COLLÈGE DE FRANCE
1530

Chaire de Physique Mésoscopique
Michel Devoret
Année 2009, 12 mai - 23 juin

CIRCUITS ET SIGNAUX QUANTIQUES (II)

QUANTUM SIGNALS AND CIRCUITS (II)

Première leçon / *First Lecture*

VISIT THE WEBSITE OF THE CHAIR OF MESOSCOPIC PHYSICS

http://www.college-de-france.fr

then follow Enseignement > Sciences Physiques > Physique Mésoscopique >

PDF FILES OF ALL LECTURES WILL BE POSTED ON THIS WEBSITE

Questions, comments and corrections are welcome!

write to "phymeso@gmail.com"

CALENDAR OF SEMINARS

May 12: Daniel Esteve, (Quantronics group, SPEC-CEA Saclay)
Faithful readout of a superconducting qubit

May 19: Christian Glattli (LPA/ENS)
Statistique de Fermi dans les conducteurs balistiques : conséquences expéri-mentales et exploitation pour l'information quantique

June 2: Steve Girvin (Yale)
Quantum Electrodynamics of Superconducting Circuits and Qubits

June 9: Charlie Marcus (Harvard)
Electron Spin as a Holder of Quantum Information: Prospects and Challenges

June 16: Frédéric Pierre (LPN/CNRS)
Energy exchange in quantum Hall edge channels

June 23: Lev Ioffe (Rutgers)
Implementation of protected qubits in Josephson junction arrays

NOTE THAT THERE IS NO LECTURE AND NO SEMINAR ON MAY 26 !

09-I-3

CONTENT OF THIS YEAR'S LECTURES

OUT-OF-EQUILIBRIUM NON-LINEAR QUANTUM CIRCUITS

1. Introduction and review of last year's course
2. Audit of information processing machines
3. Readout of qubits
4. Amplifying quantum fluctuations
5. Dynamical cooling and quantum error correction
6. Can Bloch oscillations be observed?
7. Defying the fine structure constant: Fluxonium qubit

NEXT YEAR: QUANTUM COMPUTATION WITH SOLID STATE CIRCUITS

09-I-4

LECTURE I : INTRODUCTION, THE AUDIT OF INFORMATION PROCESSING MACHINES

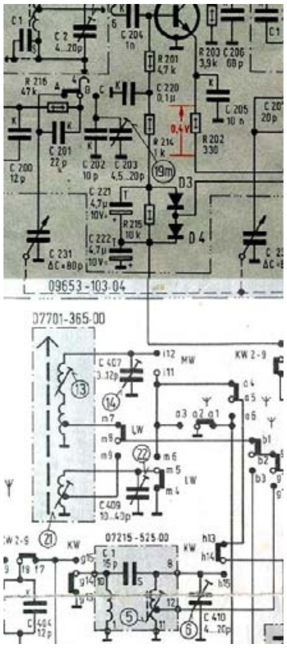
1. Limitations of information processing machines
2. Review of quantum circuits
3. Non-linearity of Josephson junctions
4. Summary of questions addressed by this course

09-I-5

OUTLINE

1. Limitations of information processing machines
2. Review of quantum circuits
3. Non-linearity of Josephson junctions
4. Summary of questions addressed by this course

09-I-5a

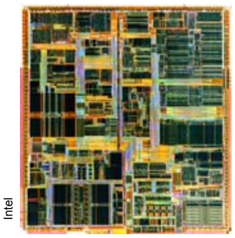


courtesy of Jens Koch

ELECTRICAL CIRCUITS

Modular architecture
- small number of basic building blocks,
- large number of combinations into useful networks

Parallel fabrication
- reliable assembly of networks with large number of elements (10^9 transistors/chip)
- uniformity of like elements
- miniaturization



Intel

Classical analysis

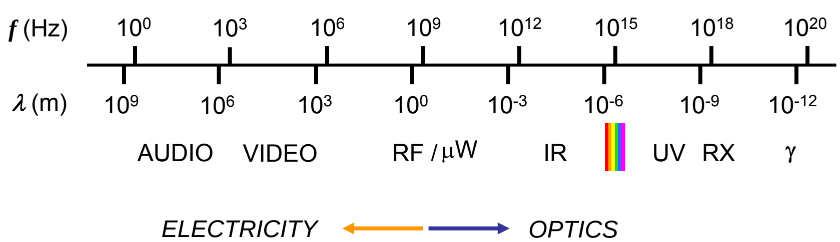
dynamics of information carrying signals like voltages and currents usually described by classical equations

(however, quantum mechanics enters at lower level in characteristics of single elements such as transistors, tunnel diodes, etc.)

