COLLÈGE DE FRANCE 1530

Chaire de Physique Mésoscopique
Michel Devoret
Année 2008, 13 mai - 24 juin

CIRCUITS ET SIGNAUX QUANTIQUES

QUANTUM SIGNALS AND CIRCUITS

Quatrième Leçon / *Fourth Lecture*

08-IV-1

VISIT THE WEBSITE OF THE CHAIR OF MESOSCOPIC PHYSICS

http://www.college-de-france.fr

and follow links to:

http://www.physinfo.fr/lectures.html

PDF FILES OF ALL LECTURES WILL BE POSTED ON THIS WEBSITE

Questions, comments and corrections are welcome!

08-IV-2

CALENDAR OF SEMINARS

May 13: Denis Vion, (Quantronics group, SPEC-CEA Saclay)
Continuous dispersive quantum measurement of an electrical circuit

May 20: Bertrand Reulet (LPS Orsay)
Current fluctuations : beyond noise

June 3: Gilles Montambaux (LPS Orsay)
Quantum interferences in disordered systems

June 10: Patrice Roche (SPEC-CEA Saclay)
Determination of the coherence length in the Integer Quantum Hall regime

June 17: Olivier Buisson, (CRTBT-Grenoble)
A quantum circuit with several energy levels

June 24: Jérôme Lesueur (ESPCI)
High Tc Josephson Nanojunctions: Physics and Applications

08-IV-3

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction and overview

Lecture II: Modes of a circuit and propagation of signals

Lecture III: The "atoms" of signal

Lecture IV: Scattering vs hamiltonian description of circuits

Lecture V: Non-linear circuit elements: length and energy scales of superconductivity

Lecture VI: Amplifying quantum signals with dispersive circuits

08-IV-4

LECTURE IV : SCATTERING vs HAMILTONIAN DESCRIPTION OF CIRCUITS

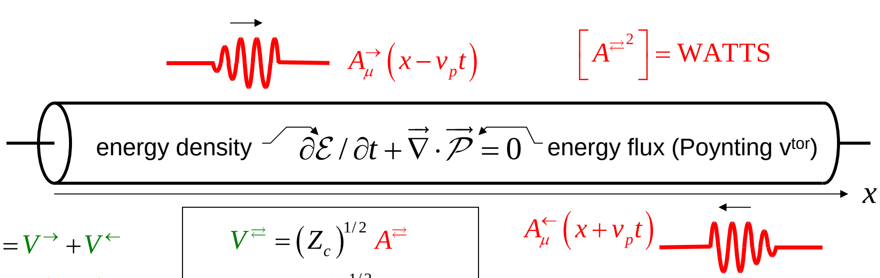
OUTLINE

1. Review of previous lecture, purpose of this lecture
2. Link between electrical quantities and photon operators
3. Quantum fluctuation-dissipation theorem
4. Fluctuations of damped harmonic oscillator

08-IV-5

PROPAGATING WAVE AMPLITUDES (CLASSICAL)

$A_\mu^{\rightarrow}(x - v_p t)$ $\qquad$ $\left[A^{\rightleftarrows 2}\right] = \text{WATTS}$



$A_\mu^{\leftarrow}(x + v_p t)$

$V = V^{\rightarrow} + V^{\leftarrow}$

$I = I^{\rightarrow} - I^{\leftarrow}$

$$V^{\rightleftarrows} = (Z_c)^{1/2} A^{\rightleftarrows}$$

$$I^{\rightleftarrows} = (Z_c)^{-1/2} A^{\rightleftarrows}$$

$$\mathcal{P}(x,t) = \left|A^{\rightarrow}(x,t)\right|^2 - \left|A^{\leftarrow}(x,t)\right|^2$$

$$\frac{\partial}{\partial x} A^{\rightleftarrows} = \mp \frac{1}{v_p}\frac{\partial}{\partial t} A^{\rightleftarrows}$$

solution:

$$A^{\rightleftarrows}(x,t) = A_0^{\rightleftarrows}(x \mp v_p t)$$

$$Z_c = \sqrt{\frac{L_\ell}{C_\ell}} = \frac{V^{\rightarrow}}{I^{\rightarrow}} = \frac{V^{\leftarrow}}{I^{\leftarrow}}$$

characteristic impedance

$$v_p = \sqrt{\frac{1}{L_\ell C_\ell}}$$

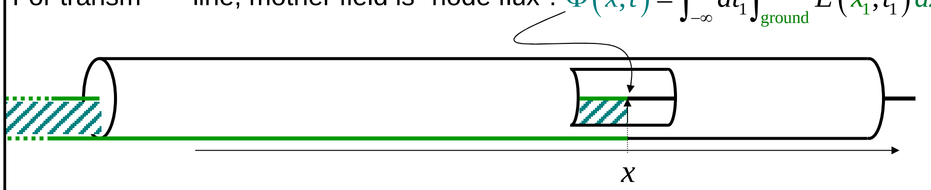
propagation velocity

LEARNING TO QUANTIZE THIS DISTRIBUTED ELEMENT SYSTEM

08-IV-6

QUANTIZING CONTINUOUS NODE FLUX OF LINE

For transmission line, mother field is "node flux": $\Phi(x,t) = \int_{-\infty}^{t} dt_1 \int_{\text{ground}}^{x} \vec{E}(x_1,t_1)\, dx_1$



