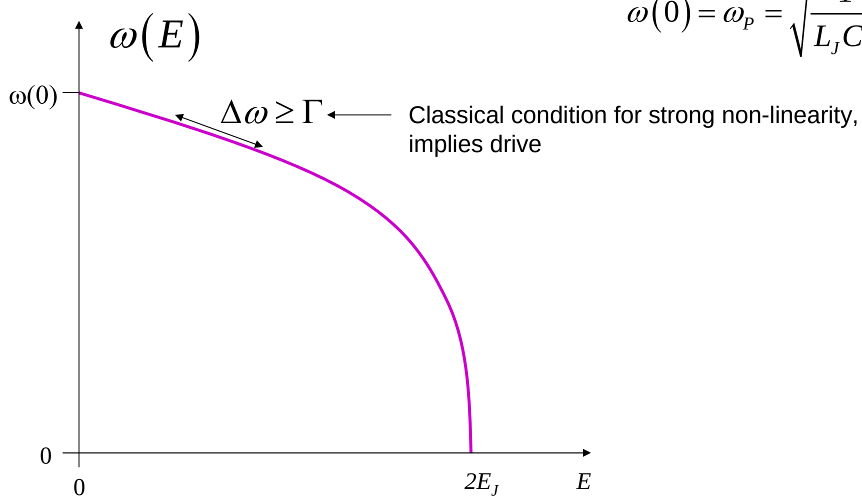
$$\omega(0) = \omega_P = \sqrt{\frac{1}{L_J C_J}}$$



09-III-24

JUNCTION NON-LINEAR INDUCTANCE

$$\omega(0) = \omega_P = \sqrt{\frac{1}{L_J C_J}}$$

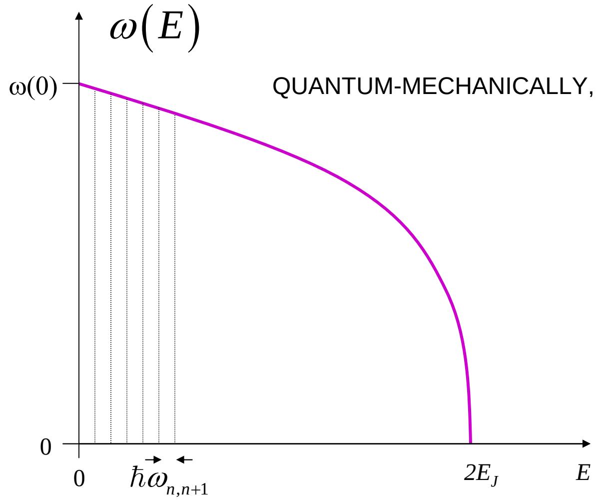


09-III-24a

JUNCTION NON-LINEAR INDUCTANCE

$$\omega(0) = \omega_P = \sqrt{\frac{1}{L_J C_J}}$$

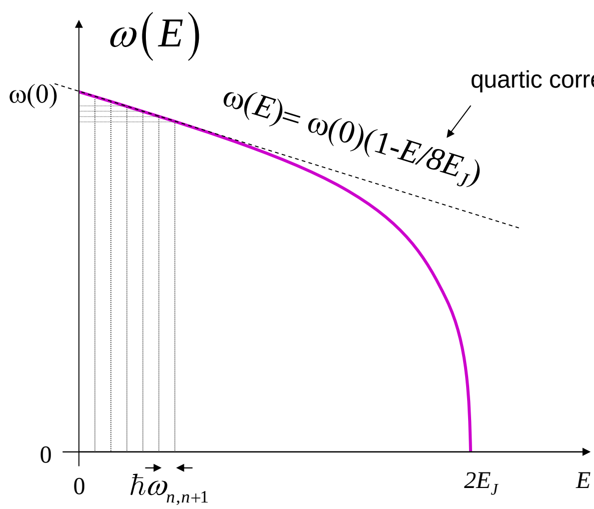
QUANTUM-MECHANICALLY, DISCRETE ENERGY LEVELS



