IMPLEMENTATION OF PROTECTED QUBITS IN JOSEPHSON JUNCTION ARRAYS

Theoretical collaborators.

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Past:

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Experimental results (Rutgers):

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PLAN

- Protection: symmetry and hardware error correction
- Josephson implementation: larger and minimalistic systems.
- Comparison between different designs
- Challenge of numerical simulations
- The most promising design (currently)
- Experiment: advances and problems.

REPETITION CODE

Classical repetition code (5 physical bits correct 2 errors):

|Logical 0> = |00000>

|Logical 1> = |11111>

Quantum repetition code:

|C 0> = |00000>

|C 1> = |11111>

- protect against the flips (X-errors)

$|+\rangle = 1/\sqrt{2}\,(|C\,0\rangle + |C\,1\rangle)$

$|-\rangle = 1/\sqrt{2}\,(|C\,0\rangle - |C\,1\rangle)$

To protect against phase errors (Z-errors) form

|Logical +> = |+++++>

|Logical - > = |- - - - ->

In matrix form:

$\vert 00000\rangle + \vert 11111\rangle$
$\vert 00000\rangle + \vert 11111\rangle$
$\vert 00000\rangle + \vert 11111\rangle$
$\vert 00000\rangle + \vert 11111\rangle$
$\vert 00000\rangle + \vert 11111\rangle$

$\vert 00000\rangle - \vert 11111\rangle$
$\vert 00000\rangle - \vert 11111\rangle$
$\vert 00000\rangle - \vert 11111\rangle$
$\vert 00000\rangle - \vert 11111\rangle$
$\vert 00000\rangle - \vert 11111\rangle$

25 physical qubits to correct two errors and store 1 logical

HARDWARE ERROR CORRECTION

Main idea:

Find the physical system in which the lowest two states are given by the same wave function as two logical states in the error correction scheme. All excited states should be separated by a large gap from two lowest logical states.

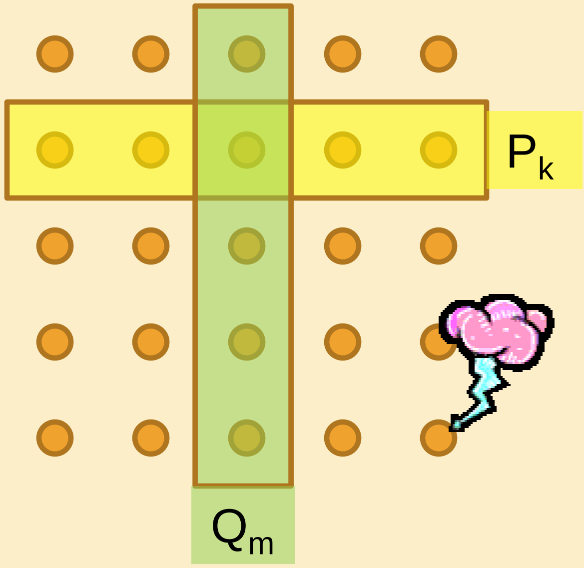
Effective noise acting on logical variable: $h_{eff} = h_1 \prod_{k>1} \frac{h_k}{\Delta E_k}$

In matrix form:

25 physical qubits to correct two errors and store 1 logical

|00000> + |11111>	|00000> - |11111>
|00000> + |11111>	|00000> - |11111>
|00000> + |11111>	|00000> - |11111>
|00000> + |11111>	|00000> - |11111>
|00000> + |11111>	|00000> - |11111>

PROTECTED QUBIT:SYMMETRY VIEW



Protected Doublet:
Special Spin Hamiltonians H with a large number of (non-local) integrals of motion P, Q:
$[H,P_k]=0$, $[H,Q_m]=0$, $[P_k,Q_m]\neq 0$

Any physical (local) noise term $\delta H(t)$ commutes with all P_k and Q_m except a O(1) number of each.
Effect of noise appears in N order of the perturbation theory:

$$\delta E \sim (\delta H(t) / \Delta)^{N-1} \delta H(t)$$

Simplest Spin Hamiltonian

$$H=\Sigma_{kl} J^x_{kl} \sigma^x_k \sigma^x_l + \Sigma_{kl} J^z_{kl} \sigma^z_k \sigma^z_l$$

Rows	Columns
$P_k=\prod_l \sigma^z_l$	$Q_k=\prod_l \sigma^x_l$

Crucial issues:
1. Which model has a large gap Δ?
2. Which model is easiest to realize in Josephson junction arrays?

EQUIVALENCE OF SYMMETRY AND ERROR CORRECTION

Symmetry:
Special Spin Hamiltonians H with a large number of (non-local) integrals of motion P, Q:
$[H,P_k]=0$, $[H,Q_m]=0$, $[P_k,Q_m]\neq 0$

Special very symmetric Hamiltonian

$$H=-\Sigma_{jkl} J^x \sigma^x_{jk} \sigma^x_{jl} - \Sigma_{kl} J^z P^z_k P^z_l$$

$P^z_k = \prod_l \sigma^z_{kl}$ - row product

Rows	Columns
$P_k=\prod_l \sigma^z_{kl}$	$Q_l=\prod_k \sigma^x_{kl}$

All P_k and Q_l anticommute

Solution of the model:
Ground states of one row:
$|GS\ 1\rangle = |\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rangle$ GS $2\rangle = |\leftarrow\leftarrow\leftarrow\leftarrow\leftarrow\leftarrow\rangle$
$|\rightarrow\rangle=(|\uparrow\rangle+|\downarrow\rangle)/\sqrt{2}$ $|\leftarrow\rangle=(|\uparrow\rangle-|\downarrow\rangle)/\sqrt{2}$
$|+\rangle=|GS\ 1\rangle+|GS\ 1\rangle$ - has even number of spins down $P^z|+\rangle=|+\rangle$
$|-\rangle=|GS\ 1\rangle-|GS\ 1\rangle$ - has odd number of spins down $P^z|-\rangle=-|-\rangle$
Ground state of the whole system:
$|1\rangle= \prod_l |+\rangle_l$ and $|0\rangle= \prod_l |-\rangle_l$

Ground state of the model Hamiltonian is the doublet of repetition code!

TWO ALTERNATIVES

- Identify and implement very symmetric Hamiltonian with a smallest possible number of Josephson junctions and islands, all very well controlled.

  Example: tetrahedral symmetry.

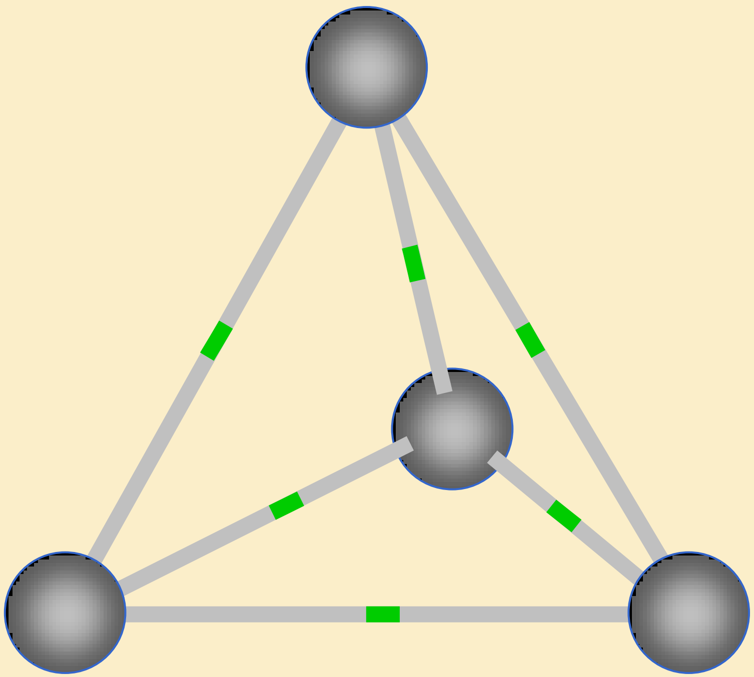
- Implement larger arrays with only approximate symmetries that compensate a lack of control by the size of the array.

TETRAHEDRAL QUBIT: SMALLEST JJ ARRAY

Josephson energy $E_J = \dfrac{I_c \Phi_0}{2\pi c}$

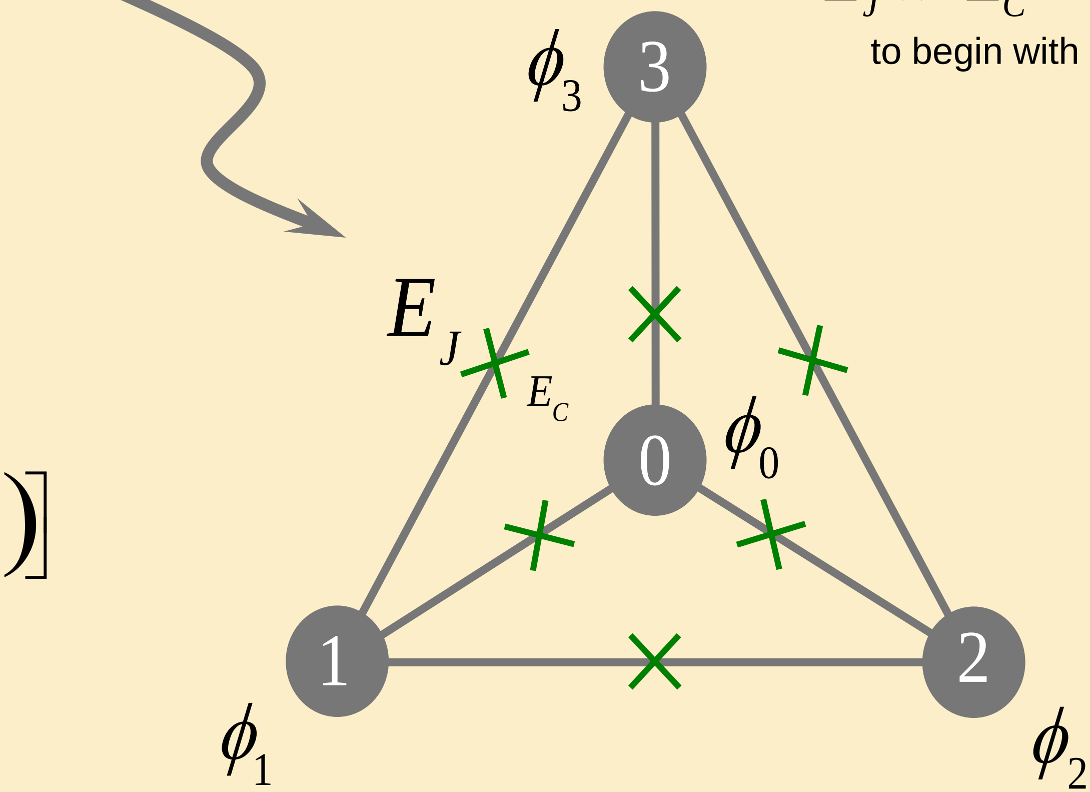
Charging/capacitive energy $E_C = \dfrac{e^2}{2C}$