Commutation relations:

$$\left[\hat{\Phi}(x_1),\hat{\Pi}(x_2)\right] = i\hbar\delta(x_1 - x_2)$$

$$\left[\hat{\Phi}(x_1),\hat{\Phi}(x_2)\right] = \left[\hat{\Pi}(x_1),\hat{\Pi}(x_2)\right] = 0$$

Fourier kingdom: $\hat{\Phi}[k] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dx\, \hat{\Phi}(x)\, e^{-ikx}$ $\quad \hat{\Pi}[k] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dx\, \hat{\Pi}(x)\, e^{-ikx}$

straight brackets

Ladder operators: $\hat{a}[k] = \frac{1}{\sqrt{2\hbar}}\left(\sqrt{\omega(|k|)C_\ell}\,\hat{\Phi}[k] + \frac{i}{\sqrt{\omega(|k|)C_\ell}}\hat{\Pi}[k]\right)$

08-IV-7

FIELD LADDER OPERATORS IN FREQ^UENCY^ DOMAIN

Introduce new notation extending frequencies to negative values, for both pos^itive^ and neg^ative^ k:

$$\hat{a}^{\rightarrow}[-\omega_0] = \hat{a}^{\dagger}\left[+\omega(k_0 > 0)\right]$$

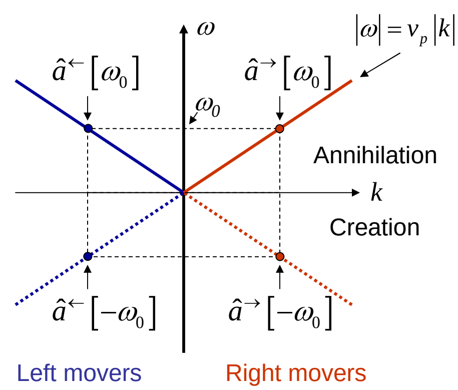
$$\hat{a}^{\leftarrow}[-\omega_0] = \hat{a}^{\dagger}\left[+\omega(-k_0 < 0)\right]$$

[Courty, Grassia and Reynaud, Europhys.Lett. 46 (1999) 31]

Makes sense since:

$$i\frac{d}{dt}\hat{a}^{\dagger}\left[|\omega|\right] = -|\omega|\hat{a}^{\dagger}\left[|\omega|\right]$$

$$i\frac{d}{dt}\hat{a}\left[|\omega|\right] = +|\omega|\hat{a}\left[|\omega|\right]$$



Commutation relations:

$$\left[\hat{a}^{\rightleftarrows}[\omega_1],\hat{a}^{\rightleftarrows}[\omega_2]\right] = \mathrm{sg}(\omega_1 - \omega_2)\,\delta(\omega_1 + \omega_2)$$

$$\left[\hat{a}^{\rightarrow}[\omega_1],\hat{a}^{\leftarrow}[\omega_2]\right] = \left[\hat{a}^{\leftarrow}[\omega_1],\hat{a}^{\rightarrow}[\omega_2]\right] = 0$$

Hamiltonian:

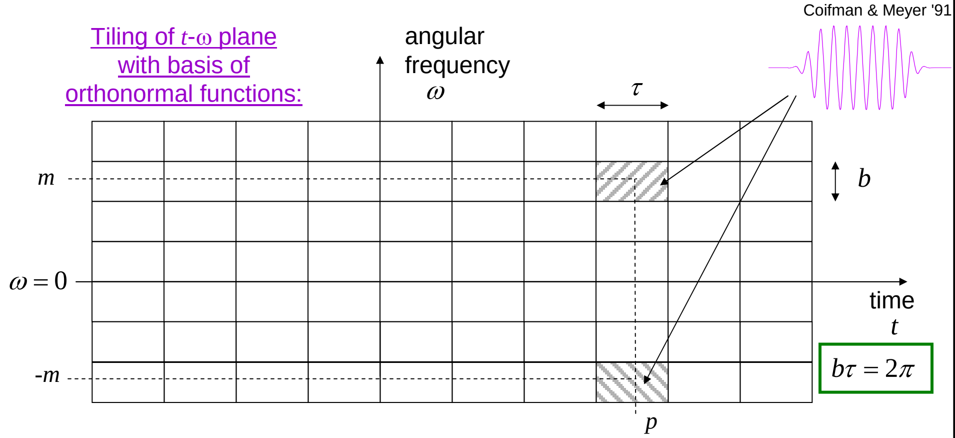
$$\hat{H} = \int_0^{\infty} \hbar\omega\, d\omega \left\{\hat{a}^{\leftarrow}[-\omega]\hat{a}^{\leftarrow}[\omega] + \hat{a}^{\rightarrow}[-\omega]\hat{a}^{\rightarrow}[\omega]\right\} \qquad \hat{H}|\text{vac}\rangle = 0$$