09-III-24b

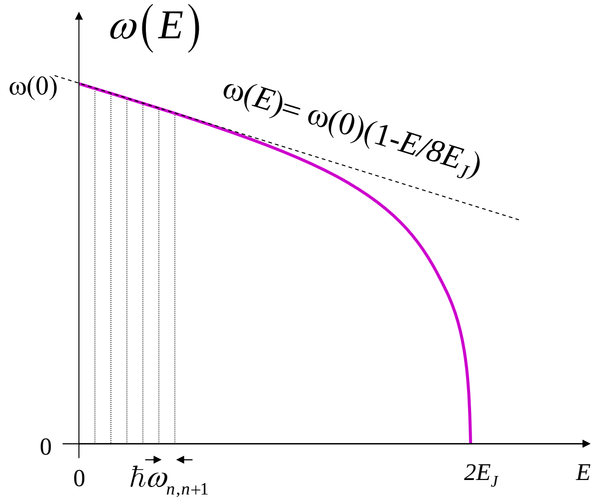
JUNCTION NON-LINEAR INDUCTANCE

quartic correction of cosine potential

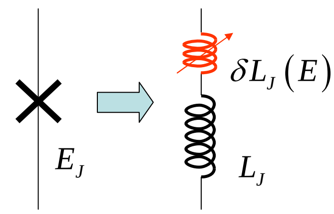


09-III-24c

JUNCTION NON-LINEAR INDUCTANCE



$$\omega(E)=\sqrt{\frac{1}{L(E)C_J}}$$



$$L_J=\frac{\hbar^2}{(2e)^2 E_J}$$

$$\delta L_J=\frac{L_J}{4}\frac{E}{E_J}=\frac{L_J}{8}\frac{\langle I_J^2\rangle}{I_0^2}$$

09-III-24d

QUANTUM NON-LINEARITY ENERGY SCALE

$$\omega_{n,n+1}=\omega_{01}\left(1-\frac{1}{8}\frac{n\hbar\omega_{01}}{E_J}\right);\, n\geq 0$$

$$\omega_{n,n+1}-\omega_{n+1,n+2}=\frac{1}{8}\frac{\hbar\omega_{01}}{E_J}$$

$$\hbar\left(\omega_{n,n+1}-\omega_{n+1,n+2}\right)=\frac{1}{8}\frac{\hbar\omega_P}{E_J}$$ neglect zero-point correction

$$\hbar\left(\omega_{n,n+1}-\omega_{n+1,n+2}\right)=\frac{(\hbar)^2}{8}\left(\sqrt{\frac{1}{L_J C_J}}\right)^2\frac{L_J}{(\hbar)^2/(2e)^2}$$

$$\hbar\left(\omega_{n,n+1}-\omega_{n+1,n+2}\right)=\frac{e^2}{2C_J}=E_C$$

weak quantum non-linearity $E_C/\hbar \ll \Gamma$ ← decay rate of quantum level

strong quantum non-linearity $E_C/\hbar \gg \Gamma$

09-III-25e

OUTLINE

1. Electrodynamics of the junction in its environment (ctn'd)
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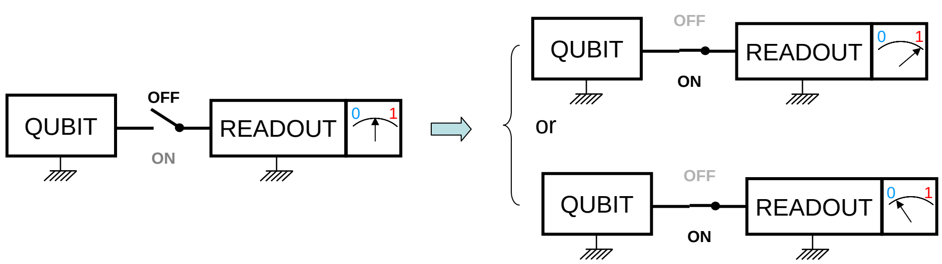
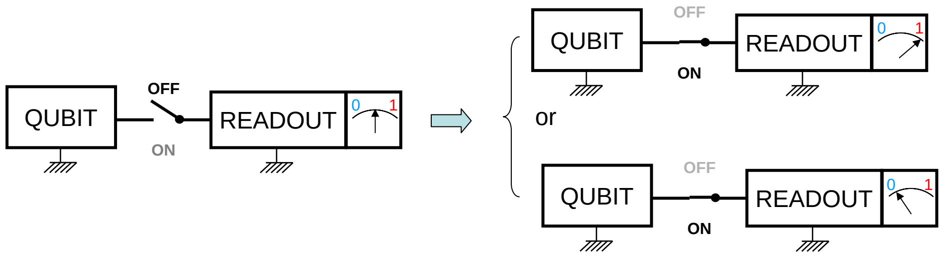
09-III-5d