► **This course deals with *quantum* electrical circuits, i.e. circuits in which information carrying signals must be treated quantum-mechanically.**

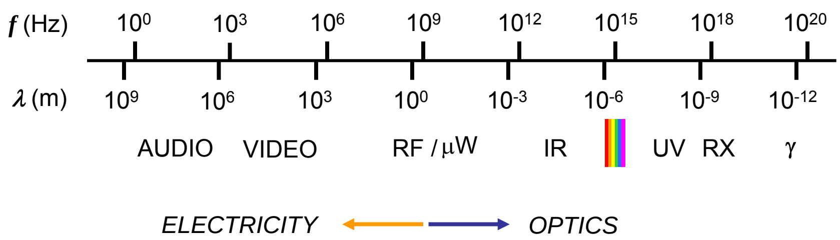
09-I-6

ELECTROMAGNETIC SPECTRUM



09-I-7

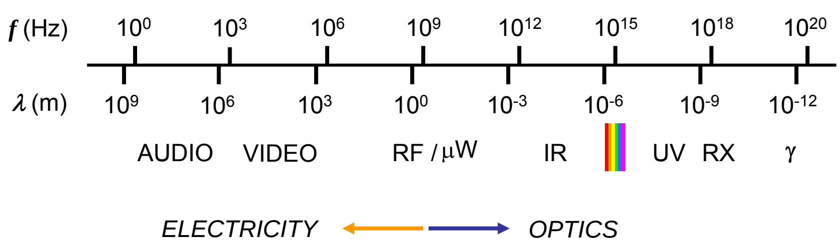
ELECTROMAGNETIC SPECTRUM



1 microwave signal with 0/1 photon → 1 bit?

09-I-7a

ELECTROMAGNETIC SPECTRUM



1 microwave signal with 0/1 photon → 1 bit?

10 GHz ~ 0.5K

09-I-7b

INFORMATION PROCESSING CIRCUITS ARE NOW SMALL AND FAST, BUT ARE THEY EFFICIENT?

Landauer, Bennett, Feynman

US Army Photo

clock speed × 10^6

volume / 10^{15}

power / 10^4

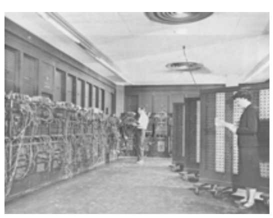
Apple

09-I-8

INFORMATION PROCESSING CIRCUITS ARE NOW SMALL AND FAST, BUT ARE THEY EFFICIENT?

Landauer, Bennett, Feynman

US Army Photo



clock speed × 10^6

volume / 10^{15}

power / 10^4

Apple

1 GOOGLE QUERY =1kJ !

"Google is fast — a typical search returns results in less than 0.2 seconds. Queries vary in degree of difficulty, but for the average query, the servers it touches each work on it for just a few thousandths of a second. Together with other work performed before your search even starts (such as building the search index) this amounts to 0.0003 kWh of energy per search, or 1 kJ. For comparison, the average adult needs about 8000 kJ a day of energy from food, so a Google search uses just about the same amount of energy that your body burns in ten seconds " OFFICIAL GOOGLE BLOG

09-I-8a

THE PROBLEM OF HEAT ENGINES EFFICIENCY GAVE BIRTH TO THERMODYNAMICS

Sadi Carnot

$$\eta < \eta_{THEOR.} = \frac{T_{HOT} - T_{COLD}}{T_{HOT}}$$

"Réflexions sur la puissance motrice du feu" (1824)

Order of magnitude $\eta_{I.C.E.} \sim 0.3$

09-I-9

[JUST AS A THERMAL ENGINES CONVERT SOURCE HEAT INTO MECHANICAL WORK AND WASTE HEAT], A COMPUTER CONVERTS FREE ENERGY INTO MATHEMATICAL WORK AND WASTE HEAT.

"The thermodynamics of computation", C. Bennett, 1982

HOW MUCH ENERGY IS NEEDED PER ELEMENTARY OPERATION?
WHAT IS ITS MINIMUM DURATION?