08-IV-8

INTRODUCING THE RF PHOTON

local cosine basis
Coifman & Meyer '91

Tiling of t-ω plane
with basis of
orthonormal functions:



The non-singular representation of photons:

$$\hat{a}_{mp} = \int_{-\infty}^{+\infty} d\omega\, \psi^*_{mp}[\omega]\, \hat{a}[\omega] \qquad \hat{a}^\dagger_{mp} = \hat{a}_{-mp}$$

THE "AMPLITUDE" OF THE CONTENT OF THE BOX IS THE PHOTON OPERATOR

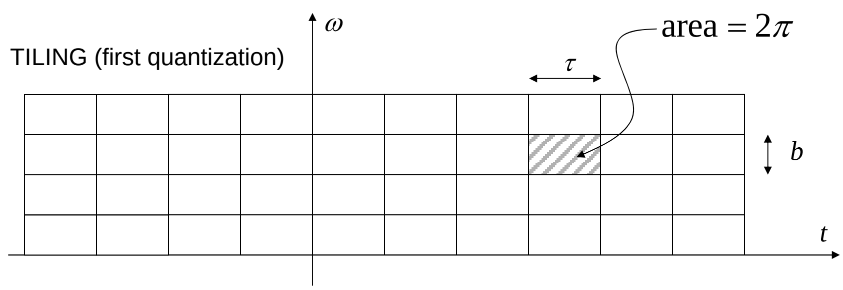
$$\left[\hat{a}_{m_1 p_1}, \hat{a}_{m_2 p_2}\right] = \mathrm{sg}(m_1 - m_2)\, \delta_{m_1+m_2}\, \delta_{p_1-p_2}$$

where $m_1, m_2, p_1, p_2 \in \mathbb{Z}$

08-IV-9

CLASSICS: STEP TILE vs LOCALIZATION BOX

TILING (first quantization)

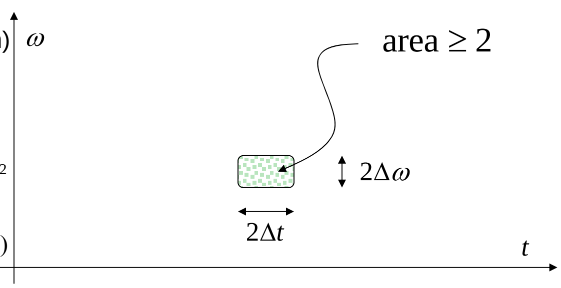


LOCALIZATION (first quantization)

$$\Delta t = \left[\int_{-\infty}^{+\infty} (t-\bar{t})^2 \left|\psi_\mu(t)\right|^2 dt\right]^{1/2}$$

$$\Delta\omega = \left[\int_{-\infty}^{+\infty} (\omega-\bar{\omega})^2 \left|\psi_\mu[\omega]\right|^2 d\omega\right]^{1/2}$$

$\Delta t \Delta\omega \geq 1/2$ $\qquad (\mu = \{m,p\})$



MAXIMAL LOCALIZATION → OVERCOMPLETENESS

08-IV-10

CLASSICAL SPECTROGRAM OF SIGNAL

SPECTRUM ANALYZER MEASURES ENERGY PER MODE

$$\langle A[\omega_1]A[\omega_2]\rangle = S_{AA}[\omega_1]\delta[\omega_1+\omega_2]$$ FORMAL!

$$A_{mp} = \int_{-\infty}^{+\infty} dt\, \psi_{mp}(t)\, A(t) = \int_{-\infty}^{+\infty} d\omega\, \psi^*_{mp}[\omega]\, A[\omega]$$

$$S_{AA}[mb=\omega](\delta\omega=b) = \langle A_{-mp}A_{mp}\rangle(\delta\omega=b)$$ Physicist [W.s]

$$\mathcal{S}_{AA}(|m|B=\nu)(\delta\nu=B) = \langle A_{-mp}A_{mp} + A_{mp}A_{-mp}\rangle(\delta\nu=B)$$

$$B = b/2\pi$$ Electrical engineer [W/Hz]

Resolution bandwidth (Hz)

08-IV-11

08-IV-12

ONE MAIN QUESTION WE WOULD LIKE TO ANSWER:

HOW DO WE GO BACK AND FORTH
BETWEEN ELECTRICAL QUANTITIES
AND FIELD MODES PHOTON OPERATORS?

$$V = V^{\rightarrow} + V^{\leftarrow} \qquad I = I^{\rightarrow} - I^{\leftarrow}$$

$\leftrightarrow$

$$V^{\rightleftarrows} = (Z_c)^{1/2} A^{\rightleftarrows} \qquad I^{\rightleftarrows} = (Z_c)^{-1/2} A^{\rightleftarrows}$$

$\leftrightarrow$

$$A^{\rightleftarrows}(t \mp x/v_p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\omega\, A^{\rightleftarrows}[\omega]\, e^{-i\omega(t \mp x/v_p)}$$

