COLLÈGE DE FRANCE
1530

Chaire de Physique Mésoscopique
Michel Devoret
Année 2008, 13 mai - 24 juin

# CIRCUITS ET SIGNAUX QUANTIQUES

# *QUANTUM SIGNALS AND CIRCUITS*

Première leçon / *First Lecture*



08-I-1

## VISIT THE WEBSITE OF THE CHAIR OF MESOSCOPIC PHYSICS

http://www.college-de-france.fr

and follow links to:

http://www.physinfo.fr/lectures.html

PDF FILES OF ALL LECTURES WILL BE POSTED ON THIS WEBSITE

Questions, comments and corrections are welcome!

08-I-2

## CALENDAR OF SEMINARS

**May 13: Denis Vion, (Quantronics group, SPEC-CEA Saclay)**
Continuous dispersive quantum measurement of an electrical circuit

**May 20: Bertrand Reulet (LPS Orsay)**
Current fluctuations : beyond noise

**June 3: Gilles Montambaux (LPS Orsay)**
Quantum interferences in disordered systems

**June 10: Patrice Roche (SPEC-CEA Saclay)**
Determination of the coherence length in the Integer Quantum Hall Regime

**June 17: Olivier Buisson, (CRTBT-Grenoble)**
A quantum circuit with several energy levels

**June 24: Jérôme Lesueur (ESPCI)**
High Tc Josephson Nanojunctions: Physics and Applications

NOTE THAT THERE IS NO LECTURE AND NO SEMINAR ON MAY 27 !

08-I-3

## PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction and overview

Lecture II: Modes of a circuit and propagation of signals

Lecture III: The "atoms" of signal

Lecture IV: Quantum fluctuations in transmission lines

Lecture V: Introduction to non-linear active circuits

Lecture VI: Amplifying quantum signals with dispersive circuits

**NEXT YEAR: STRONGLY NON-LINEAR AND/OR DISSIPATIVE CIRCUITS**

08-I-4

# LECTURE I : INTRODUCTION AND OVERVIEW

1. Review of classical radio-frequency circuits
2. Quantum information processing
3. Quantum-mechanical LC oscillator
4. A non-dissipative, non-linear element: the Josephson junction
5. Energy levels and transitions of the Cooper Pair Box
6. Summary of questions addressed by this course

08-I-5

# CLASSICAL RADIO-FREQUENCY CIRCUITS

Communications

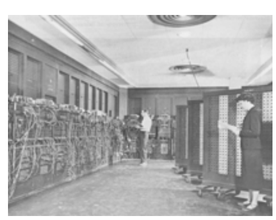
Hergé, Moulinsart

→

1940's : MHz

2000's : GHz

Computation



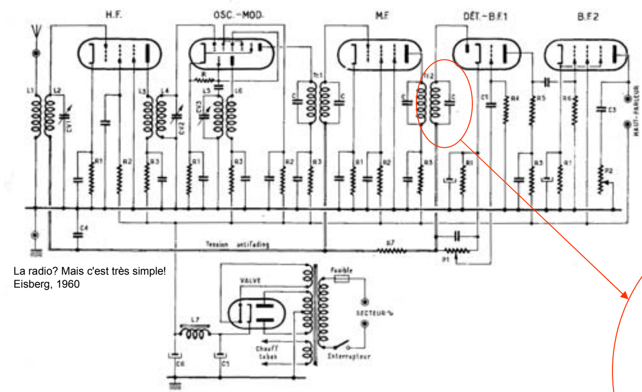
US Army Photo

→

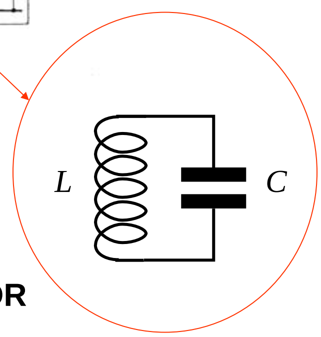
Sony

08-I-6

# SIMPLEST EXAMPLE :




La radio? Mais c'est très simple!
Eisberg, 1960

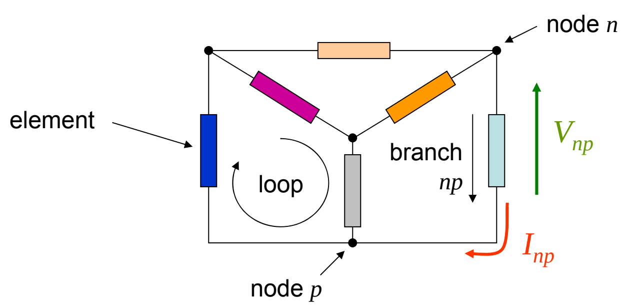



**THE LC OSCILLATOR**

08-I-7

# WHAT IS A RADIO-FREQUENCY CIRCUIT?

A RF CIRCUIT IS A NETWORK OF ELECTRICAL ELEMENTS




TWO DYNAMICAL VARIABLES CHARACTERIZE THE STATE OF EACH DIPOLE ELEMENT AT EVERY INSTANT:

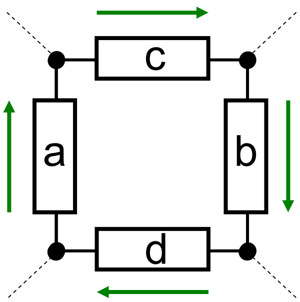
Voltage across the element: $V_{np}(t) = \int_n^p \vec{E} \cdot \overrightarrow{d\ell}$

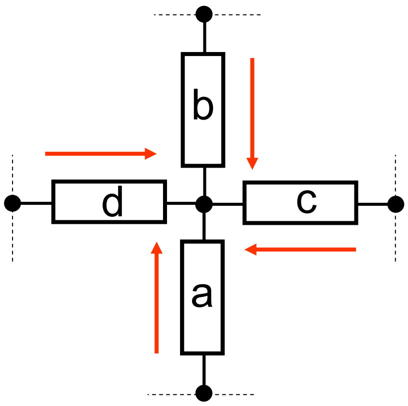
Current through the element: $I_{np}(t) = \iint \vec{j} \cdot \overrightarrow{d\sigma}_{np}$

Signals: any linear combination of these variables

08-I-8

# KIRCHHOFF'S LAWS




$$\sum_{\text{branches } \lambda \text{ around loop}} V_\lambda = 0$$

branches $\lambda$
around loop

$$\sum_{\text{branches } \nu \text{ tied to node}} I_\nu = 0$$

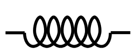
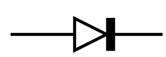
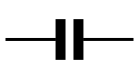
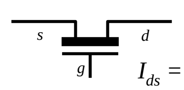
branches $\nu$
tied to node

08-I-9

# CONSTITUTIVE RELATIONS

EACH ELEMENT IS TAKEN FROM A FINITE SET OF ELEMENT TYPES

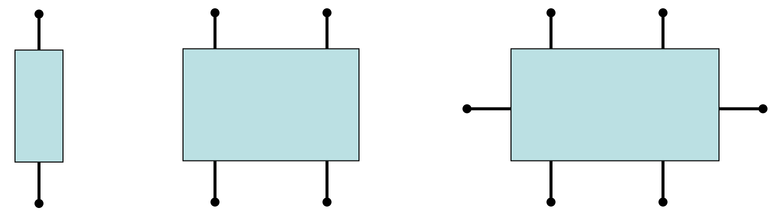
EACH ELEMENT TYPE IS CHARACTERIZED BY A RELATION BETWEEN VOLTAGE AND CURRENT

| LINEAR | | | NON-LINEAR | | |
|---|---|---|---|---|---|
| inductance: |  | $V = L\, dI/dt$ | diode: |  | $I = G(V)$ |
| capacitance: |  | $I = C\, dV/dt$ | transistor: |  | $I_{ds} = G(I_{gs}, V_{ds})\, V_{ds}$ <br> $I_{gs} = 0$ |
| resistance: |  | $V = R\, I$ | | | |
| ⋮ | | | ⋮ | | |

08-I-10

# GENERALIZATIONS: MULTIPOLES, PORTS

AN ELEMENT CAN BE CONNECTED TO MORE THAN TWO NODES



dipole quadrupole hexapole etc....

n poles → { n-1 currents, n-1 voltages }

ALSO, CONSTITUTIVE RELATION OF ELEMENT NEED NOT BE LOCAL IN TIME

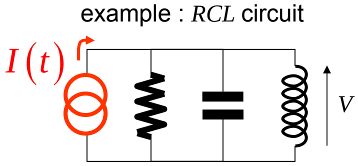
WILL DEAL WITH THESE HIGHER DEGREES OF COMPLEXITY LATER

08-I-11

# COMPUTING THE DYNAMICAL STATE OF A CIRCUIT IN CLASSICAL PHYSICS

specify circuit with sources → choose set of independent variables → constitutive relations + Kirchhoff's laws → solve for currents and voltages in circuit

example : *RCL* circuit




$\phi(t)$ : flux thru inductance

$\dot{\phi}(t)$ : voltage $V$ across inductance

current thru inductance $\phi / L$

current thru resistance $\dot{\phi} / R$

current thru capacitance $C\ddot{\phi}$

$$C\ddot{\phi} + \frac{\dot{\phi}}{R} + \frac{\phi}{L} = I(t)$$

ETC...