09-I-10

IN PRINCIPLE, COMPUTATION CAN BE REVERSIBLE.
THEN COST IS ONLY THE ERASURE OF INFORMATION
IN OUTPUT REGISTER, i.e. $k_B T \ln 2$ PER BIT

1	0	1	1	0	0	1	0	1	0	0	1	1	1	0

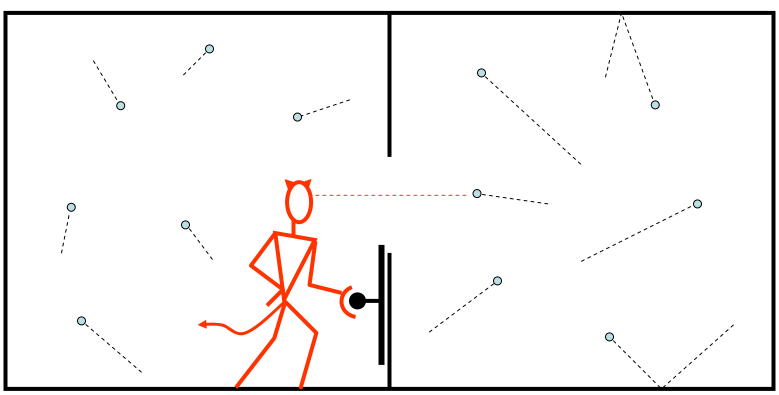
↓

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$k_B T \ln 2 \sim 10^{-20}$J @ 300K

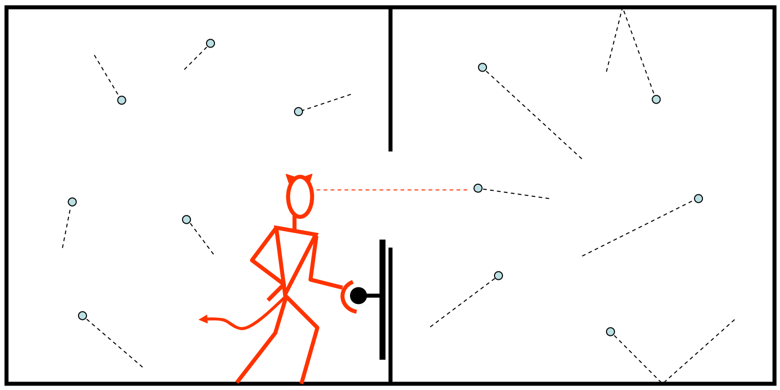
09-I-11

MAXWELL'S DEMON

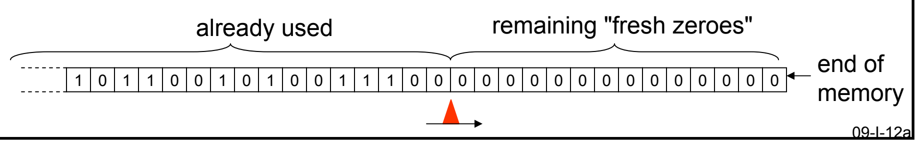


09-I-12

MAXWELL'S DEMON



DEMON'S MEMORY DURING HIS WORK:



09-I-12a

THE ULTIMATE COST OF COMMUNICATION

maximum channel capacity for signal power P:

$$C \leq \sqrt{\frac{\pi}{3}\frac{P}{\hbar}}$$

Gordon (1964), Lebedev and Levitin (1966)

100μW ~ 10^{15} bits/s!

09-I-13

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09-I-5b

QUANTUM INFORMATION PROCESSING

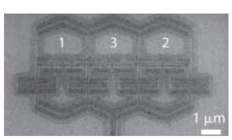
of atoms

micro

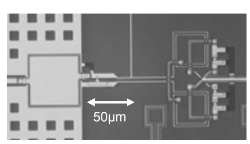
MACRO

- OPTICAL PHOTONS
- NUCLEAR SPINS
- IONS
- ATOMS
- MOLECULES
- QUANTUM DOTS
- SUPERCONDUCTING JOSEPHSON TUNNEL CIRCUITS

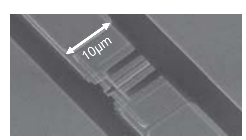
coupling with env^t.