?

$$\hat{a}_\mu = \int_{-\infty}^{+\infty} d\omega\, \psi^*_{mp}[\omega]\, \hat{a}^{\rightleftarrows}[\omega]$$

$\leftrightarrow$

$$\left[\hat{a}^{\rightleftarrows}[\omega_1], \hat{a}^{\rightleftarrows}[\omega_2]\right] = \mathrm{sg}(\omega_1-\omega_2)\,\delta(\omega_1+\omega_2)$$

$\left[\hat{A}[\omega]\right]$ = [power $^{1/2}$ × time] = [action]$^{1/2}$

$\left[\hat{a}[\omega]\right]$ = [time]$^{1/2}$

IMPLEMENTING SECOND QUANTIZATION

ANSWER:

$$\hat{A}^{\rightleftarrows}[\omega] = \sqrt{\frac{\hbar|\omega|}{2}}\, \hat{a}^{\rightleftarrows}[\omega]$$

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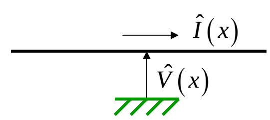
08-IV-5b

BACK TO VOLTAGES AND CURRENTS

Want expressions for physical variables:



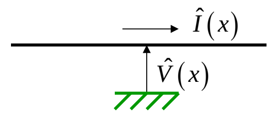
Flux:

$$\hat{\Phi}(x,t) = \sqrt{\frac{\hbar Z_c}{4\pi}} \int_{-\infty}^{+\infty} \frac{d\omega}{\sqrt{|\omega|}} \left\{ \hat{a}^{\rightarrow}[\omega] e^{-i\omega(t-x/c)} + \hat{a}^{\leftarrow}[\omega] e^{-i\omega(t+x/c)} \right\}$$

08-IV-13

BACK TO VOLTAGES AND CURRENTS

Want expressions for physical observables:



Flux:

$$\hat{\Phi}(x,t)=\sqrt{\frac{\hbar Z_c}{4\pi}}\int_{-\infty}^{+\infty}\frac{d\omega}{\sqrt{|\omega|}}\left\{\hat{a}^{\rightarrow}[\omega]e^{-i\omega(t-x/c)}+\hat{a}^{\leftarrow}[\omega]e^{-i\omega(t+x/c)}\right\}$$

Voltage:

$$\hat{V}(x,t)=\hat{V}^{\rightarrow}(x,t)+\hat{V}^{\leftarrow}(x,t)$$

Current:

$$\hat{I}(x,t)=\hat{I}^{\rightarrow}(x,t)-\hat{I}^{\leftarrow}(x,t)$$

$$\hat{V}^{\rightleftarrows}(x,t)=\sqrt{\frac{\hbar Z_c}{4\pi}}\int_{-\infty}^{+\infty}\frac{d\omega}{\sqrt{|\omega|}}\,i\omega\hat{a}^{\rightleftarrows}[\omega]e^{-i\omega(t\mp x/c)}$$

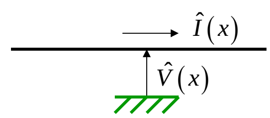
$$\hat{I}^{\rightleftarrows}(x,t)=\sqrt{\frac{\hbar}{4\pi Z_c}}\int_{-\infty}^{+\infty}\frac{d\omega}{\sqrt{|\omega|}}\,i\omega\hat{a}^{\rightleftarrows}[\omega]e^{-i\omega(t\mp x/c)}$$

08-IV-13a

08-IV-13b

BACK TO VOLTAGES AND CURRENTS

Want expressions for physical variables:



Flux:

$$\hat{\Phi}(x,t)=\sqrt{\frac{\hbar Z_c}{4\pi}}\int_{-\infty}^{+\infty}\frac{d\omega}{\sqrt{|\omega|}}\left\{\hat{a}^{\rightarrow}[\omega]e^{-i\omega(t-x/c)}+\hat{a}^{\leftarrow}[\omega]e^{-i\omega(t+x/c)}\right\}$$

Voltage:

$$\hat{V}(x,t)=\hat{V}^{\rightarrow}(x,t)+\hat{V}^{\leftarrow}(x,t)$$

$$\hat{V}^{\rightleftarrows}(x,t)=\sqrt{\frac{\hbar Z_c}{4\pi}}\int_{-\infty}^{+\infty}\frac{d\omega}{\sqrt{|\omega|}}\,i\omega\hat{a}^{\rightleftarrows}[\omega]e^{-i\omega(t\mp x/c)}$$

Current:

$$\hat{I}(x,t)=\hat{I}^{\rightarrow}(x,t)-\hat{I}^{\leftarrow}(x,t)$$

$$\hat{I}^{\rightleftarrows}(x,t)=\sqrt{\frac{\hbar}{4\pi Z_c}}\int_{-\infty}^{+\infty}\frac{d\omega}{\sqrt{|\omega|}}\,i\omega\hat{a}^{\rightleftarrows}[\omega]e^{-i\omega(t\mp x/c)}$$