$E_J \gg E_C$ to begin with



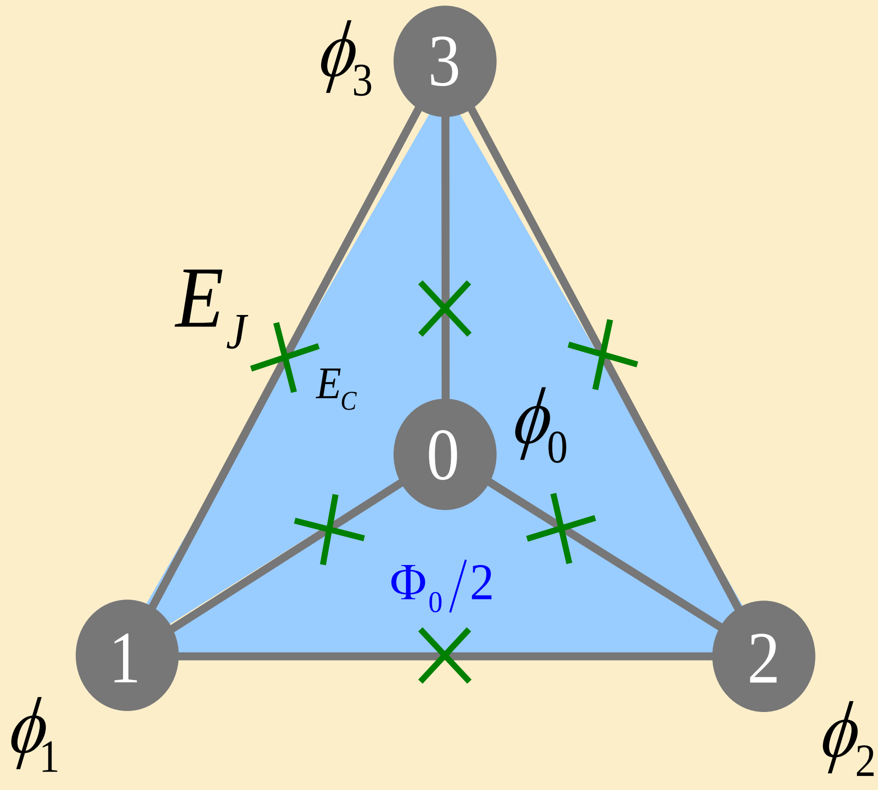
Classical energy

$$V = E_J \sum_{i<j} \left[1 - \cos\left(\phi_j - \phi_i \right) \right]$$



FRUSTRATION

Threading a flux $\Phi_0/2$ through each triangle



Choose a symmetric gauge

$$\phi_{ij} = \phi_j - \phi_i \rightarrow \phi_{ij} + \pi$$

Classical energy

$$V_\pi = E_J \sum_{i<j} \cos\phi_{ij}$$

reversed signs

$$= \frac{E_J}{2}\left[\left|\sum_j e^{i\phi_j}\right|^2 - 4\right]$$

➡ minimization gives one complex equation defining a line in 3D phase space.

NEW PROPERTIES

Symmetry

The tetrahedral symmetry group

T_d or S_4

is non-Abelian and contains nontrivial representations,

$$24 = g = \sum_{k=1}^{5} d_k^2 = 1^2 + 1^2 + 2^2 + 3^2 + 3^2.$$

via **electric frustration**

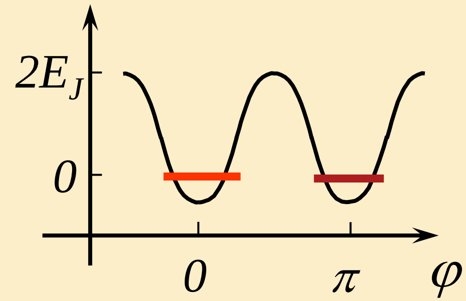
Push this **doublet** to become the **ground state** and use it as a quantum bit.

This emulates a

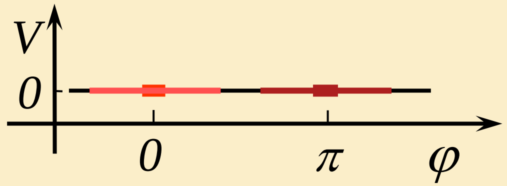
spin 1/2 in a **zero magnetic field**,

the ideal starting point for the construction of a qubit.

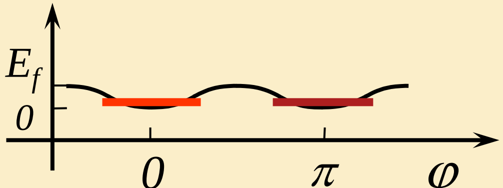
Usually, we deal with two states separated by a **classical** barrier, e.g., in a 2φ–junction,



In the **magnetically frustrated** tetrahedron with conventional cos φ–junctions, we encounter a continuous **classical** degeneracy,

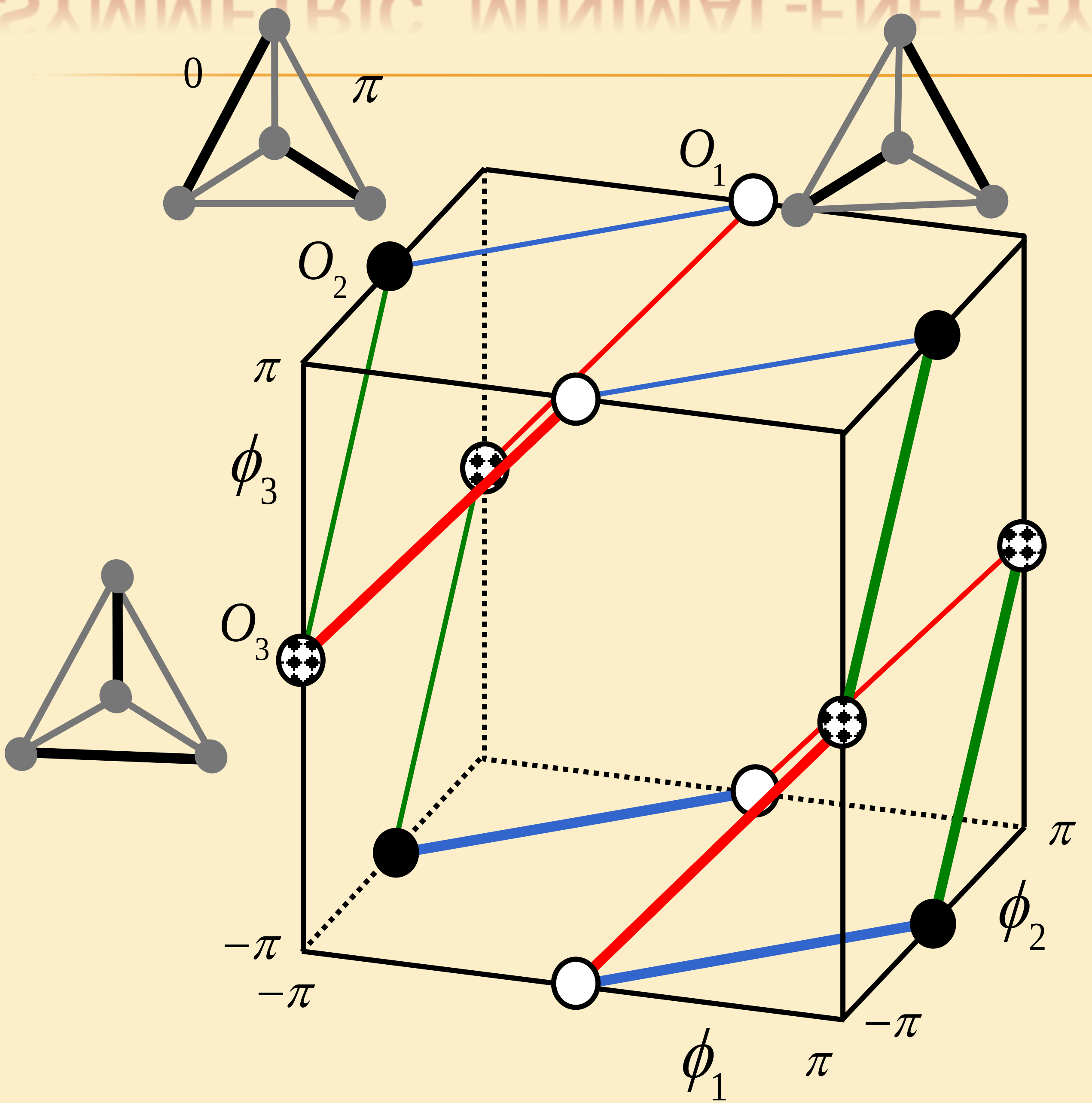


Only when quantum fluctuations are accounted for, we obtain a **fluctuation-induced** weak potential,

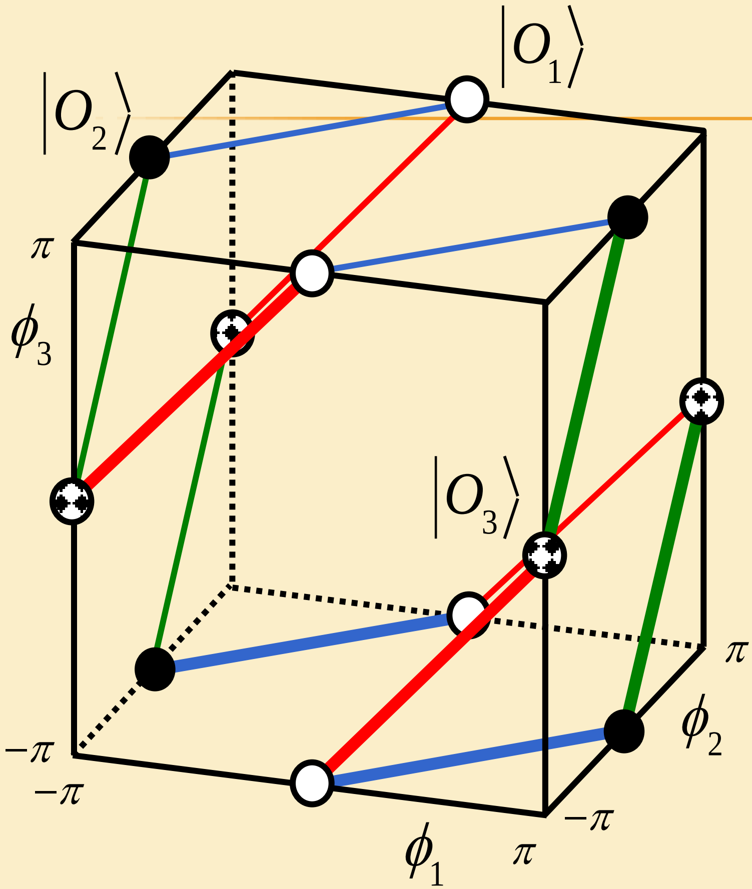


This enhances charge/quantum fluctuations without the need to go to ultra-small junctions

SYMMETRIC MINIMAL-ENERGY STATES



MIXING



Within a full
quantum mechanical
description,
all the points

are mixed through tunneling.