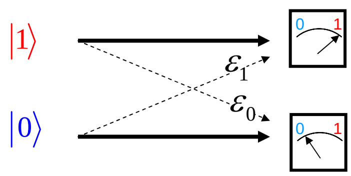
THE QUBIT MEMORY READOUT PROBLEM



09-III-26

THE QUBIT MEMORY READOUT PROBLEM



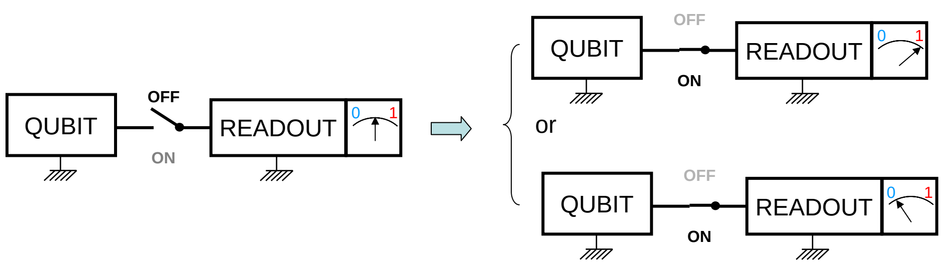


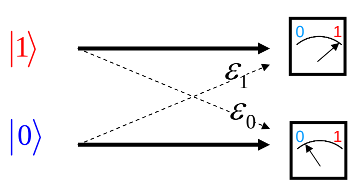
FIDELITY:

$$F = 1 - \varepsilon_0 - \varepsilon_1$$

09-III-26a

THE QUBIT MEMORY READOUT PROBLEM





FIDELITY:

$$F = 1 - \varepsilon_0 - \varepsilon_1$$

WANT:

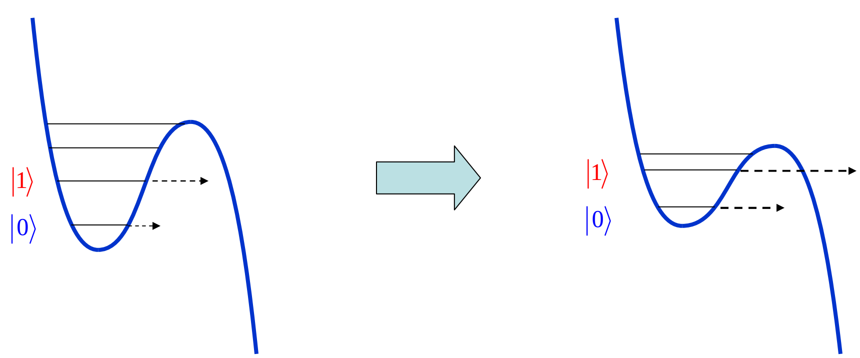
1) SWITCH WITH ON/OFF RATIO AS LARGE AS POSSIBLE
2) READOUT WITH F AS CLOSE TO 1 AS POSSIBLE