Characteristic scales for $Z_c \sim 50\Omega$:

$$\sqrt{\hbar Z_c}\sim 0.036\Phi_0 \qquad \sqrt{\frac{\hbar}{Z_c}}\sim 9e$$

WAVE AMPLITUDE vs PHOTON OPERATORS

propagation direction

angular frequency (positive or negative)

$$\hat{A}^{\rightleftarrows}[\omega] = \sqrt{\frac{\hbar|\omega|}{2}}\,\hat{a}^{\rightleftarrows}[\omega]$$

operator of wave amplitude in Fourier domain

field ladder operator

Total energy transported by line: $\int_{-\infty}^{+\infty} d\omega \hat{A}^{\rightleftarrows}[-\omega]\hat{A}^{\rightleftarrows}[\omega] = \int_{-\infty}^{+\infty} dt \left|\hat{A}^{\rightleftarrows}(t)\right|^2$ (Parseval)

Mode amplitude:

$$\hat{A}_{\mu} = \int_{-\infty}^{+\infty} d\omega \sqrt{\frac{\hbar|\omega|}{2}}\,\psi^{*}_{mp}[\omega]\,\hat{a}^{\rightleftarrows}[\omega]$$

$$\bar{\omega}_m = \int_{-\infty}^{+\infty} d\omega\, \omega \left|\psi_{mp}[\omega]\right|^2$$

$$\left[\hat{A}_{m_1 p_1}, \hat{A}_{m_2 p_2}\right] = \frac{\hbar\bar{\omega}_{m_1}}{2}\delta_{m_1+m_2}\delta_{p_1-p_2}$$

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QUANTUM STATE OF THE LINE

Each mode $\mu = \{(m,p),(-m,p)\}$ can be excited with an arbitrary number of photons

We obtain the general photon state:

$$|n_1, n_2, ..., n_\mu, ...\rangle = \prod_\mu a_{-\mu}^{n_\mu} |vac\rangle$$

$$n_\mu = a_\mu^\dagger a_\mu$$

Most general pure state of the line:

$$|\Psi\rangle = c_1 |n_1^1, n_2^1, ..., n_\mu^1, ...\rangle + c_2 |n_1^2, n_2^2, ..., n_\mu^2, ...\rangle + ... + c_\Omega |n_1^\Omega, n_2^\Omega, ..., n_\mu^\Omega, ...\rangle$$

$$|\Psi\rangle = \sum_h c_h \prod_\mu a_{-\mu}^{n(h,\mu)} |vac\rangle$$

Contains all possible forms of entanglement, correlations, etc...

08-IV-15

ANTICOMMUTATOR FOR PURE PHOTON STATE

$$\left\langle \left\{ \hat{A}_{m_1 p_1}, \hat{A}_{m_2 p_2} \right\} \right\rangle = \hbar\bar{\omega}_{m_1} \left(n_{m_1 p_1} + \frac{1}{2} \right) \delta_{m_1 + m_2} \delta_{p_1 - p_2}$$

ANTICOMMUTATOR FOR THERMAL STATE

$$\rho = \frac{\exp\left[-\frac{1}{k_B T} \sum_\mu \hbar\omega_\mu n_\mu \right]}{\mathrm{Tr}\exp\left[-\frac{1}{k_B T} \sum_\mu \hbar\omega_\mu n_\mu \right]} \qquad \langle n \rangle_T + \frac{1}{2} = \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} + \frac{1}{2} = \frac{1}{2}\coth\frac{\hbar\omega}{2k_B T}$$

$$\left\langle \left\{ \hat{A}_{m_1 p_1}, \hat{A}_{m_2 p_2} \right\} \right\rangle_T = \frac{\hbar\bar{\omega}_{m_1}}{2} \coth\frac{\hbar\bar{\omega}_{m_1}}{2k_B T} \delta_{m_1 + m_2} \delta_{p_1 - p_2}$$

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OUTLINE

1. Review of previous lecture, purpose of this lecture
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08-IV-5c

QUANTUM FLUCTUATION-DISSIPATION THEOREM

$$\frac{1}{2}\left\langle\left\{a[\omega_1],a[\omega_2]\right\}\right\rangle_T = N_T\left(|\omega_1|\right)\delta\left(\omega_1+\omega_2\right)$$

$$N_T\left(|\omega|\right)=\frac{1}{2}\coth\left(\frac{\hbar|\omega|}{2k_BT}\right)$$

$$N_T\left(|\omega|\right)\xrightarrow[T\to\infty]{}\frac{k_BT}{\hbar|\omega|}$$

$$\left\langle\left\{\hat{V}^{\rightleftharpoons}(t_1),\hat{V}^{\rightleftharpoons}(t_2)\right\}\right\rangle = -\left\langle\frac{\hbar Z_c}{4\pi}\int_{-\infty}^{+\infty}d\omega_1\frac{\omega_1}{\sqrt{|\omega_1|}}\mathrm{e}^{-i\omega_1 t_1}\int_{-\infty}^{+\infty}d\omega_2\frac{\omega_2}{\sqrt{|\omega_2|}}\mathrm{e}^{-i\omega_2 t_2}\left\{\hat{a}^{\rightleftharpoons}[\omega_1],\hat{a}^{\rightleftharpoons}[\omega_2]\right\}\right\rangle$$