The Hamiltonian describing the mixing between the semi-classical states

$$|O_1\rangle, |O_2\rangle, |O_3\rangle$$

takes the form

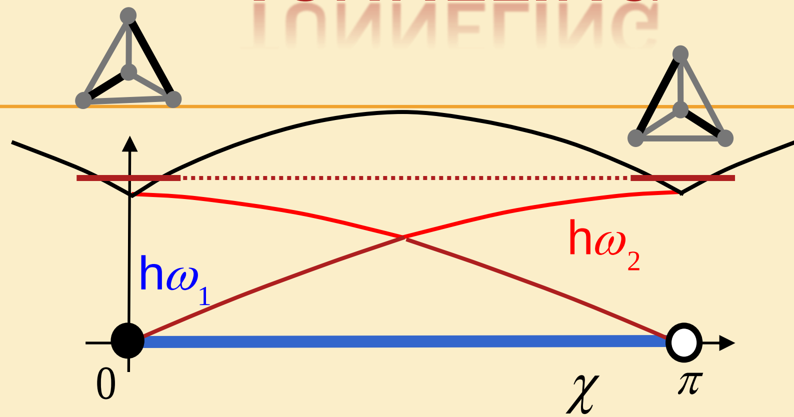
$$H_t = \begin{pmatrix} 0 & t & t \\ t & 0 & t \\ t & t & 0 \end{pmatrix}$$

and produces the eigenvalues

$$E_d = -t \quad \text{and} \quad E_s = 2t.$$

The tunneling amplitude t involves a non-trivial **phase** and a **modulus** to be calculated within a semi-classical approximation

TUNNELING



Two inequivalent tunneling trajectories produce an Aharononv-Bohm-Casher phase

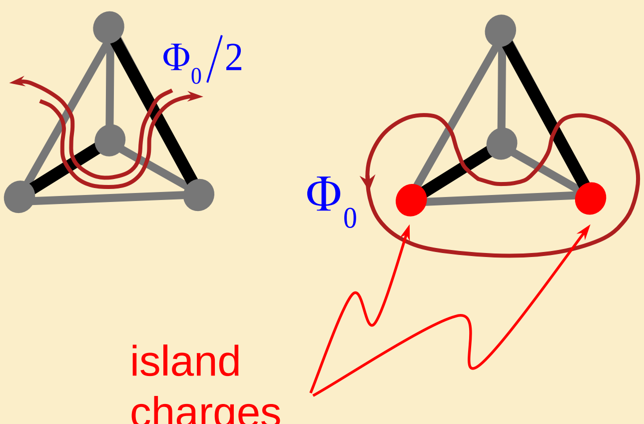
$$\exp\left[2\pi i\left(Q_1+Q_2\right)/2e\right].$$

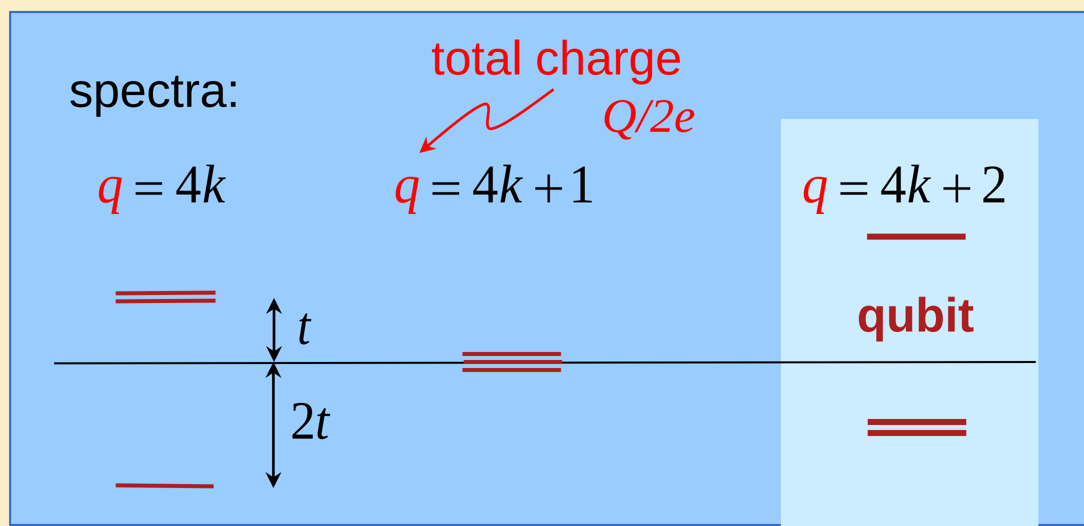
Combining with the tunneling action

$$S_s \sim 1.88\left(E_J/E_C\right)^{1/4}$$

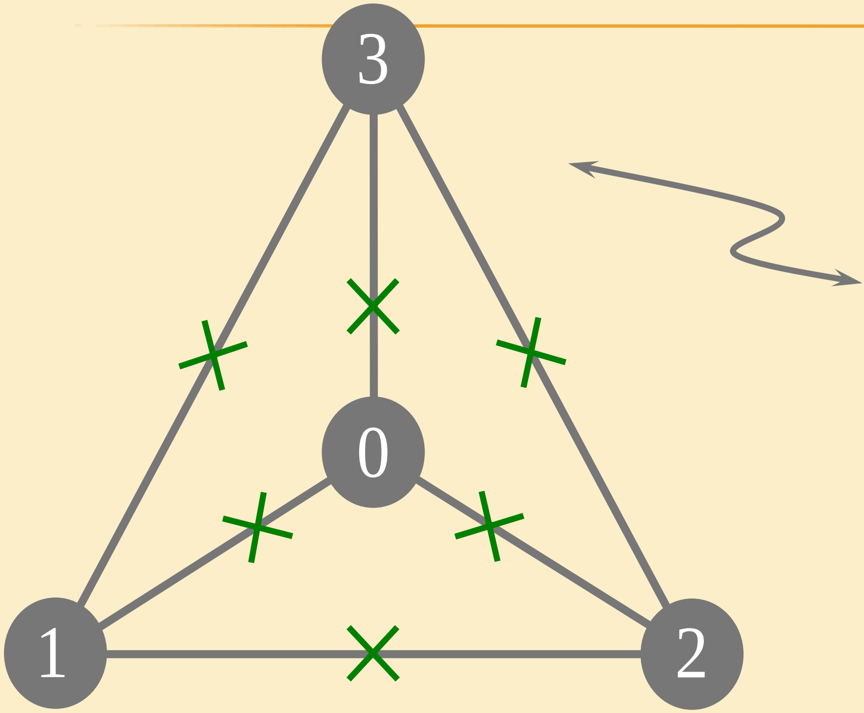
we find the tunneling amplitude

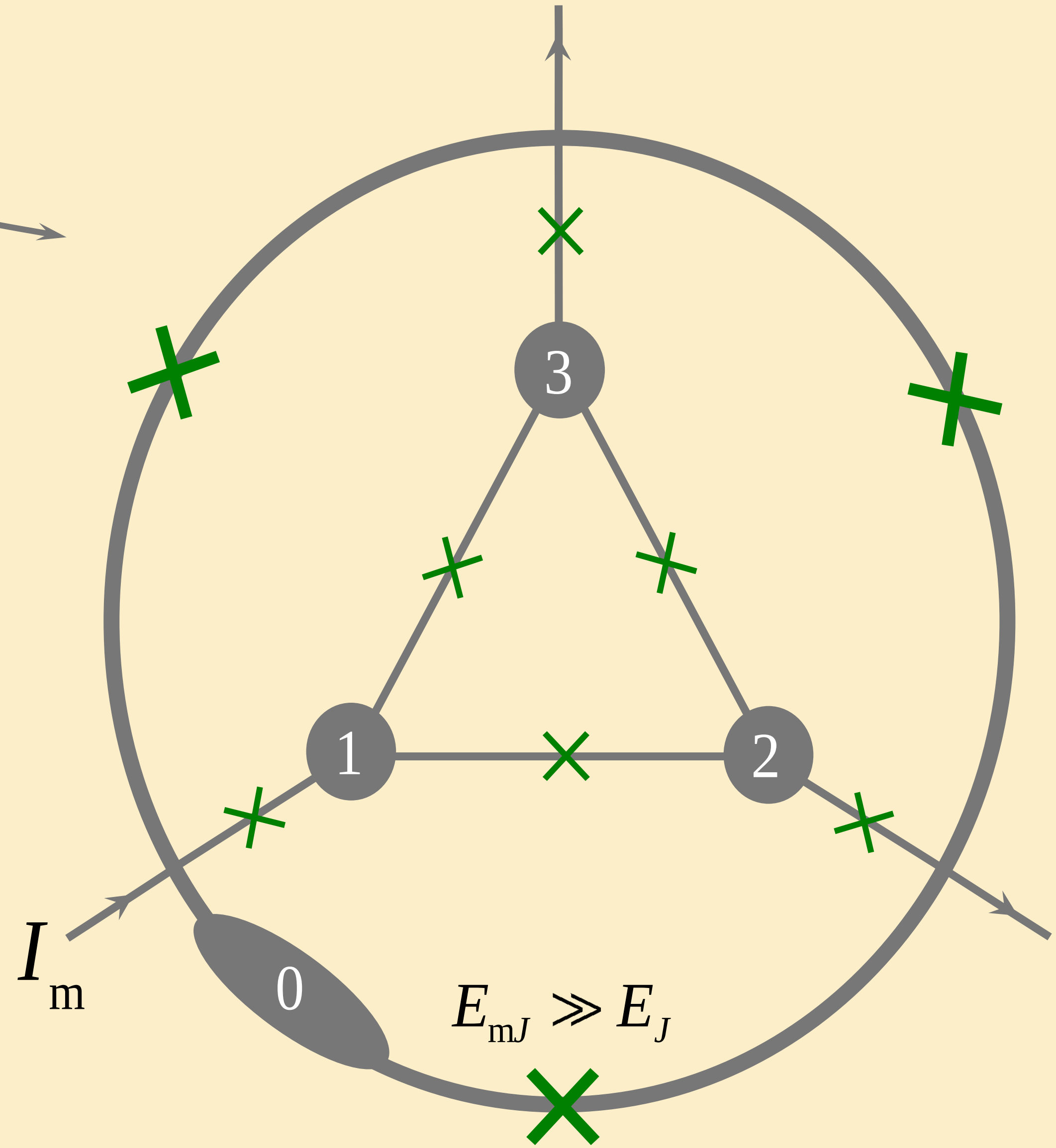
$$t \approx -\hbar/T\,\exp\left(-S_s\right)\cos\left[\pi\left(q_1+q_2\right)\right].$$