from A. O. Niskanen et al. Science 316, 723 (2007)



courtesy of J. Martinis, 2009

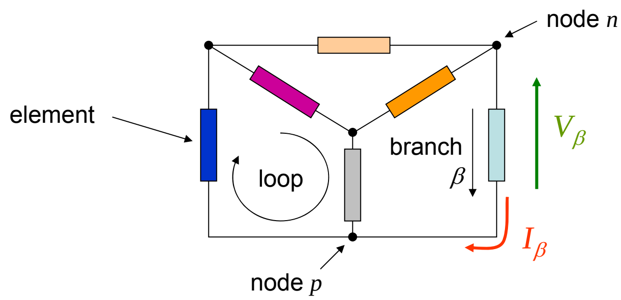


from Metcalfe et al., 2007

09-I-14

DESCRIPTION OF RADIO-FREQUENCY CIRCUITS

A RF CIRCUIT IS A NETWORK OF ELECTRICAL ELEMENTS



TWO DYNAMICAL VARIABLES CHARACTERIZE THE STATE OF EACH DIPOLE ELEMENT AT EVERY INSTANT:

Voltage across the element: $V_\beta(t) = \int_n^p \vec{E} \cdot \vec{d\ell}$

Current through the element: $I_\beta(t) = \iint \vec{j} \cdot \vec{d\sigma}_{np}$

Signals: time dependence of circuit variables

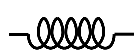
09-I-15

CONSTITUTIVE RELATIONS OF ELEMENTS

EACH ELEMENT IS TAKEN FROM A FINITE SET OF ELEMENT TYPES

EACH ELEMENT TYPE IS CHARACTERIZED BY A RELATION BETWEEN VOLTAGE AND CURRENT

Examples:

inductance:  $V - L\, dI/dt = 0$

capacitance:  $I - C\, dV/dt = 0$

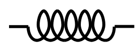
09-I-16

CONSTITUTIVE RELATIONS OF ELEMENTS

EACH ELEMENT IS TAKEN FROM A FINITE SET OF ELEMENT TYPES

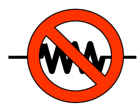
EACH ELEMENT TYPE IS CHARACTERIZED BY A RELATION BETWEEN VOLTAGE AND CURRENT

Examples:

inductance:  $V - L\,dI/dt = 0$

capacitance:  $I - C\,dV/dt = 0$

Caveat:

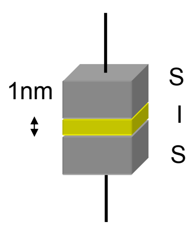
resistance:  ~~$V - R\,I = 0$~~

tend to kill quantum coherence (resistance)

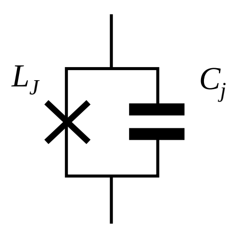
not the correct constitutive relation of a resistance anyway (see next lecture) ($V - R\,I = 0$)

09-I-16a

JOSEPHSON JUNCTION PROVIDES A NON-LINEAR INDUCTOR

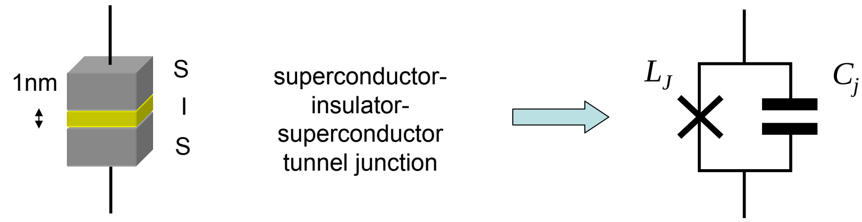


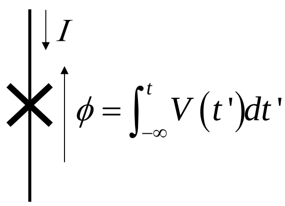
superconductor-insulator-superconductor tunnel junction