$$=\frac{\hbar Z_c}{4\pi}\int_{-\infty}^{+\infty}d\omega\;\omega\mathrm{e}^{-i\omega(t_1-t_2)}\coth\left(\frac{\hbar\omega}{2k_BT}\right)$$

$$\left\langle\hat{V}^{\rightleftharpoons}[\omega_1]\hat{V}^{\rightleftharpoons}[\omega_2]\right\rangle = S_{VV}^{\rightleftharpoons}[\omega_1]\delta(\omega_1+\omega_2)$$

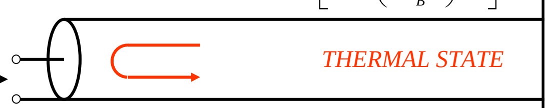
$$S_{VV}^{\rightleftharpoons}[\omega]=\frac{Z_c}{4}\hbar\omega\left[\coth\left(\frac{\hbar\omega}{2k_BT}\right)+1\right]$$

QUANTUM FLUCTUATION-DISSIPATION THEOREM (CNTD)

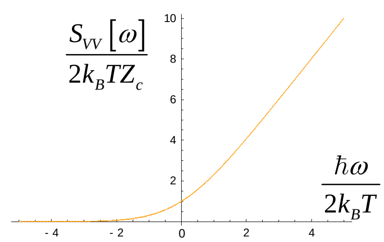
$$\left\langle\hat{V}^{\rightleftharpoons}[\omega_1]\hat{V}^{\rightleftharpoons}[\omega_2]\right\rangle = S_{VV}^{\rightleftharpoons}[\omega_1]\delta(\omega_1+\omega_2)$$

$$S_{VV}^{\rightleftharpoons}[\omega]=\frac{Z_c}{4}\hbar\omega\left[\coth\left(\frac{\hbar\omega}{2k_BT}\right)+1\right]$$

$$S_{VV}[\omega]=Z_c\hbar\omega\left[\coth\left(\frac{\hbar\omega}{2k_BT}\right)+1\right]$$



$$S_{VV}[\omega]=4S_{VV}^{\rightleftharpoons}[\omega]$$



ω > 0 : emission

ω < 0 : absorption

Recover results of Johnson-Nyquist noise:

Voltage fluctuations in classical regime:

$$\mathcal{S}_{VV}[\nu]=4k_BTR$$

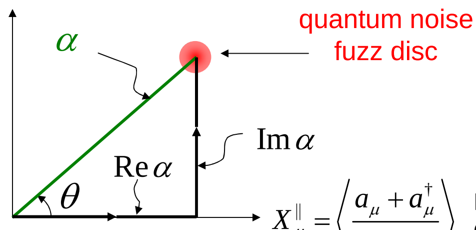
Power flow in classical regime:

$$P=k_BTB$$

SEMI-CLASSICAL (GLAUBER) SIGNAL MODE

$$X_\mu^\perp = \left\langle \frac{a_\mu - a_\mu^\dagger}{2i} \right\rangle$$

QUADRATURE COMPNENT



$$X_\mu^\parallel = \left\langle \frac{a_\mu + a_\mu^\dagger}{2} \right\rangle$$

IN-PHASE COMPNENT

α = signal mode complex amplitude

$|\alpha|^2$ = signal mode mean energy in photon number

θ = signal mode mean phase

$$a_\mu |\alpha\rangle = \alpha |\alpha\rangle$$

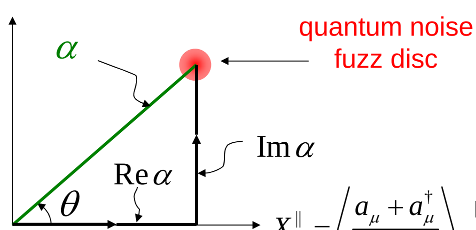
$$|\alpha\rangle = e^{\alpha a_\mu^\dagger - \alpha^* a_\mu} |0\rangle$$