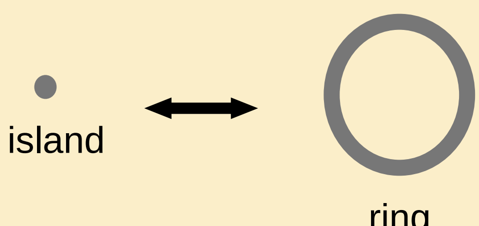


ISOLATED VS. CONNECTED TETRAHEDRON

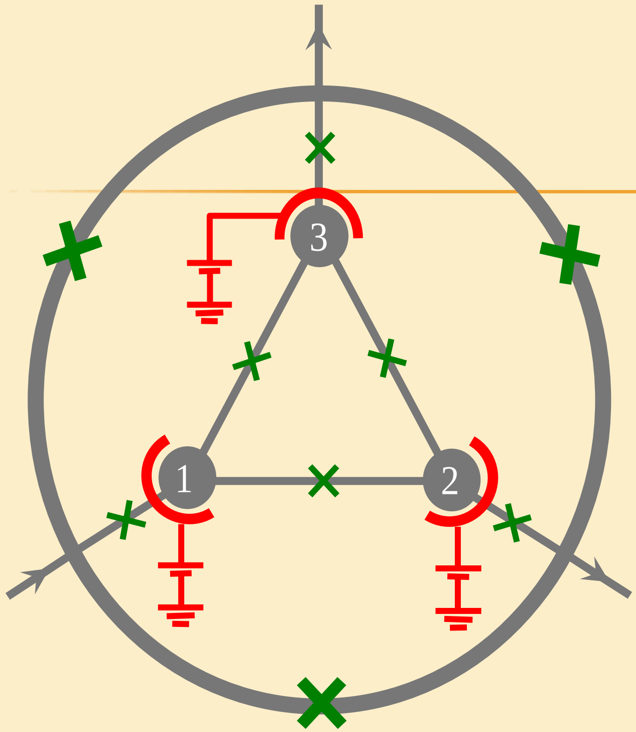




Invert the inner island `0'
for symmetric measurement



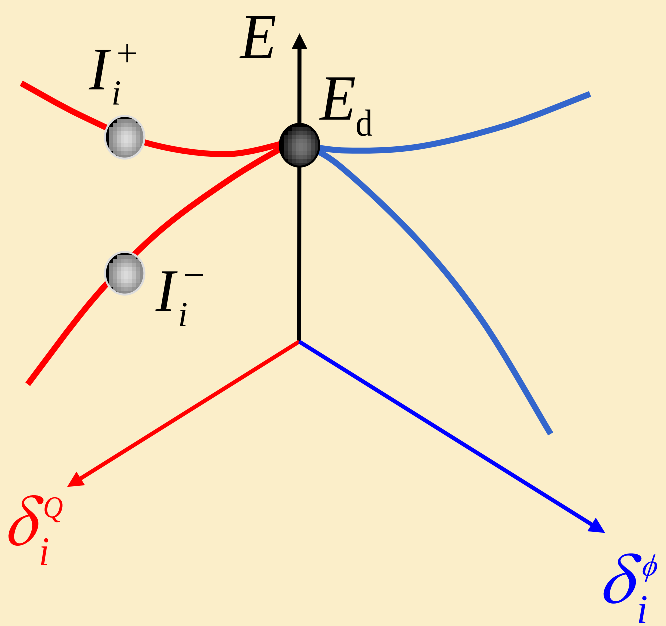
MEASUREMENT: OPERATOR σ_Z



Unbiased state, carries **no currents** on links and no polarization charges on the islands.

Charge-biased state, carries **currents** on links differentiating between the qubit-states $|\pm\rangle$

Similarly flux bias distinguishes states |0> and |1>



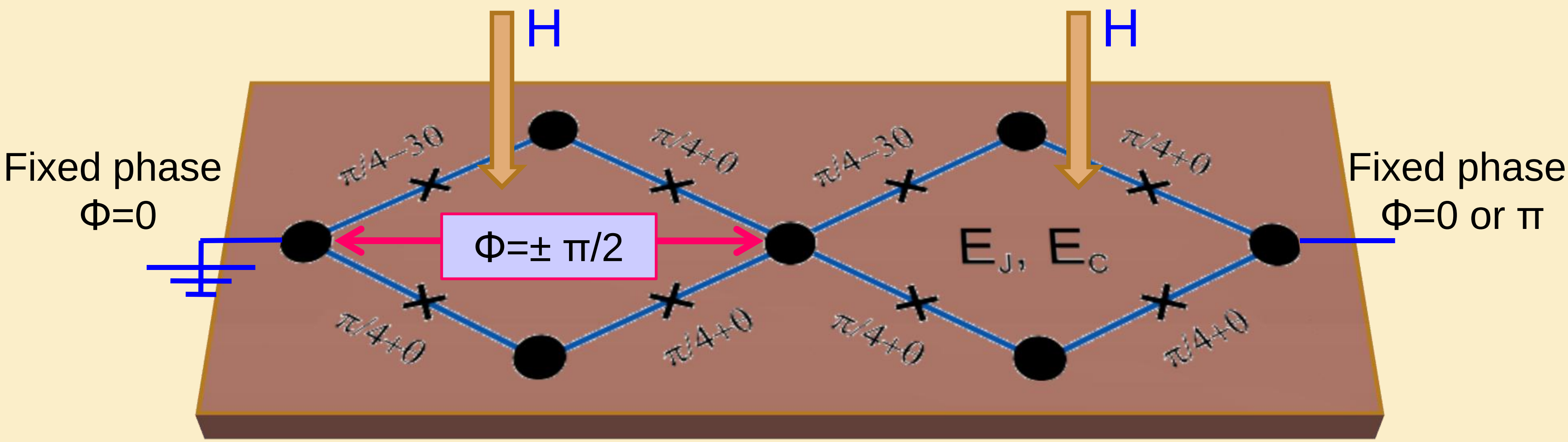
$$I_i^{\pm} = \frac{2e}{\hbar}\frac{\partial \delta E^{\pm}}{\partial \delta_i^{\Phi}}$$

$$= \pm\frac{2\pi est}{\sqrt{3}\hbar}\left(2\delta_i^Q + \delta_j^Q + \delta_k^Q\right).$$

TETRAHEDRAL QUBIT: CONCLUSION

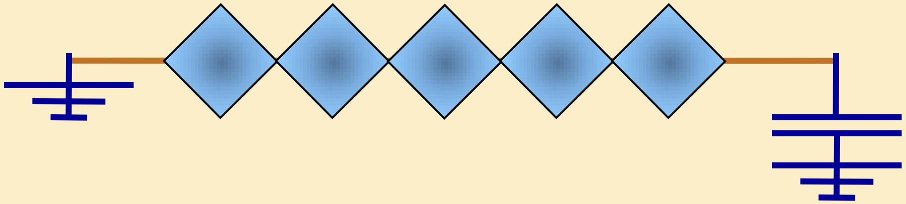
- Minimal system: three islands, 6 junctions.
- Tetrahedral group contains many ‘redundant’ symmetries $\longrightarrow$ protection
- Needs both charge and flux frustration
- Allows measurement in charge and phase basis.

REALIZATION OF INDIVIDUAL SPINS/BITS AND THEIR INTERACTION: MODULAR APPROACH



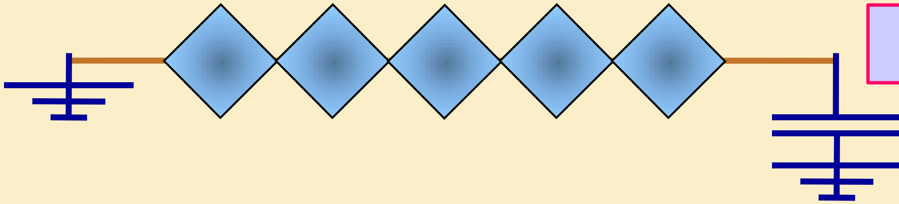
Discrete states of each rhombus: $|\varphi = +\pi/2\rangle$, $|\varphi = -\pi/2\rangle$,
Only simultaneous flips are possible: $H = t\,\sigma^x_k\,\sigma^x_l$

Longer chains: $H=t\,\Sigma_{k,m}\,\sigma^x_k\,\sigma^x_m$ + constraint $\prod_k \sigma^z_k$=const



Large capacitor preventing phase changes of the end point.

JOSEPHSON IMPLEMENTATION OF REPETITION CODE

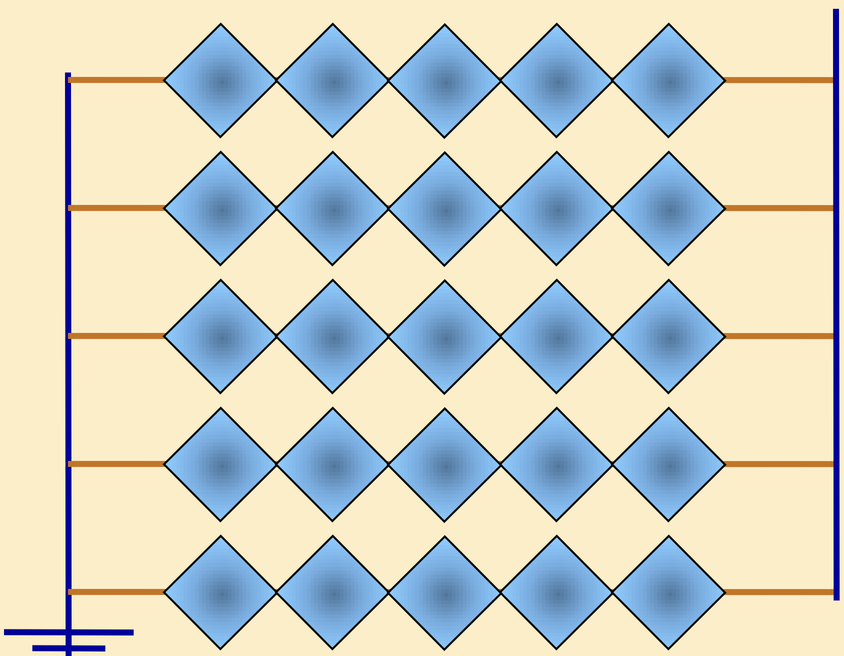


Phase at the end of the chain: $\psi = \pm \pi/2$

$H = t\ \Sigma_{k,m}\ \sigma^x_k \sigma^x_m$ + constraint $\prod_k \sigma^z_k$=const

|GS 1> = |→→→→→>   GS 2> = |←←←←←>

Protecting against the phase errors (in original basis) no protection against flip errors: $V(\psi) = - V_2 \cos(2\psi)$: V_2 is too small



$\Psi = \pm\pi/2$

WHERE IS THE CATCH?

- Josephson elements are not discrete.

Noise suppression contains

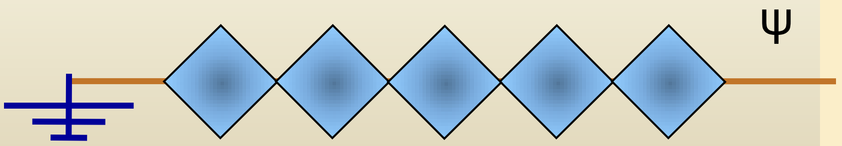
$$\left(\frac{\delta\Phi}{\Phi_0}\frac{E_J}{\Delta}\right)^{k-1}\left(\gamma\frac{(\delta E_J)^2}{\Delta E_J}\right)^{[(k+1)/2]}$$

Δ ~ transition amplitude $\quad t \sim \exp(-\sqrt{2E_{2J}/E_C})$

⟶ we need large quantum fluctuations, i.e. $E_{2J}/E_C \sim 1$.