08-I-12a

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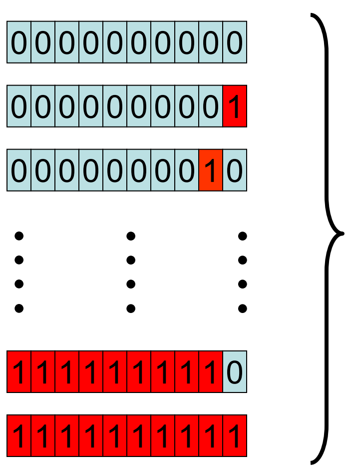
08-I-5b

## THE POWER OF QUANTUM SUPERPOSITION

**REGISTER WITH N=10 BITS:**




**$2^N$ = 1024 POSSIBLE CONFIGURATIONS**

classically, can store and work only on one number between 0 et 1023

**"quantally", can store and work on an arbitrary superposition of these numbers!**

$$|\Psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle + ..... + \alpha_{2^N-1}|2^N-1\rangle$$

08-I-13a

## QUANTUM PARALLELISM

suppose a function $f$ $\quad j \in \{0,1023\} \rightarrow n = f(j) \in \{0,1023\}$

Classically, need 1000 ×10-bit registers (10,000 bits) to store information about this function and to work on it.

Quantum-mechanically, a 20-qubit register can suffice!

$$|\Psi\rangle = \frac{1}{2^{N/2}} \sum_{j=0}^{2^N-1} |j\rangle |f(j)\rangle$$

Function encoded in a superposition of states of register

08-I-14

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08-I-5c

## A CIRCUIT BEHAVING QUANTUM-MECHANICALLY AT THE LEVEL OF CURRENTS AND VOLTAGES ?

SIMPLEST EXAMPLE: <u>SUPERCONDUCTING</u> **LC** OSCILLATOR CIRCUIT

d ~ 0.5 mm

MICROFABRICATION ➡ L ~ 3nH, C ~ 1pF, $\omega_r/2\pi$ ~ 5GHz

d << $\lambda$: LUMPED ELEMENT REGIME

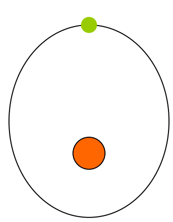
SUPERCONDUCTIVITY ➡ ONLY ONE COLLECTIVE VARIABLE

INCOMPRESSIBLE ELECTRONIC FLUID SLOSHES BETWEEN PLATES.
NO INTERNAL DEGREES OF FREEDOM.

08-I-15c

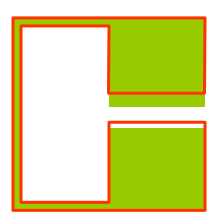
## DEGREE OF FREEDOM IN ATOM vs CIRCUIT: SEMI-CLASSICAL DESCRIPTION

Rydberg atom



Superconducting
LC oscillator

L



C

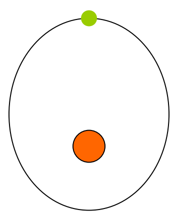
08-I-16

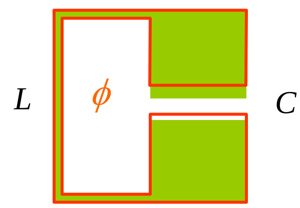
# DEGREE OF FREEDOM IN ATOM vs CIRCUIT

semiclassical picture

Rydberg atom

Superconducting LC oscillator






2 possible correspondences:

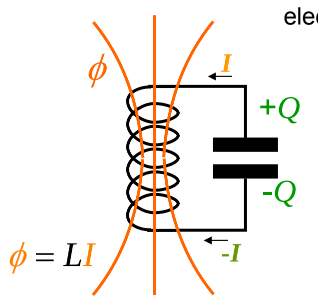
charge dynamics
- velocity of electron → current through inductor
- force on electron → voltage across capacitor

field dynamics
- velocity of electron → voltage across capacitor
- force on electron → current through inductor

08-I-16b

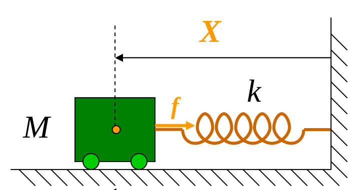
# FLUX AND CHARGE IN LC OSCILLATOR

electrical world




$\phi = LI$ $\quad Q = CV$

mechanical world




$x = -\frac{1}{k} f$ $\quad P = MV$

| | | | |
|---|---|---|---|
| position variable: | $\phi$ | ⟷ | $x$ |
| momentum variable: | $Q$ | ⟷ | $P$ |
| generalized force on mass : | $I$ | ⟷ | $f$ |
| generalized velocity: | $V$ | ⟷ | $V$ |
| generalized mass : | $C$ | ⟷ | $M$ |
| generalized spring constant: | $1/L$ | ⟷ | $k$ |