09-III-26b

TWO MAIN STRATEGIES

09-III-27

STATE DECAY STRATEGY

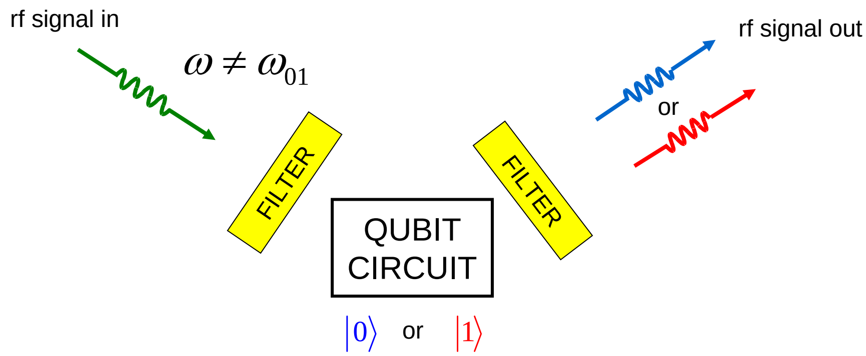


Martinis, Devoret and Clarke, PRL **55** (1985)
Martinis, Nam, Aumentado and Urbina, PRL **89** (2002)

09-III-28

DISPERSIVE READOUT STRATEGY

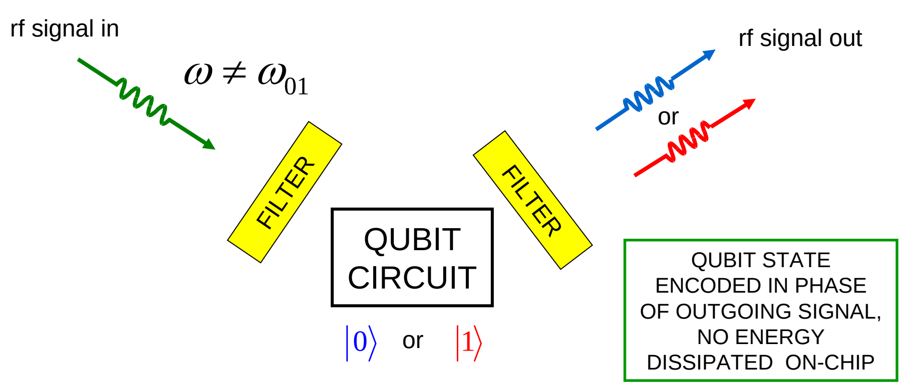
Blais et al. PRA 2004, Walraff et al., Nature 2004



09-III-29

DISPERSIVE READOUT STRATEGY

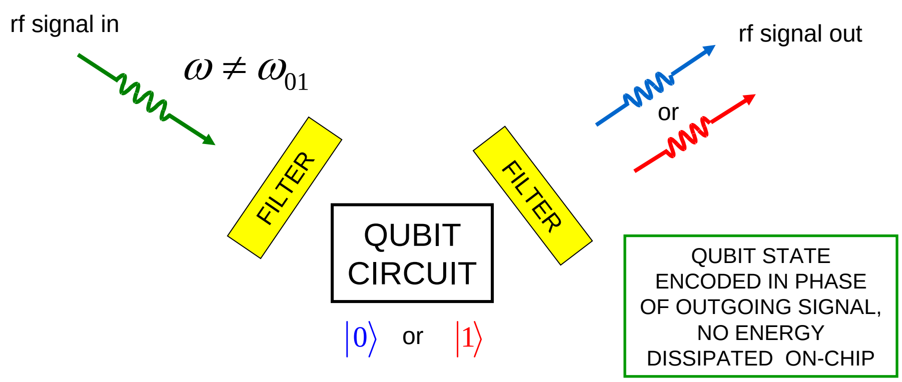
Blais et al. PRA 2004, Walraff et al., Nature 2004



09-III-29a

DISPERSIVE READOUT STRATEGY

Blais et al. PRA 2004, Walraff et al., Nature 2004



A) SHELTER QUBIT FROM ALL RADIATION EXCEPT READOUT RF
B) AMPLIFY OUTGOING SIGNAL WITH LOWEST NOISE POSSIBLE
C) SEND ENOUGH PHOTONS TO BEAT NOISE

09-III-29b

END OF LECTURE