But large quantum fluctuations ⟶ low phase rigidity across the chain $V(\psi) = -V_2\cos(2\psi)$ with

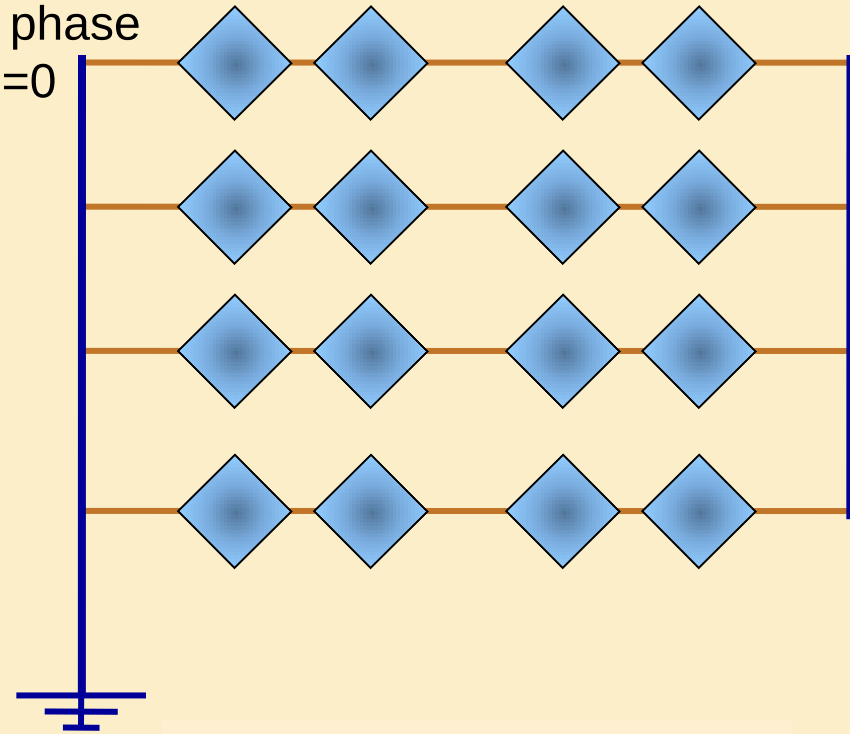
$$V_2 \sim \exp(-N\sqrt{2E_C/E_J})$$



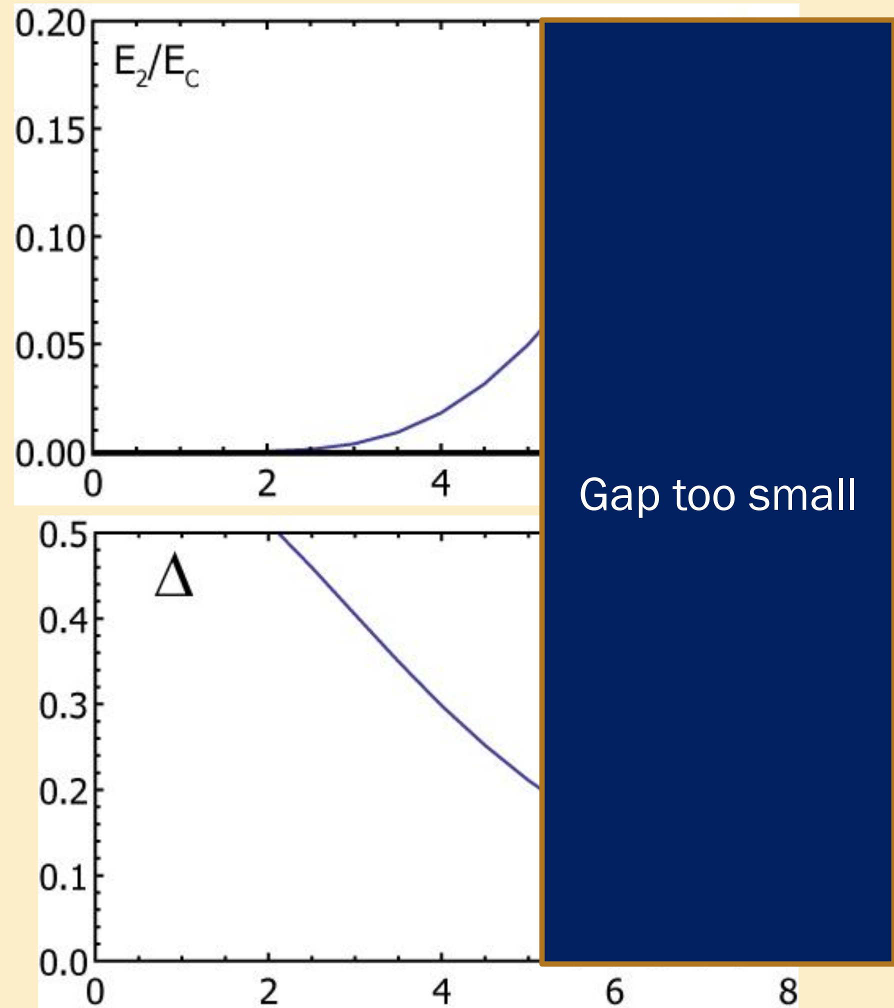
⟶ for long chains V_2 becomes too small even for large number K of parallel chains.