09-I-17

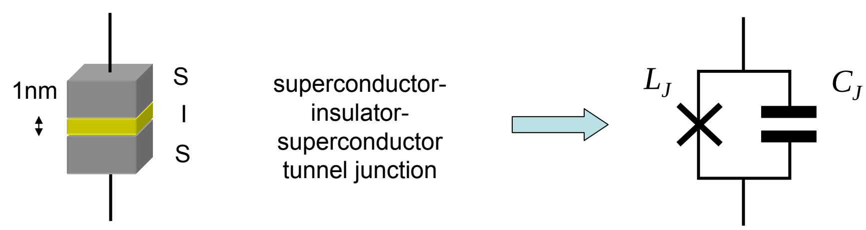
JOSEPHSON JUNCTION PROVIDES A NON-LINEAR INDUCTOR

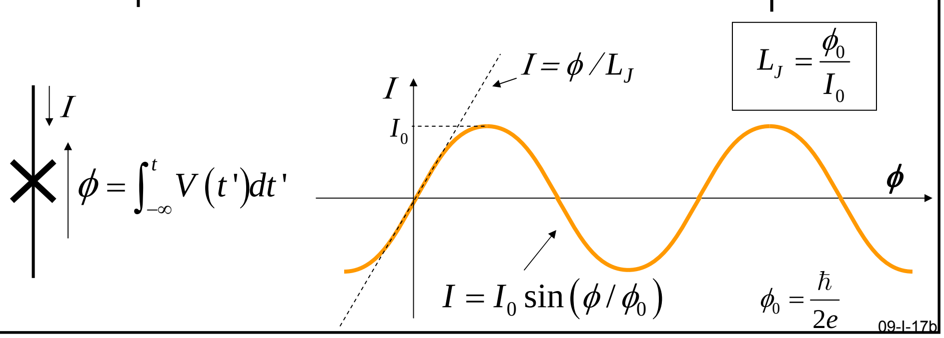




09-I-17a

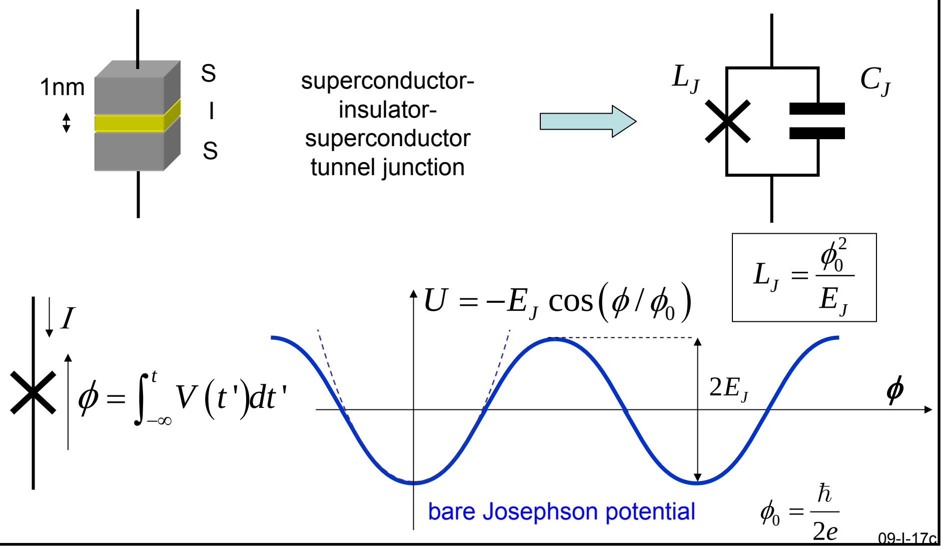
JOSEPHSON TUNNEL JUNCTION PROVIDES A NON-LINEAR INDUCTOR WITH NO DISSIPATION





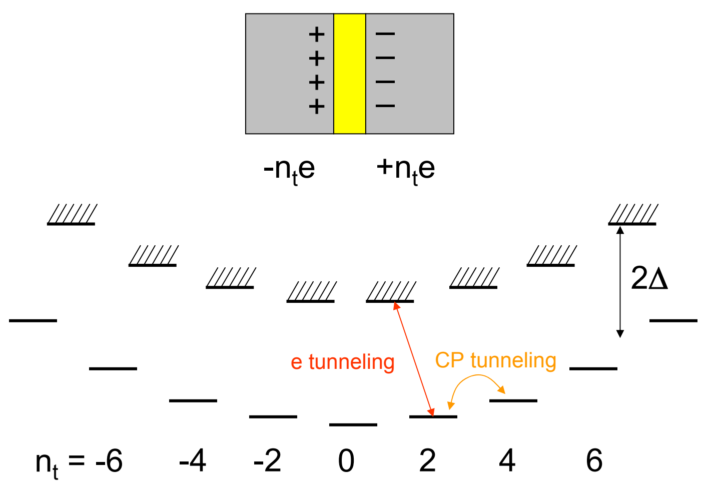
09-I-17b

JOSEPHSON TUNNEL JUNCTION PROVIDES A NON-LINEAR INDUCTOR WITH NO DISSIPATION



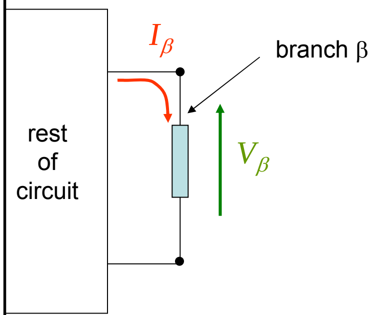
09-I-17c

LOW-LYING EXCITATIONS OF A JUNCTION: CHARGE STATES



09-I-18

QUANTUM TREATMENT OF CIRCUITS



Need to take branch flux and branch charge as basic variables:

$$\phi_\beta(t) = \int_{-\infty}^{t} V_\beta(t')dt'$$

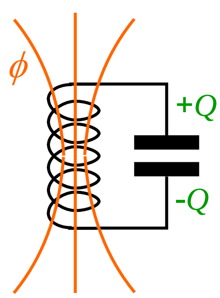
$$Q_\beta(t) = \int_{-\infty}^{t} I_\beta(t')dt'$$

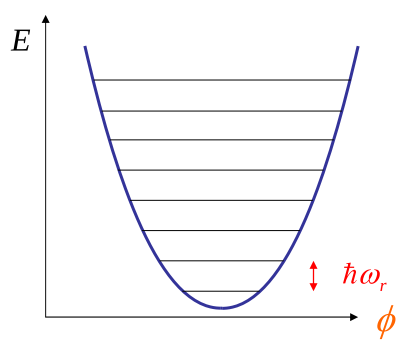
For every branch β in the circuit:

$$\left[\hat{\phi}_\beta, \hat{Q}_\beta\right] = i\hbar$$

09-I-19

LC CIRCUIT AS A QUANTUM HARMONIC OSCILLATOR





$$\hat{a} = \frac{\hat{\phi}}{\phi_{ZPF}} + i\frac{\hat{Q}}{Q_{ZPF}}; \quad \hat{a}^\dagger = \frac{\hat{\phi}}{\phi_{ZPF}} - i\frac{\hat{Q}}{Q_{ZPF}}$$

← annihilation and creation operators

$$\left[\hat{a}, \hat{a}^\dagger\right] = 1$$

$$\phi_{ZPF} = \sqrt{2\hbar\omega_r L}$$

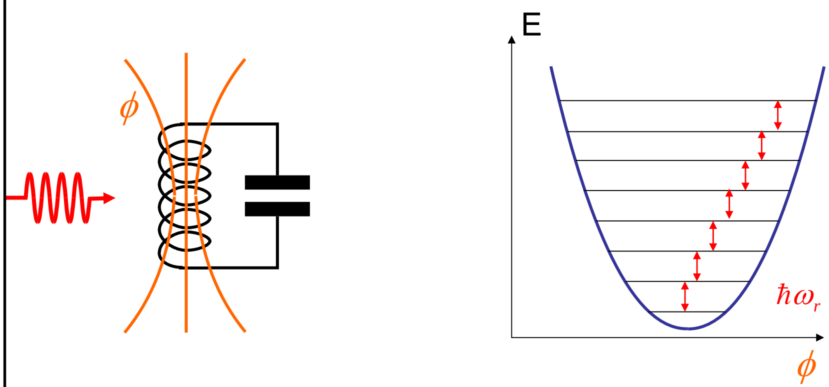
$$Q_{ZPF} = \sqrt{2\hbar\omega_r C}$$

trapped photons!

$$\hat{H} = \hbar\omega_r\left(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}\right)$$

09-I-20

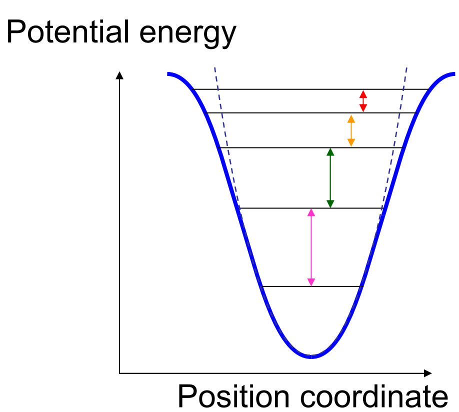
ALL TRANSITIONS BETWEEN QUANTUM LEVELS ARE DEGENERATE IN PURELY LINEAR CIRCUITS!