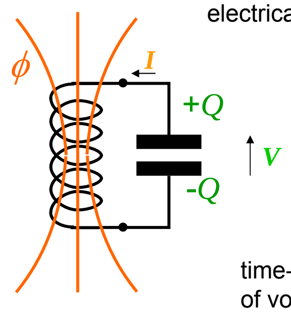
$x = X - X_0$

↑ equilibrium position of spring

08-I-17a

# HAMILTONIAN FORMALISM

electrical world



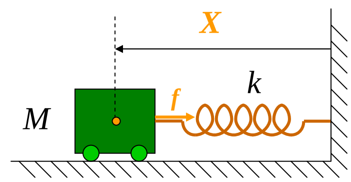

time-integral of voltage on inductor = flux in inductor

charge on capacitor

$$H(\phi, Q) = \frac{Q^2}{2C} + \frac{\phi^2}{2L}$$

conjugate variables

mechanical world




$x = X - X_0$

momentum of mass

elongation of spring

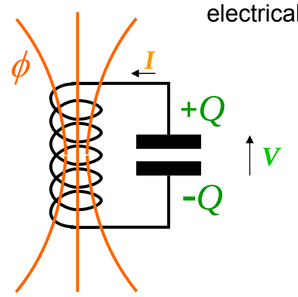
$$H(x, P) = \frac{P^2}{2M} + \frac{kx^2}{2}$$

conjugate variables

08-I-18

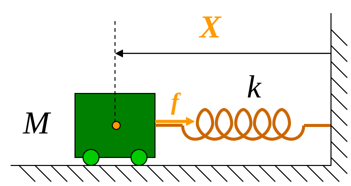
# HAMILTON'S EQUATION OF MOTION

electrical world




$$\left.\begin{aligned} \dot{\phi} &= \frac{\partial H}{\partial Q} = \frac{Q}{C} \\ \dot{Q} &= -\frac{\partial H}{\partial \phi} = -\frac{\phi}{L} \end{aligned}\right\} \Rightarrow \omega_r = \sqrt{\frac{1}{LC}}$$

mechanical world




$x = X - X_0$

$$\left.\begin{aligned} \dot{x} &= \frac{\partial H}{\partial P} = \frac{P}{M} \\ \dot{P} &= -\frac{\partial H}{\partial x} = -kx \end{aligned}\right\} \Rightarrow \omega_r = \sqrt{\frac{k}{M}}$$

08-I-19

## FROM CLASSICAL TO QUANTUM PHYSICS: CORRESPONDENCE PRINCIPLE

$X$ and $P$ are conjugate variables
$$\begin{cases} X \rightarrow \hat{X} & \text{position operator} \\ P \rightarrow \hat{P} & \text{momentum operator} \end{cases}$$

$$H\left(X,P\right) \rightarrow \hat{H}\left(\hat{X},\hat{P}\right)$$ hamiltonian operator

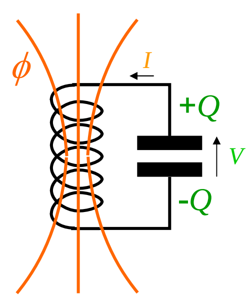
$$\frac{\partial A \partial B}{\partial X \partial P} - \frac{\partial A \partial B}{\partial P \partial X} = \left\{A,B\right\}_{PB} \rightarrow \left[\hat{A},\hat{B}\right]/i\hbar$$ commutator

Poisson bracket
$$\left\{X,P\right\}_{PB} = 1 \rightarrow \left[\hat{X},\hat{P}\right] = i\hbar$$

position and momentum operators do not commute

08-I-20

## FLUX AND CHARGE DO NOT COMMUTE




$$\left[\hat{\phi},\hat{Q}\right] = i\hbar$$

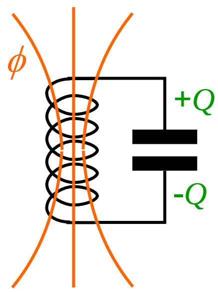
This fundamental result arises from the correspondence principle applied to the LC oscillator

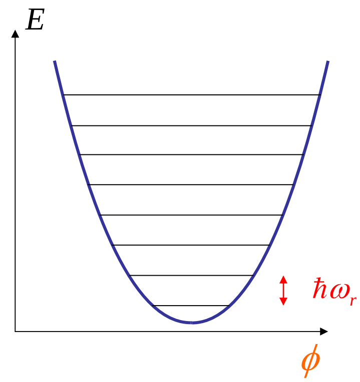
Not every pairs of variables can satisfy such commutation relation. As *A.J. Leggett* once remarked in a seminar presenting his seminal work on macroscopic quantum mechanics: " We cannot quantize the equations of the stock market!"

We can also obtain this result from quantum field theory through the commutation relations of the electric and magnetic field. See third lecture in this course.

08-I-21

## LC CIRCUIT AS QUANTUM HARMONIC OSCILLATOR




$$\hat{H} = \hbar\omega_r\left(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}\right)$$

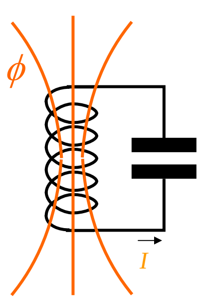
$$\hat{a} = \frac{\hat{\phi}}{\phi_r} + i\frac{\hat{Q}}{Q_r}; \quad \hat{a}^\dagger = \frac{\hat{\phi}}{\phi_r} - i\frac{\hat{Q}}{Q_r}$$ ← annihilation and creation operators