RESOLUTION: FIRST ATTEMPT.
FEW (K>1) PARALLEL CHAINS FOR N=2-4

Fixed phase
$\Phi=0$



Dynamical phase Φ



$V(\Phi) = K\,V_{chain}(\Phi)$

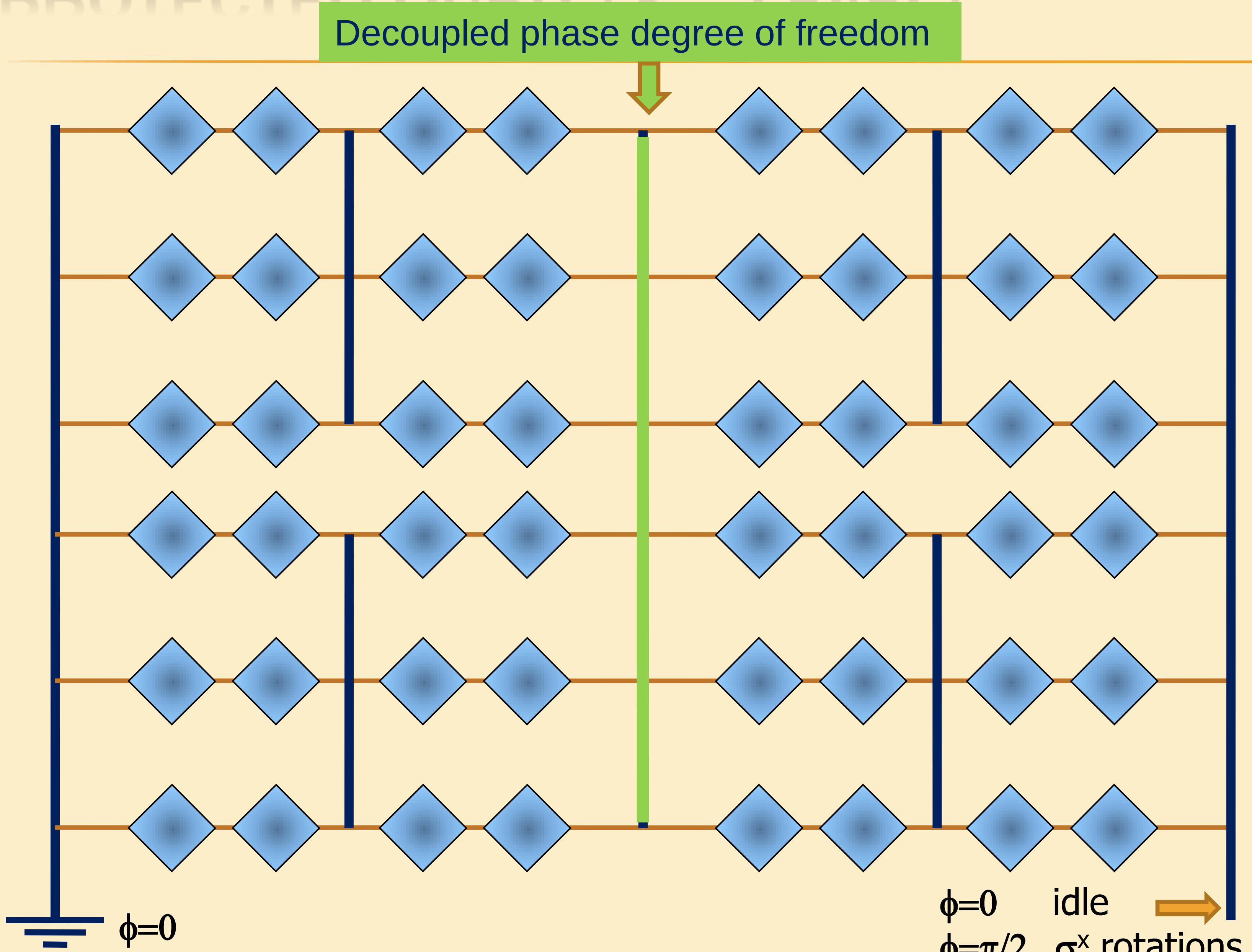
$C_{eff} = K\,C_{chain}$

Need $K^2\,\Delta v_{chain} / E_{c\ chain} >> 1$

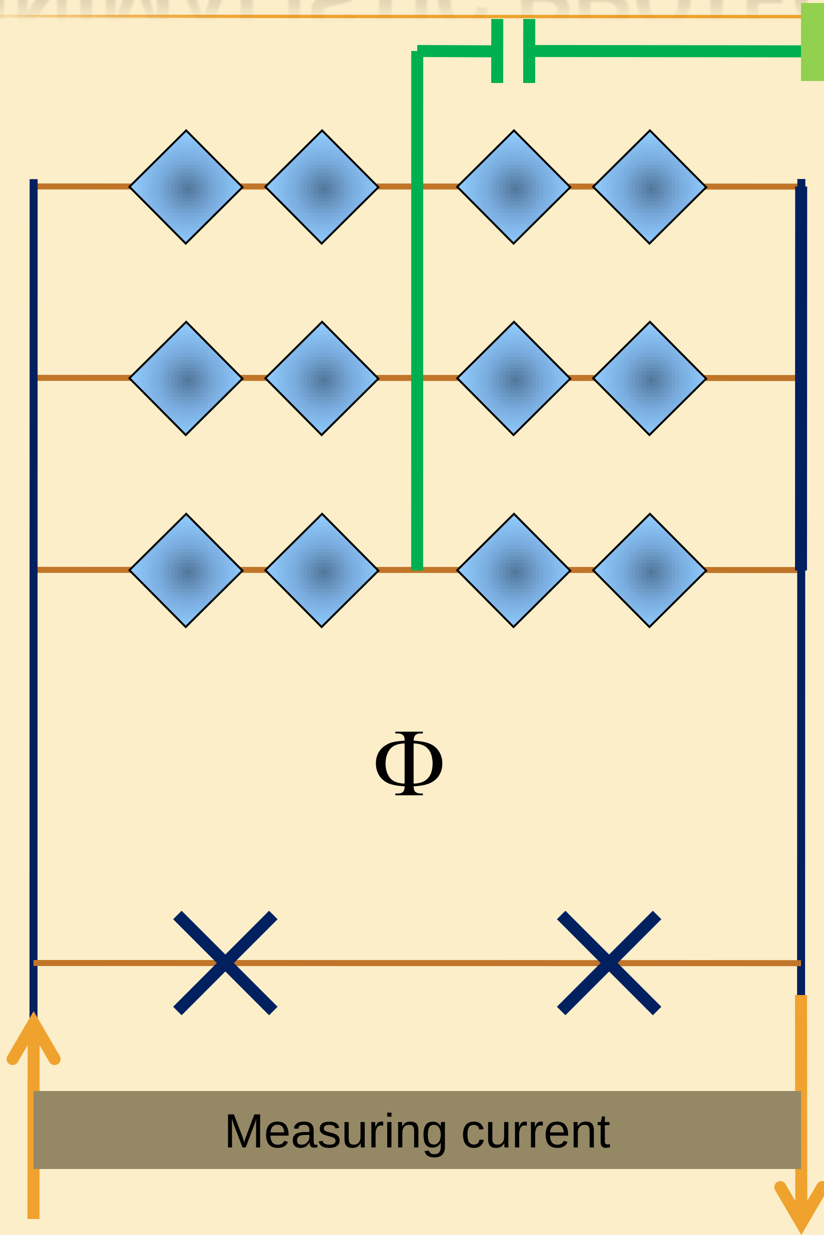
$E_{c\ 4\ rhombi\ chain} \sim E_c$

Need K~10-20

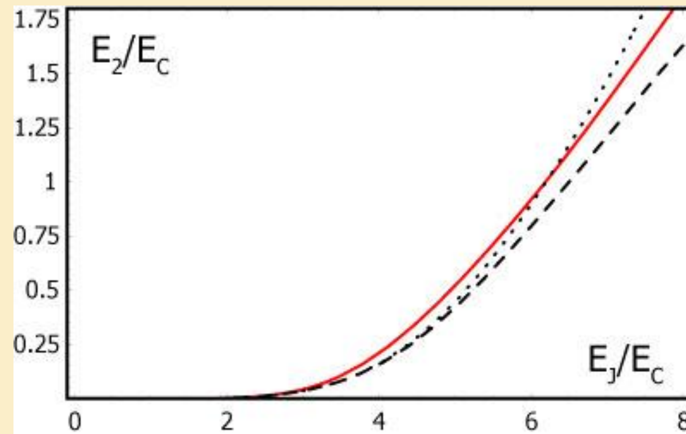
PROTECTED QUBIT (3RD LEVEL)



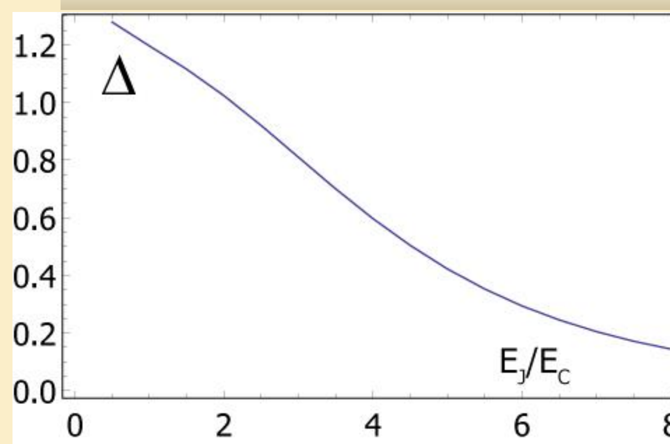
MINIMALISTIC PROTECTED SYSTEM (1)



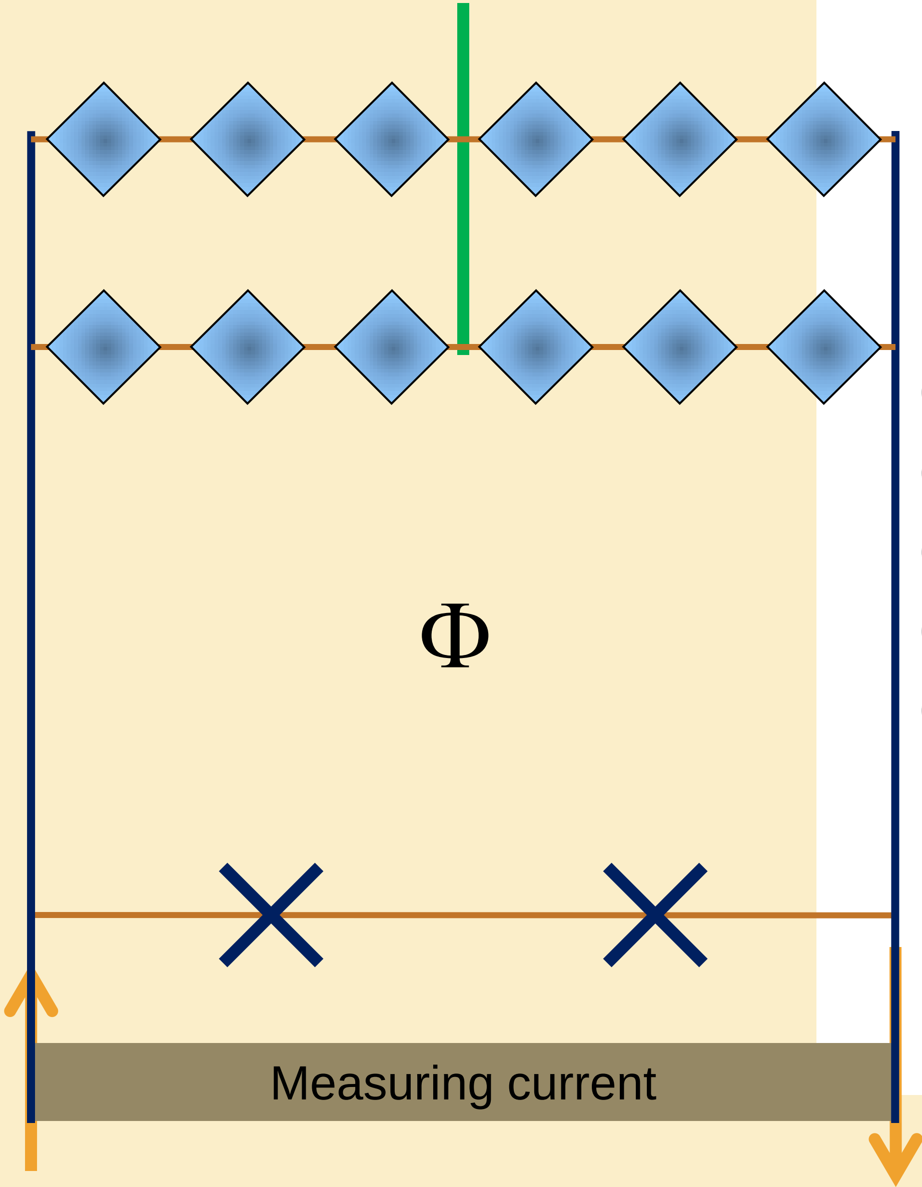
Josephson energy $E_2\cos\phi$ dependence on junction parameters



Gap to lowest excitation

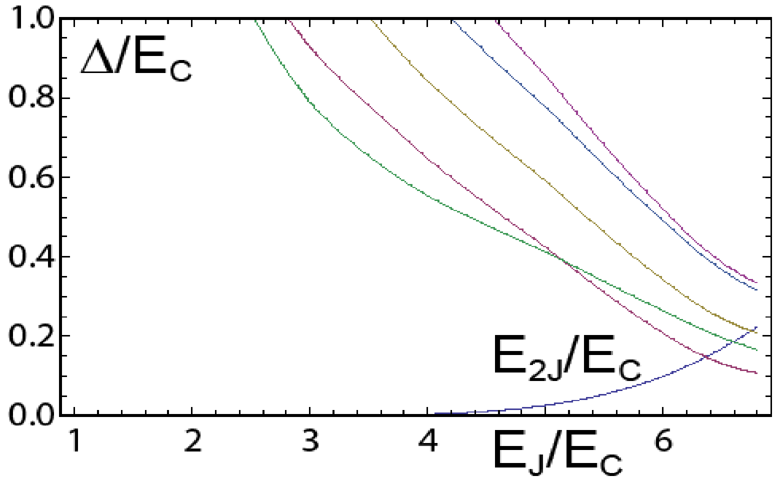


MINIMALISTIC PROTECTED SYSTEM (2)



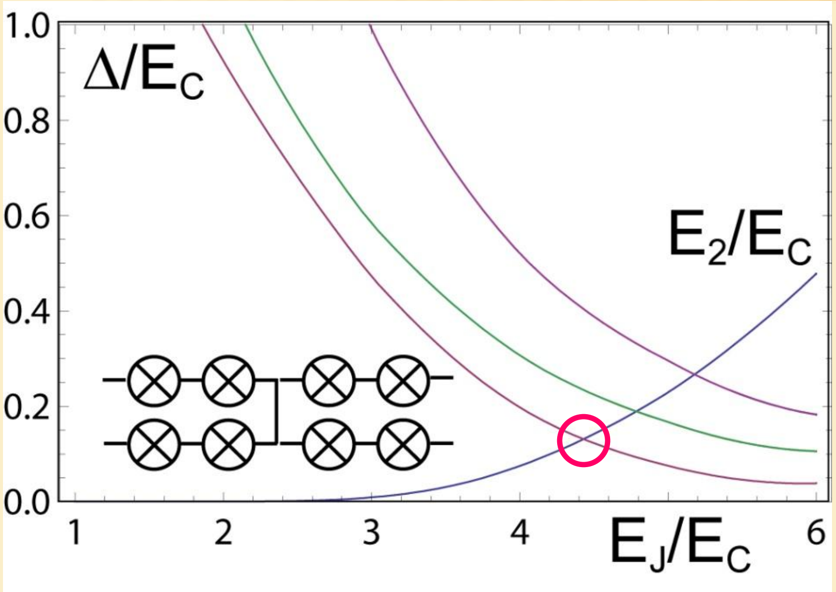
Measuring current

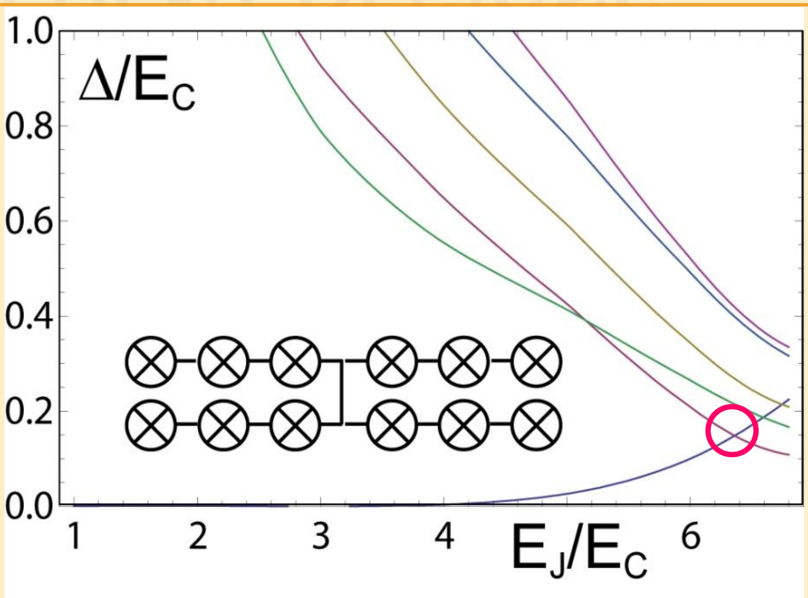
Gap to lowest excitation

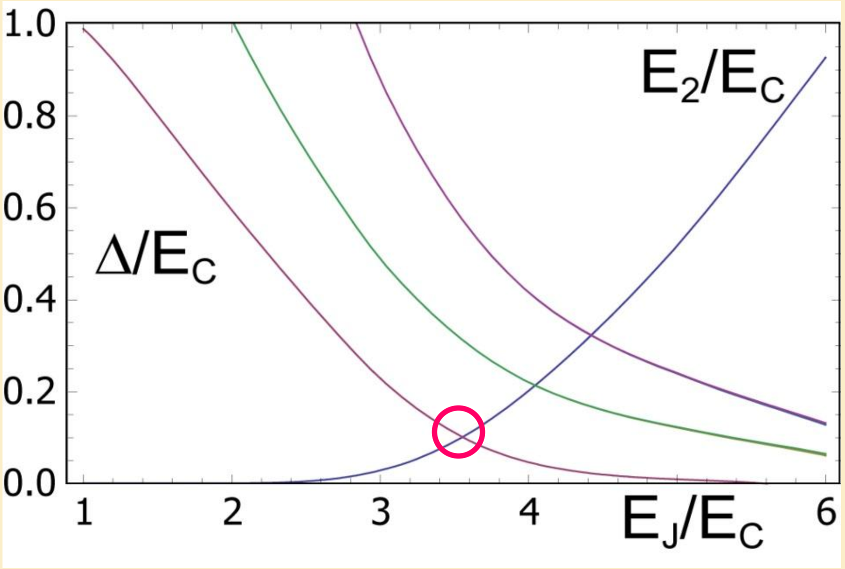


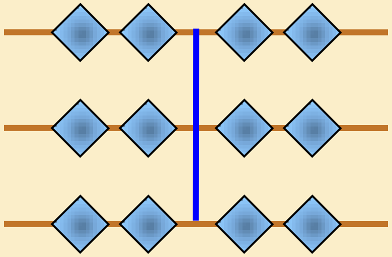
Josephson energy $E_2\cos\phi$ dependence on junction parameters