08-IV-19

SEMI-CLASSICAL (GLAUBER) SIGNAL MODE

$$X_\mu^\perp = \left\langle \frac{a_\mu - a_\mu^\dagger}{2i} \right\rangle$$

QUADRATURE COMPNENT



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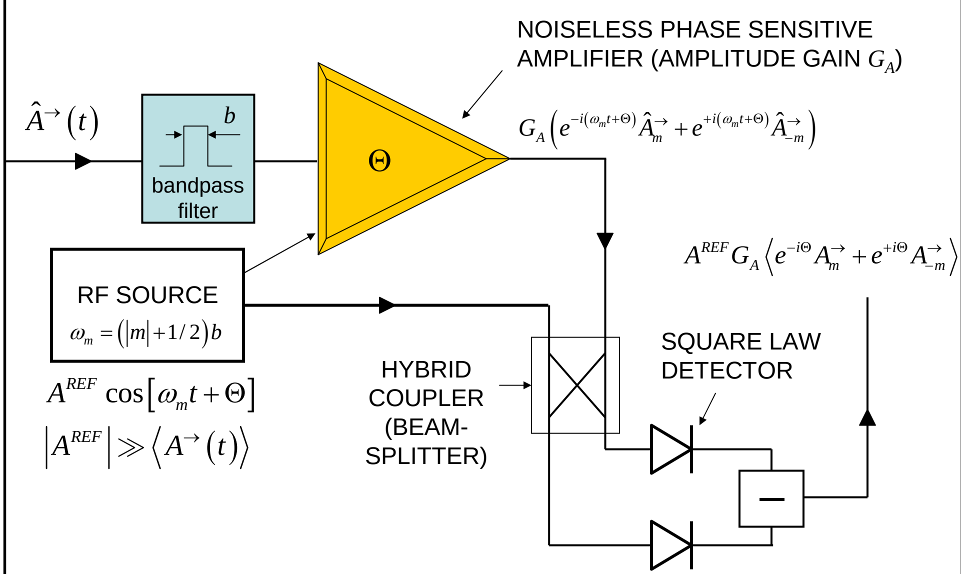
$$a_\mu |\alpha\rangle = \alpha |\alpha\rangle$$

$$|\alpha\rangle = e^{\alpha a_\mu^\dagger - \alpha^* a_\mu} |0\rangle$$

Fresnel vector ⇒ Fresnel "lollypop"

08-IV-19a

MEASURING MODE WAVE AMPLITUDE WITH HOMODYNE DETECTION



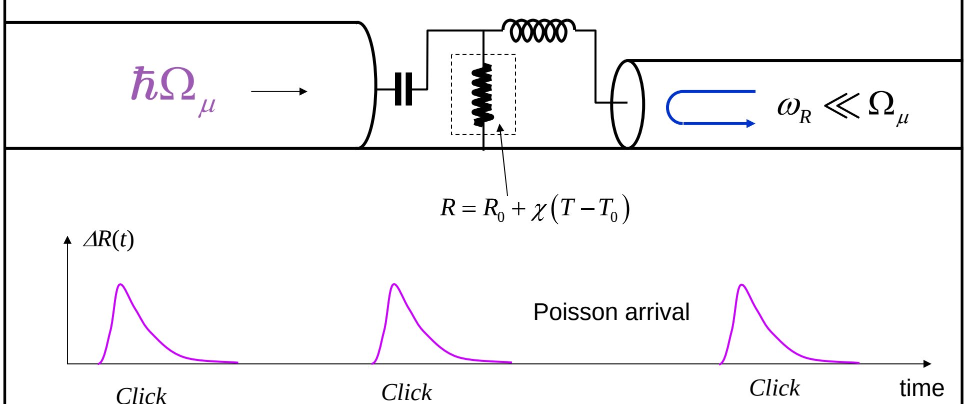
See M. A. Castellanos-Beltran & K. W. Lehnert, arXiv:0706.2373v1 (2007)

08-IV-20

RF PHOTOMULTIPLIER

Courtesy of D. Prober (2008)

Can we measure $n_\mu = a_\mu^\dagger a_\mu$?



Energy of resistor monitored much more quickly than coherence time of weak incoming signal

WORK IN PROGRESS

08-IV-21

OUTLINE

1. Review of previous lecture, purpose of this lecture
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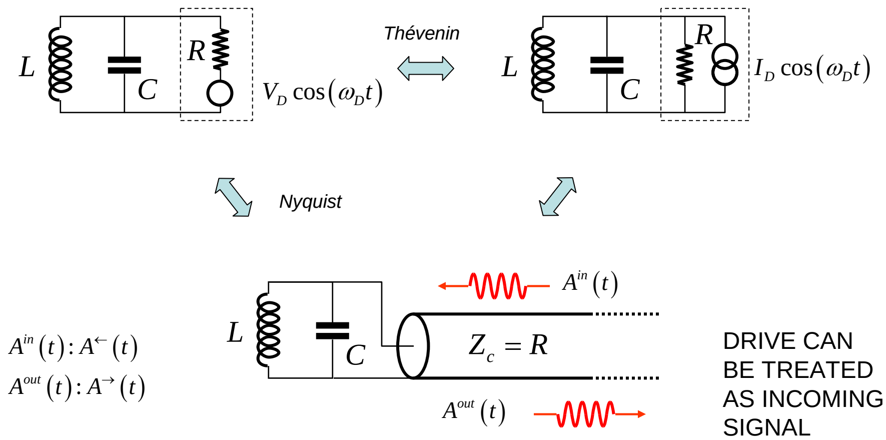
08-IV-5d