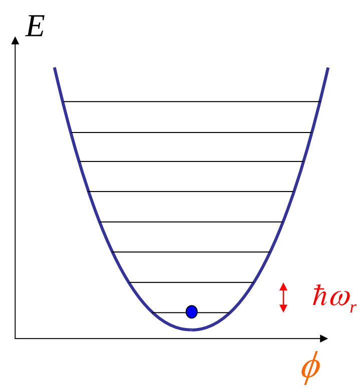
$$\phi_r = \sqrt{2\hbar\omega_r L}$$

$$Q_r = \sqrt{2\hbar\omega_r C}$$

08-I-22

## THERMAL EXCITATION OF LC CIRCUIT



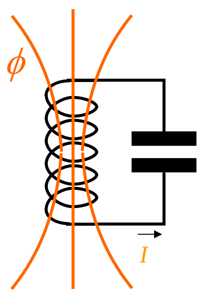

Can place the circuit in its ground state

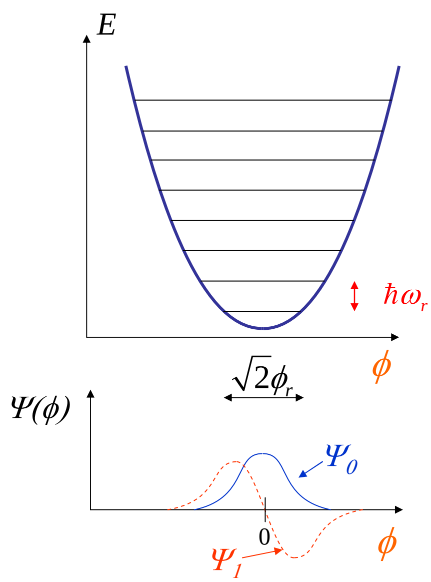
$$\hbar\omega_r \gg k_B T$$

5 GHz ↗ $\hbar\omega_r$; $k_B T$ ↖ 10mK

08-I-22a

# WAVEFUNCTIONS OF LC CIRCUIT

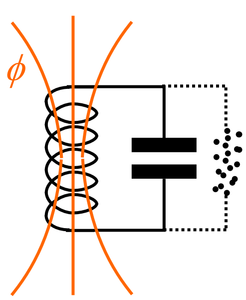


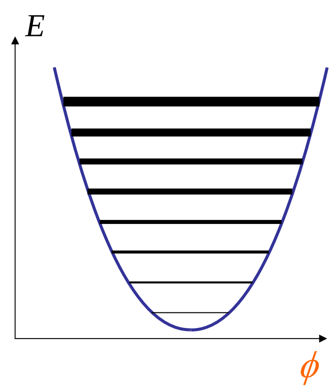




In every energy eigenstate,
(photon state)
current flows in opposite
directions simultaneously!

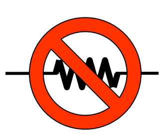
08-I-23

# EFFECT OF DAMPING




important:
negligible dissipation



dissipation broadens energy levels

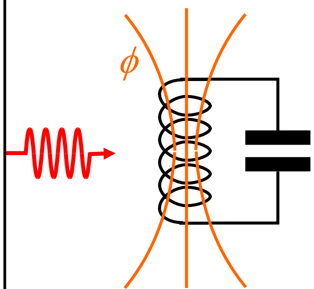
$$E_n = \hbar\omega_r\left[n\left(1+\frac{i}{2Q}\right)+\frac{1}{2}\right]$$