COMPARISON OF DIFFERENT DESIGNS



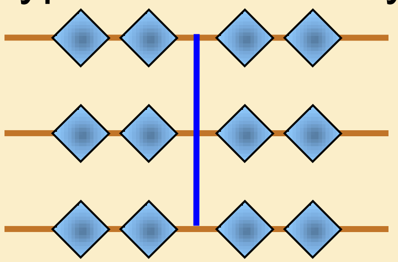






CHALLENGE OF NUMERICAL SIMULATIONS.

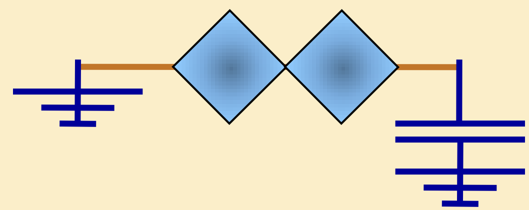
Typical ‘small’ array: 12 rhombi (48 junctions), 31 island.



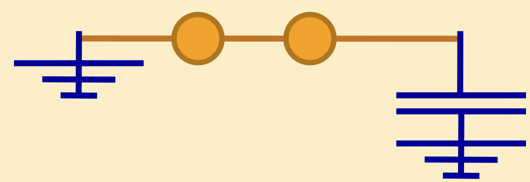
Need at least 5 charge states on each island (better 7-9)
Total number of states: $>5^{31}=10^{20} \rightarrow$ impossible for any classical computer.

The idea of computations:

1. Diagonalize small chains, such as

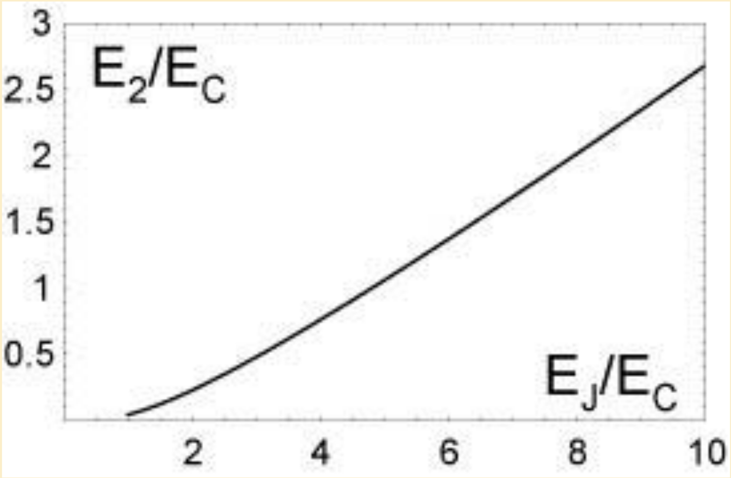


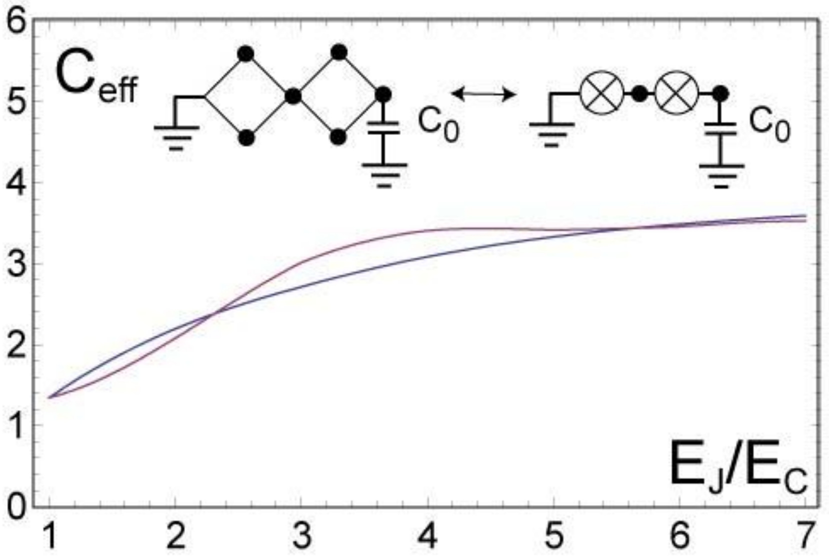
2. Compare the result with the ones obtained for chain of effective junctions
Find parameters (E_2,E_C) of effective junctions.