CANNOT STEER THE SYSTEM TO AN ARBITRARY STATE IF PERFECTLY LINEAR

09-I-21

NEED NON-LINEARITY TO FULLY REVEAL QUANTUM MECHANICS



09-I-22

1. Limitations of information processing machines
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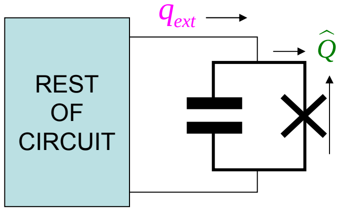
09-I-5c

ENERGY SCALES
OF THE JOSEPHSON JUNCTION "ATOM"



$q_{ext} \rightarrow$

$\hat{Q} = \int_{-\infty}^{t} \hat{I}(t')dt'$

$\hat{\phi} = \int_{-\infty}^{t} \hat{V}(t')dt'$

$\hat{\varphi} = \frac{2e\hat{\phi}}{\hbar}$

$\hat{N} = \frac{\hat{Q}}{2e}$

$\left[\hat{\varphi}, \hat{N}\right] = i$

Hamiltonian: $$\hat{H}_J = 8E_C \frac{\left(\hat{N} - N_{ext}\right)^2}{2} - E_J \cos\hat{\varphi}$$

Coulomb charging energy for 1e

$$E_C = \frac{e^2}{2C_j}$$

reduced offset charge

$$N_{ext} = \frac{q_{ext}}{2e}$$

Josephson energy

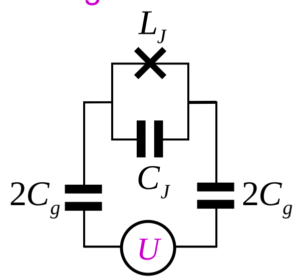
$$E_J = \frac{1}{8} \mathcal{N}\mathcal{T}\Delta$$

barrier transp^{cy} ($\mathcal{T}$); gap (Δ); # cond^{ion} channels ($\mathcal{N}$); valid for opaque barrier

09-I-23

3 TYPES OF BIASES

charge



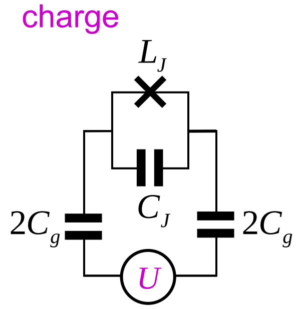
$$8E_C \frac{\left(\hat{N} - C_g U/2\right)^2}{2} - E_J \cos\hat{\varphi}$$

"Cooper pair box"
$\hat{\varphi}$ lives on circle
$\hat{N}$ integer

09-I-24

3 TYPES OF BIASES

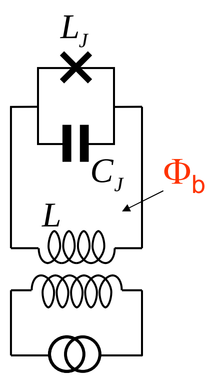
charge



$$8E_C \frac{\left(\hat{N} - C_g U/2\right)^2}{2} - E_J \cos\hat{\varphi}$$

"Cooper pair box"
$\hat{\varphi}$ lives on circle
$\hat{N}$ integer

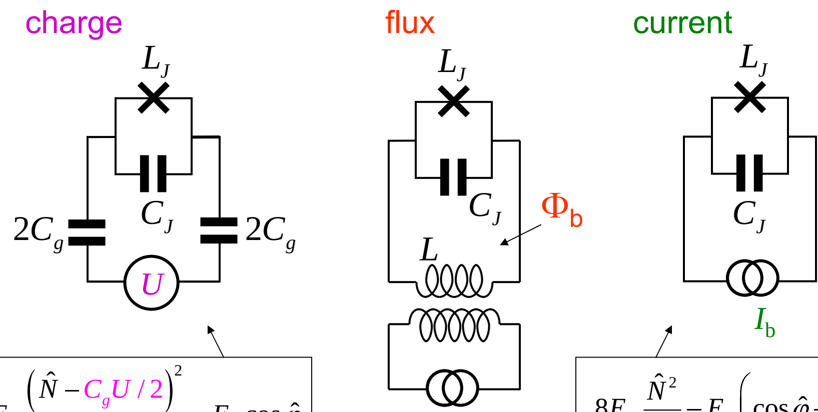
flux



$$8E_C \frac{\hat{N}^2}{2} + E_L \frac{\left(\hat{\varphi} - \frac{2e\Phi_b}{\hbar}\right)^2}{2} - E_J \cos\hat{\varphi}$$

09-I-24a

3 TYPES OF BIASES



charge: $8E_C \dfrac{\left(\hat{N} - C_g U/2\right)^2}{2} - E_J \cos\hat{\varphi}$

"Cooper pair box"