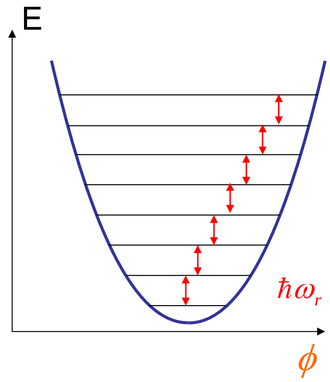
$$Q = RC\omega_r$$

} see why later

08-I-24

# ALL TRANSITIONS ARE DEGENERATE

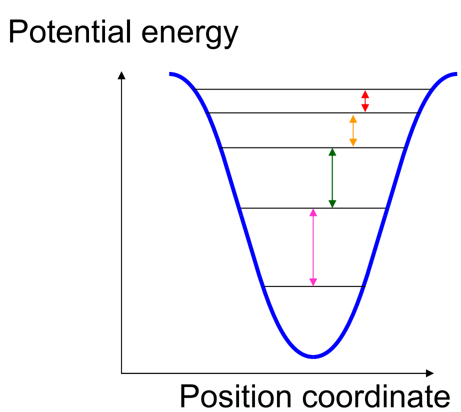



**CANNOT STEER THE SYSTEM TO AN ARBITRARY STATE IF PERFECTLY LINEAR**



# NEED NON-LINEARITY TO FULLY REVEAL QUANTUM MECHANICS



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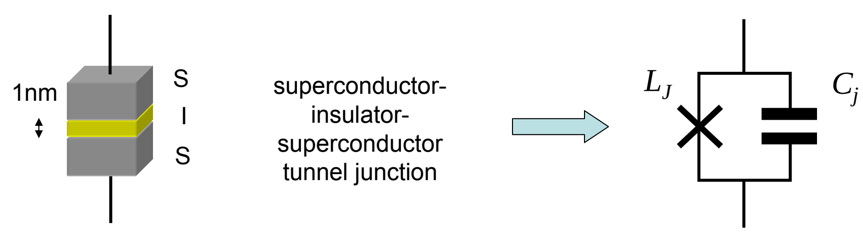
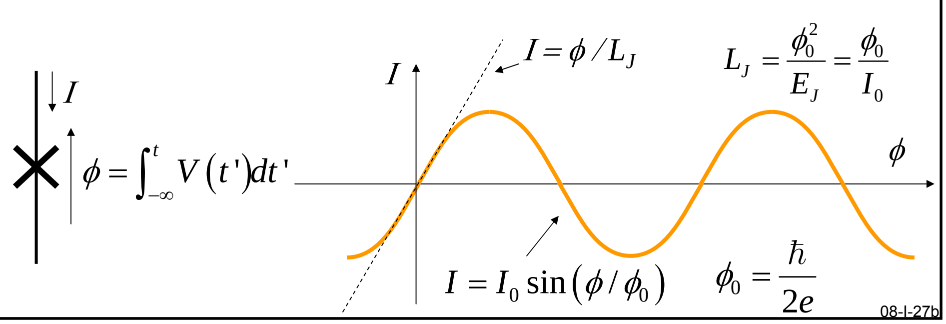
08-I-5d

# JOSEPHSON JUNCTION PROVIDES A NON-LINEAR INDUCTOR

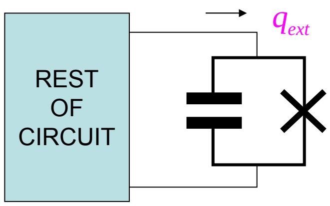



08-I-27b

08-I-28a

## COUPLING PARAMETERS OF THE JOSEPHSON JUNCTION "ATOM"




THE hamiltonian: (we mean it!)

$$\widehat{H}_j = \frac{1}{2C_j}\left(\widehat{Q} - q_{ext}\right)^2 - E_J \cos\frac{2e\widehat{\phi}}{\hbar}$$

Comparable with lowest order model for hydrogen atom →

$$\widehat{H} = \frac{1}{2m_e}\left(\hat{p} - \frac{eA}{\hbar}\right)^2 - \frac{e^2}{4\pi\varepsilon_0}\frac{1}{\hat{r}}$$

## TWO ENERGY SCALES

Two dimensionless variables:

$$\hat{\varphi} = \frac{2e\hat{\phi}}{\hbar}$$

$$\hat{N} = \frac{\hat{Q}}{2e}$$

$$\Rightarrow \quad \left[\hat{\varphi}, \hat{N}\right] = i$$

Hamiltonian becomes :

$$\widehat{H}_j = 8E_C \frac{\left(\widehat{N} - N_{ext}\right)^2}{2} - E_J \cos\hat{\varphi}$$

Coulomb charging energy for 1e

$$E_C = \frac{e^2}{2C_j}$$

Josephson energy

$$E_J = \frac{1}{8}\mathcal{N}\mathcal{T}\Delta$$

barrier transp$^{cy}$ ($\mathcal{T}$)

gap ($\Delta$)

# cond$^{ion}$ channels ($\mathcal{N}$)

valid for opaque barrier

reduced offset charge

$$N_{ext} = \frac{q_{ext}}{2e}$$

08-I-29

## HARMONIC APPROXIMATION

$$\widehat{H}_j = 8E_C \frac{\left(\widehat{N} - N_{ext}\right)^2}{2} - E_J \cos\hat{\varphi}$$

⇓

$$\widehat{H}_{j,h} = 8E_C \frac{\left(\widehat{N} - N_{ext}\right)^2}{2} + E_J \frac{\hat{\varphi}^2}{2}$$

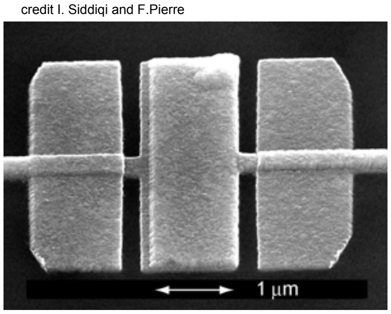
Josephson plasma frequency $\omega_P = \frac{\sqrt{8E_C E_J}}{\hbar}$