SCATTERING APPROACH TO DRIVEN DISSIPATIVE CIRCUITS

MAIN IDEA: RESISTANCE IS EQUIVALENT TO SEMI-INFINITE TRANSMISSION LINE

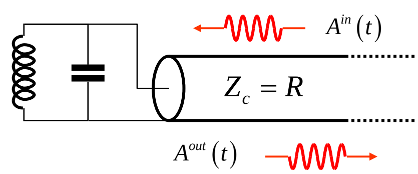


DRIVE CAN BE TREATED AS INCOMING SIGNAL

08-IV-22

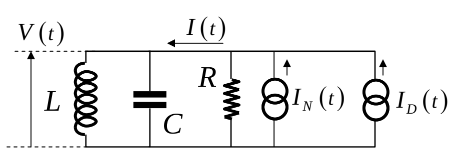
A KEY POINT

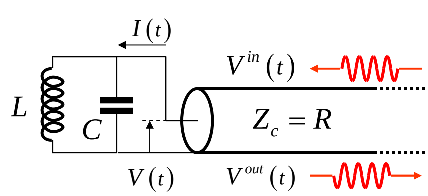
UNLIKE $A^{\leftarrow}(t)$ AND $A^{\rightarrow}(t)$, $A^{in}(t)$ AND $A^{out}(t)$ ARE NOT INDEPENDENT



TERMINATING CIRCUIT IMPOSES A STRICT RELATIONSHIP BETWEEN INCOMING AND OUTGOING WAVES

RESISTANCE-TRANSMISSION LINE EQUIVALENCE





$$\begin{cases} V(t) = \dot{\Phi}(t) = R\left(I_D(t) + I_N(t) - I(t)\right) \\ C\ddot{\Phi} + \dfrac{\dot{\Phi}}{R} + \dfrac{\Phi}{L} = I_D(t) + I_N(t) \end{cases}$$

$$\ddot{\Phi} + \frac{\dot{\Phi}}{RC} + \frac{\Phi}{LC} = \frac{I_D(t) + I_N(t)}{C}$$

$$\boxed{V(t) + Z_c I(t) = 2V^{in}(t)}$$

$$\begin{cases} V(t) = V^{in}(t) + V^{out}(t) \\ V^{in/out}(t) = Z_c^{1/2}\, A^{in/out}(t) \\ I(t) = I^{in}(t) - I^{out}(t) \\ I^{in/out}(t) = Z_c^{-1/2}\, A^{in/out}(t) \end{cases}$$

$$\boxed{\ddot{\Phi} + 2\Gamma\dot{\Phi} + \omega_0^2 \Phi = 4\Gamma V^{in}(t)}$$

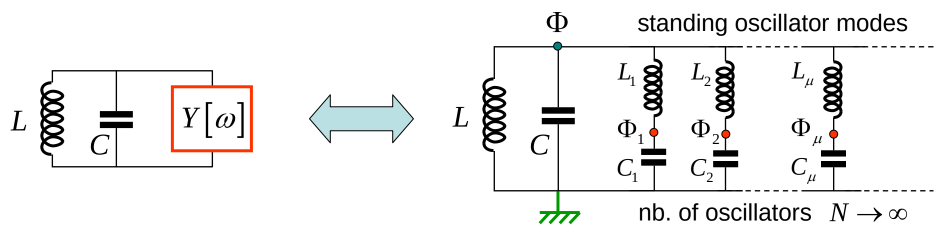
$$\boxed{V^{out}(t) = \dot{\Phi} - V^{in}(t)}$$

$\omega_0 = \dfrac{1}{\sqrt{LC}}$ resonance frequency

$\Gamma = \dfrac{1}{2RC}$ amplitude damping rate (HWHM)

$\dfrac{\omega_0}{2\Gamma} = Q \gg 1$ quality factor

ANOTHER POINT OF VIEW: HAMILTONIAN APPROACH OF CALDEIRA AND LEGGETT



$$\hat{H} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C} + \sum_{\mu=1}^{N} \left[\frac{\hat{Q}_\mu}{2C_\mu} + \frac{\left(\hat{\Phi}_\mu - \hat{\Phi}\right)^2}{2L_\mu} \right]$$

$$\mathrm{Re}\left[Y\left[\omega_\mu\right]\right] = \sqrt{\frac{C_\mu}{L_\mu}} \qquad \omega_\mu = \sqrt{\frac{1}{L_\mu C_\mu}}$$

$$Z_{tot}[\omega] = \frac{1}{\frac{1}{jL\omega} + jC\omega + Y[\omega]} \qquad (j = -i)$$

$$\left\langle \hat{\Phi}^2 \right\rangle = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega \frac{\hbar Z_T[\omega]}{2\omega} \coth\left(\frac{\hbar\omega}{2k_B T}\right)$$

$$\left\langle \hat{Q}^2 \right\rangle = \frac{C^2}{2\pi} \int_{-\infty}^{+\infty} d\omega \frac{\hbar\omega Z_T[\omega]}{2} \coth\left(\frac{\hbar\omega}{2k_B T}\right)$$

08-IV-29

NEXT LECTURE:

1) NON-LINEARITY INTRODUCED IN CIRCUITS BY JOSEPHSON TUNNELING

2) ACTIVE QUANTUM CIRCUITS

08-IV-30