$\hat{\varphi}$ lives on circle

$\hat{N}$ integer

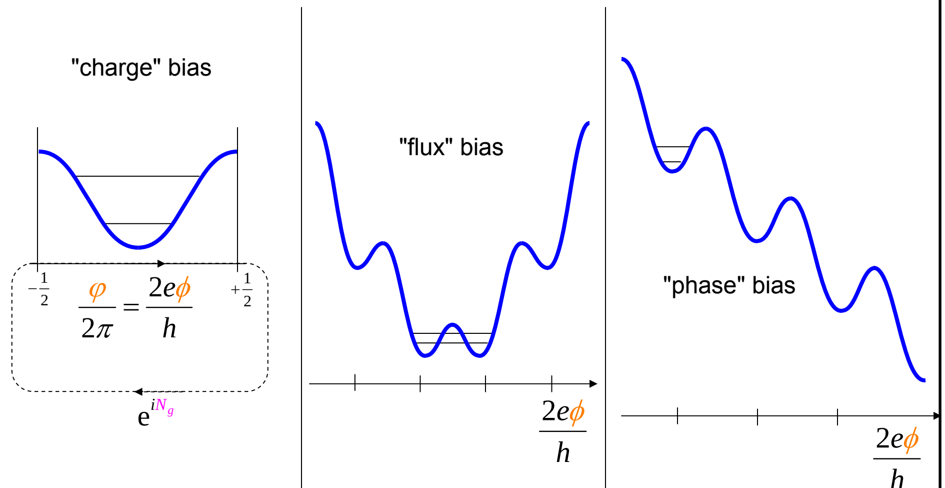
flux: $8E_C \dfrac{\hat{N}^2}{2} + E_L \dfrac{\left(\hat{\varphi} - \dfrac{2e\Phi_b}{\hbar}\right)^2}{2} - E_J \cos\hat{\varphi}$

current: $8E_C \dfrac{\hat{N}^2}{2} - E_J \left(\cos\hat{\varphi} - \dfrac{I_b}{I_0}\hat{\varphi}\right)$

09-I-24b

EFFECTIVE POTENTIAL OF 3 MAIN BIAS SCHEMES

see also proposals for topologically protected qubits, for example Feigelman et al. PRL 92, 098301 (2004)



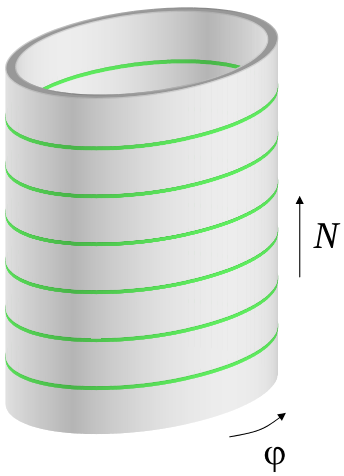
CEA Saclay, NEC, Yale Chalmers, JPL, ...

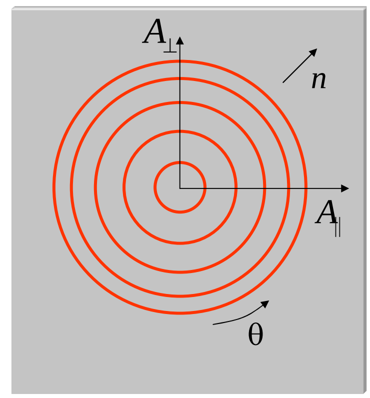
TU Delft, NEC, NTT, IBM, MIT, UC Berkeley, SUNY, IPHT Jena ....

UC Berkeley, NIST, UCSB, U. Maryland, I. Neel Grenoble...

09-I-25

CHARGE STATES vs PHOTON STATES





$A_\parallel = \mathrm{Re}[a]$
$A_\perp = \mathrm{Im}[a]$

Hilbert spaces have different topologies

09-I-26

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09-I-5b

IN PRINCIPLE, IT IS POSSIBLE TO ACCOUNT FOR EVERY PIECE OF ENERGY AND ENTROPY IN THE WORK OF A MACHINE PROCESSING QUANTUM INFORMATION.

THERE ARE PRINCIPLES GOVERNING THE ULTIMATE PERFORMANCE OF THESE MACHINES

CAN WE SEE THESE PRINCIPLES IN ACTION IN THE PRACTICAL DESIGN OF QUANTUM CIRCUITS?

09-I-27

HOW DO WE TREAT QUANTUM-MECHANICALLY A DISSIPATIVE, NON-LINEAR, OUT-OF-EQUILIBRIUM ENGINEERED SYSTEM?

09-I-28

SELECTED BIBLIOGRAPHY

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