Spectrum independent of DC value of $N_{ext}$

08-I-30

## TUNNEL JUNCTIONS IN REAL LIFE

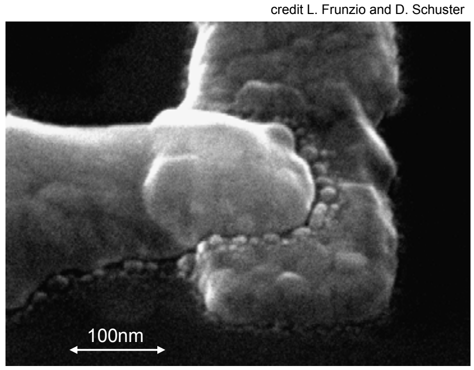
credit I. Siddiqi and F.Pierre




$E_J$ ~ 50K

$\omega_p$ ~ 30-40GHz

credit L. Frunzio and D. Schuster




$E_J$ ~ 0.5K

08-I-31

# OUTLINE

1. Review of classical radio-frequency circuits
2. Quantum information processing
3. Quantum-mechanical LC oscillator
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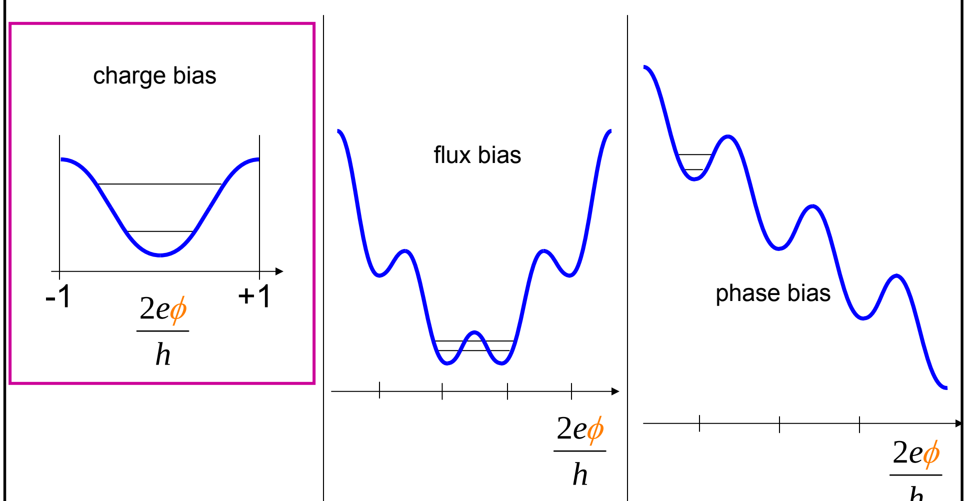
6. Summary of questions addressed by this course

08-I-5e

# EFFECTIVE POTENTIAL OF 3 BIAS SCHEMES



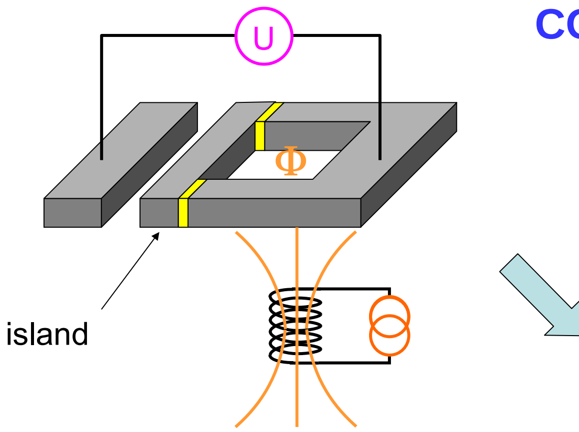

CEA Saclay, Yale
NEC, Chalmers, JPL, ...

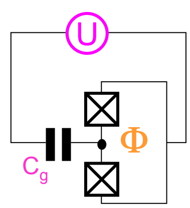
TU Delft, NEC, NTT, MIT
UC Berkeley, IBM, SUNY
IPHT Jena ....

UC Berkeley, NIST, UCSB,
U. Maryland, CRTBT Grenoble...

08-I-32a

## COOPER PAIR "BOX"



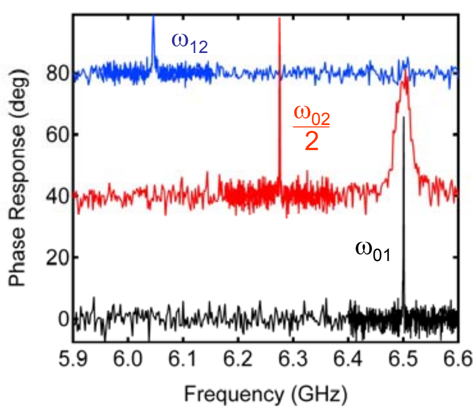

2 control parameters $C_g U/2e$ and $\Phi/\Phi_0$

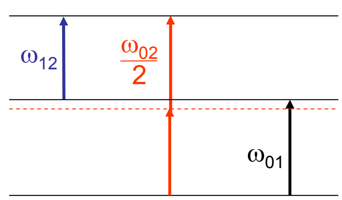
Bouchiat et al. 97
Nakamura, Pashkin & Tsai 99
Vion et al. 2002



## SPECTROSCOPY OF A COOPER PAIR BOX

J. Schreier et al. 08




Anharmonicity:

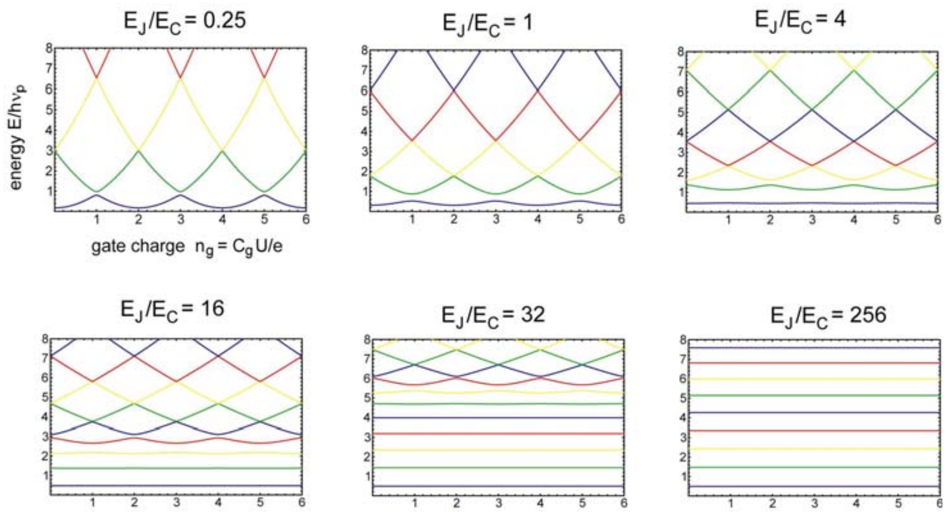
$$\omega_{01} - \omega_{12} = 455\text{MHz} \simeq E_C$$

Sufficient to control the junction as a two level system

*Slide courtesy of J. Schreier and R. Schoelkopf*

## ANHARMONICITY vs CHARGE SENSITIVITY

Cooper pair box soluble in terms of Mathieu functions (A. Cottet, PhD thesis, Orsay, 2002)




08-I-35

## ANHARMONICITY vs CHARGE SENSITIVITY IN THE LIMIT $E_C/E_J << 1$

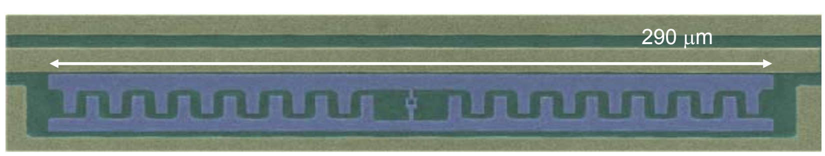
anharmonicity: $$\frac{\omega_{12}-\omega_{01}}{(\omega_{12}+\omega_{01})/2} \rightarrow \sqrt{\frac{E_C}{8E_J}}$$

peak-to-peak charge modulation amplitude of level m:

$$\epsilon_m \rightarrow (-1)^m E_C \frac{2^{4m+5}}{m!} \sqrt{\frac{2}{\pi}} \left(\frac{E_J}{2E_C}\right)^{\frac{m}{2}+\frac{3}{4}} e^{-\sqrt{8E_J/E_C}}$$

J. Koch et al. 07

TRANSMON: SHUNT JUNCTION WITH CAPACITANCE




*Courtesy of J. Schreier and R. Schoelkopf*

08-I-36

## OUTLINE

1. Review of classical radio-frequency circuits
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08-I-5f

**CAN WE BUILD ALL THE QUANTUM INFORMATION PROCESSING PRIMITIVES OUT OF SIMPLE CIRCUITS AND CONNECT THESE CIRCUITS TO PERFORM ANY DESIRED FUNCTION?**

08-I-37

| QUANTUM OPTICS | QUANTUM RF CIRCUITS |
|---|---|
| FIBERS, BEAMS | TRANSM. LINES, WIRES |
| BEAM-SPLITTERS | COUPLERS |
| MIRRORS | CAPACITORS |
| LASERS | GENERATORS |
| PHOTODETECTORS | AMPLIFIERS |
| ATOMS | JOSEPHSON JUNCTIONS |

DRAWBACKS OF CIRCUITS: ARTIFICIAL ATOMS PRONE TO VARIATIONS

ADVANTAGES OF CIRCUITS:
- PARALLEL FABRICATION METHODS
- LEGO BLOCK CONSTRUCTION OF HAMILTONIAN
- ARBITRARILY LARGE ATOM-FIELD COUPLING

08-I-38

**HOW DO WE TRANSLATE FROM THE LANGUAGE OF ELECTRICAL SIGNALS COUPLED IN A NON-LINEAR CIRCUIT INTO THE LANGUAGE OF PHOTONS INTERACTING WITH ATOMS (EMISSION, ABSORPTION, SCATTERING, DETECTION)?**

**HOW DO WE DO TREAT QUANTUM-MECHANICALLY A DISSIPATIVE, NON-LINEAR, OUT-OF-EQUILIBRIUM ENGINEERED SYSTEM?**

08-I-39

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08-I-40

END OF LECTURE