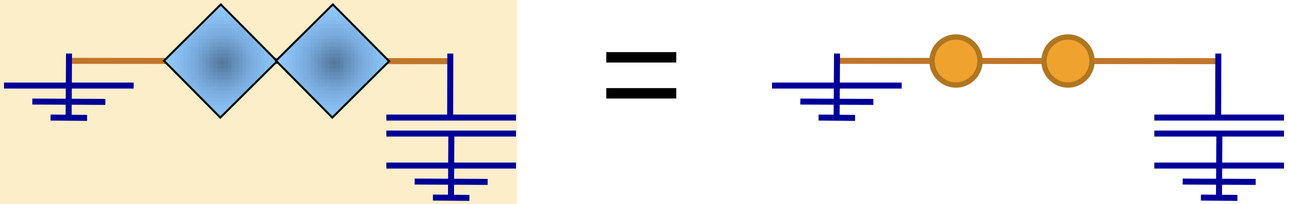


3. Use effective junctions to reduce the number of degrees of freedom in the arrays.

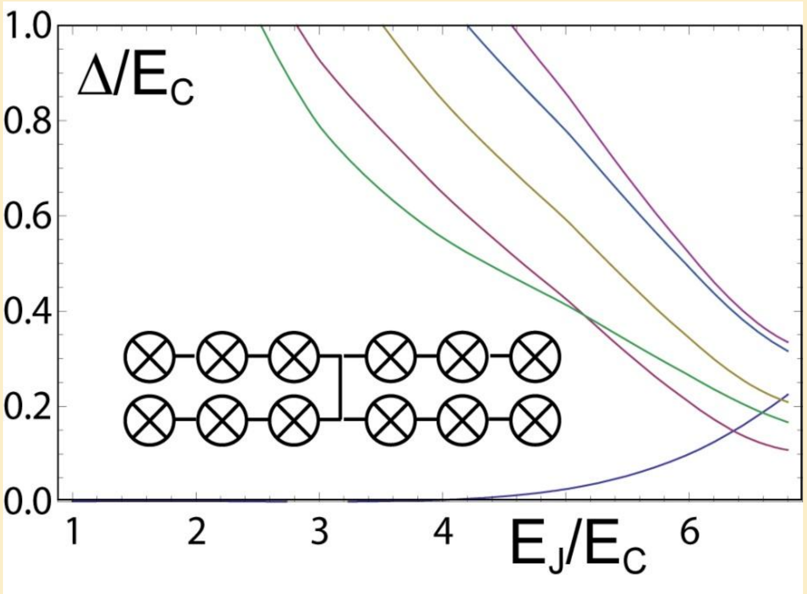
1 RHOMBUS APPROXIMATION



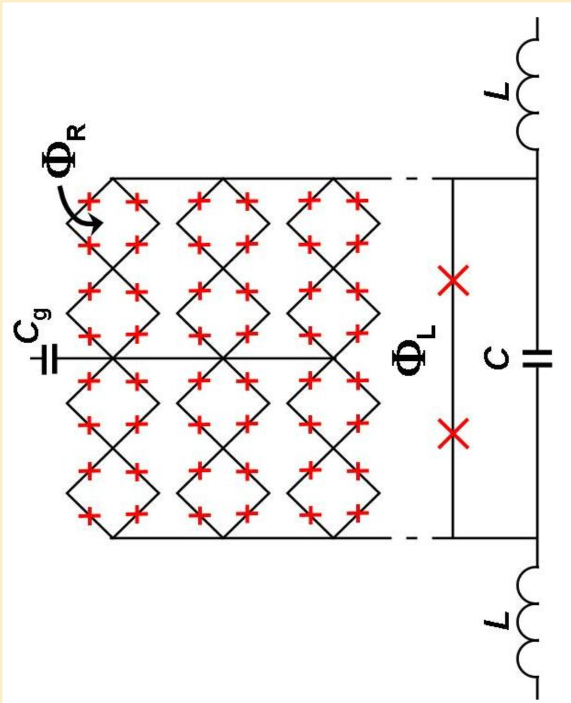




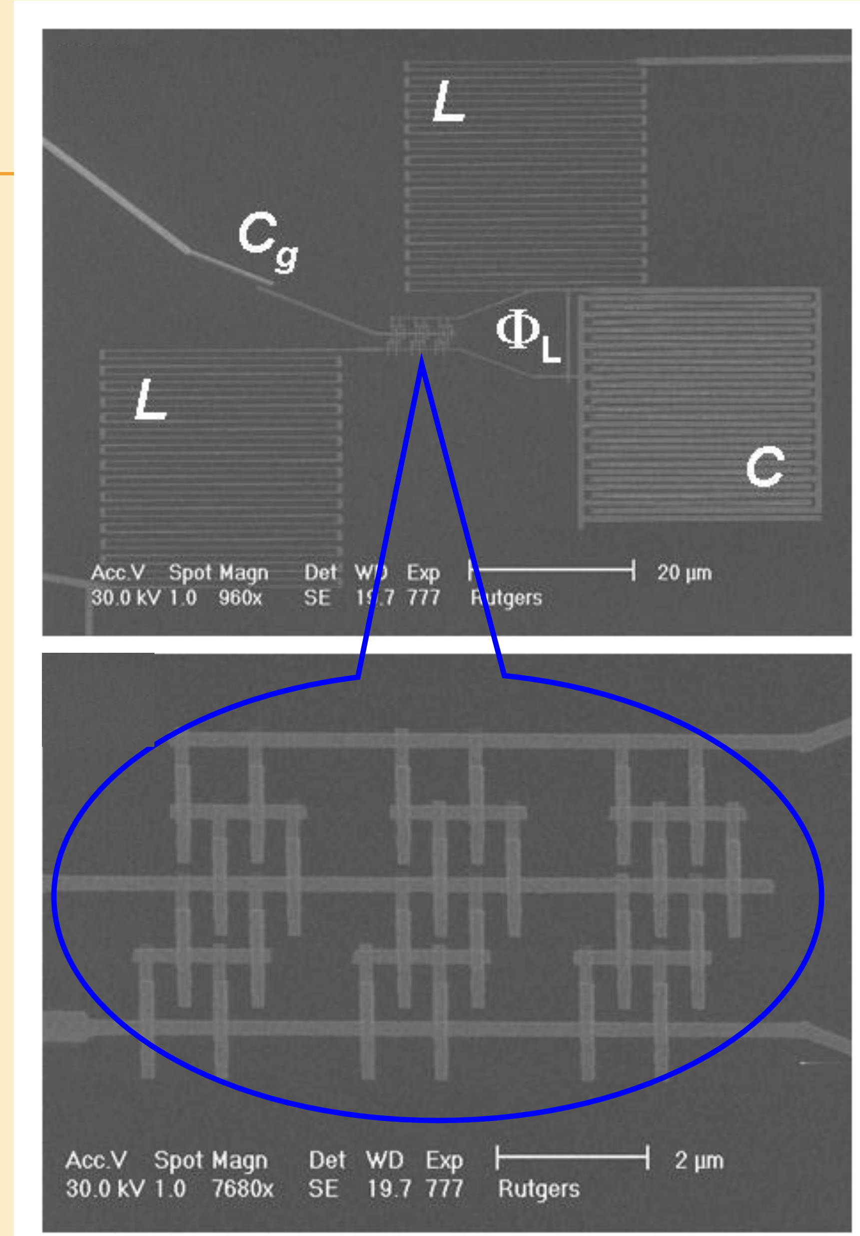
CURRENTLY THE MOST PROMISING DESIGN.



DEVICE

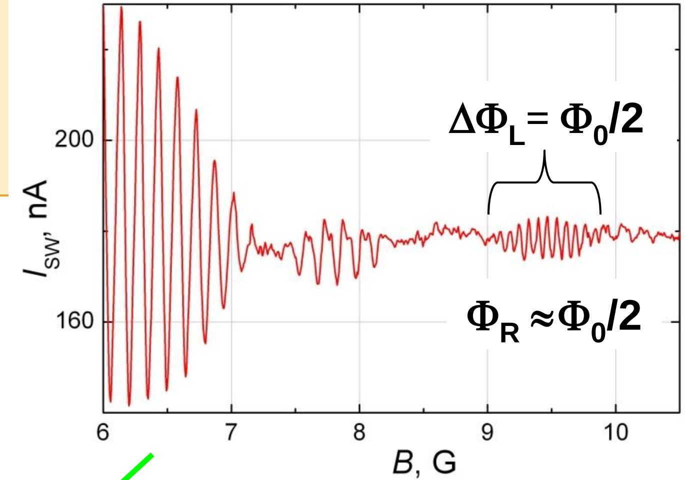


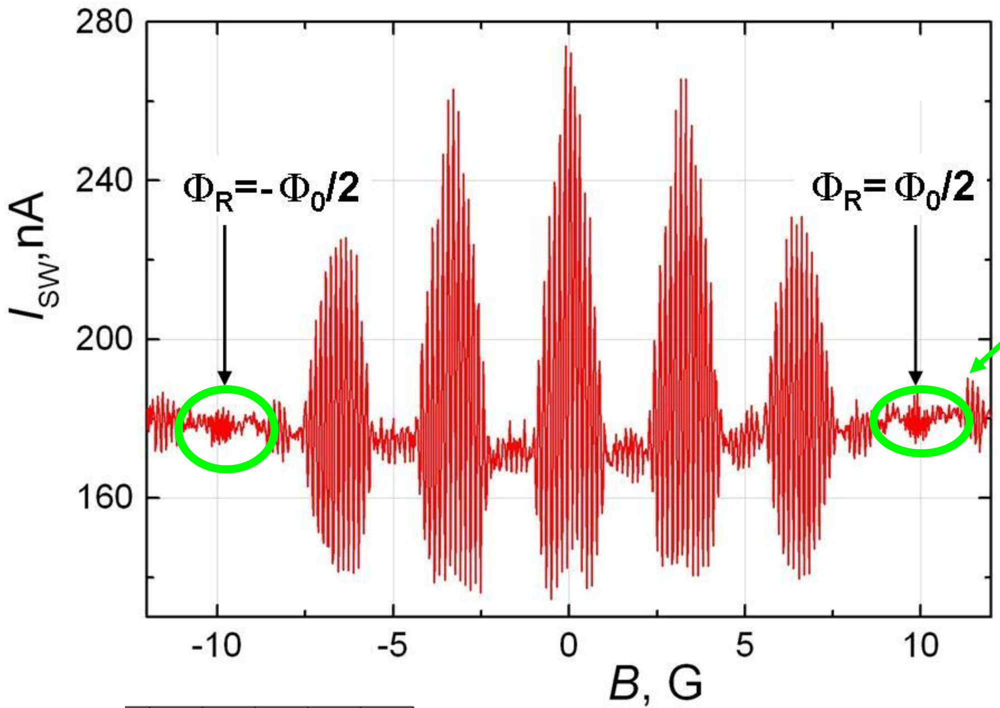
Φ_R
C_g
Φ_L
C
L
L



L
C_g
Φ_L
L
C
Acc.V Spot Magn Det WD Exp
30.0 kV 1.0 960x SE 19.7 777
20 µm
Acc.V Spot Magn Det WD Exp
30.0 kV 1.0 7680x SE 19.7 777
2 µm
Rutgers

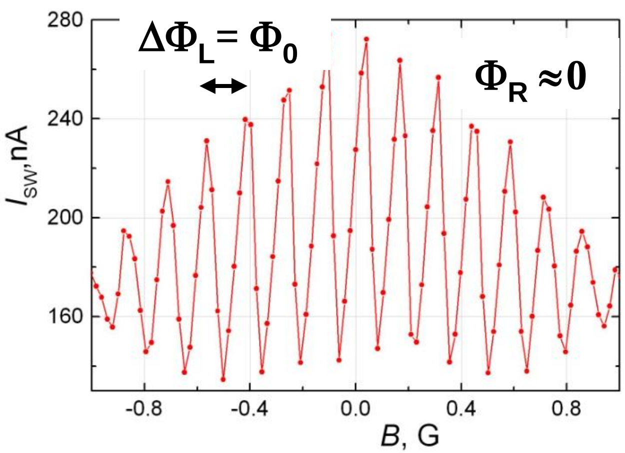
OSCILLATIONS OF SWITCHING CURRENT



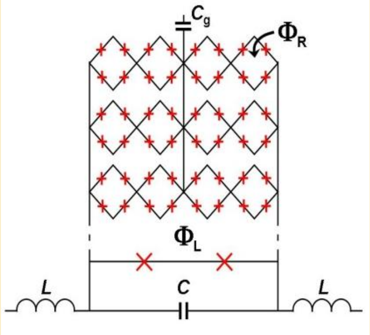


"Frustrated" regime [$\Phi_R \cong (n+1/2)\Phi_0$]: effective E_J is small, quantum fluctuations are large

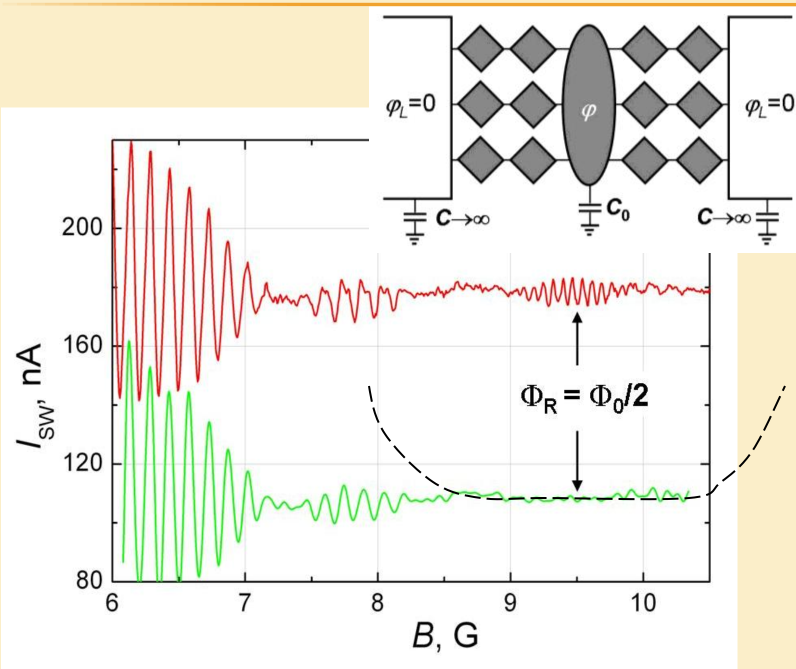
The beating pattern - due to the intermediate-size loops between adjacent rhombi chains with an area 4×(rhombus area)

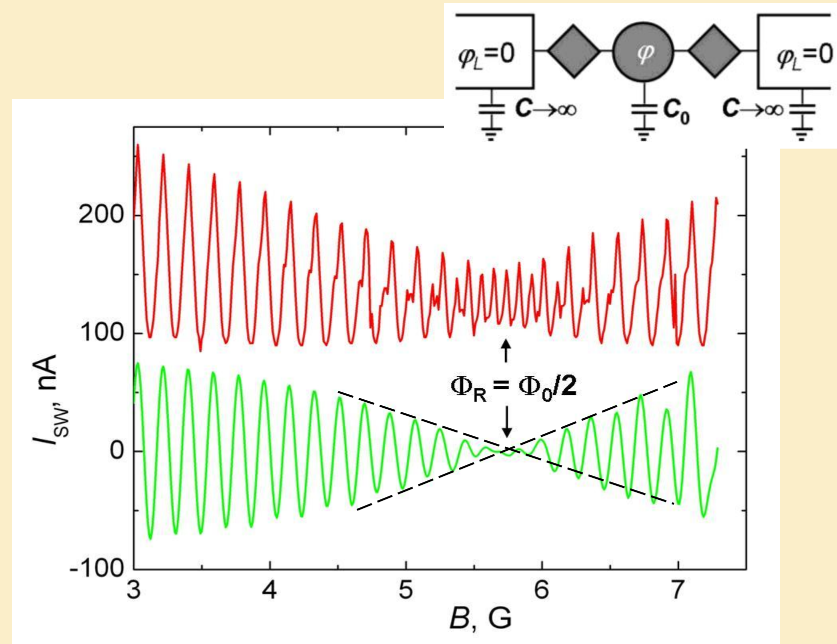


"Non-frustrated" regime [Φ_R far from $(n+1/2)\Phi_0$]: effective E_J is large, quantum fluctuations are small



OSCILLATIONS IN THE FRUSTRATED REGIME $\Phi_R \cong (N+1/2)\,\Phi_0$: CORRELATED TRANSPORT OF PAIRS OF COOPER PAIRS



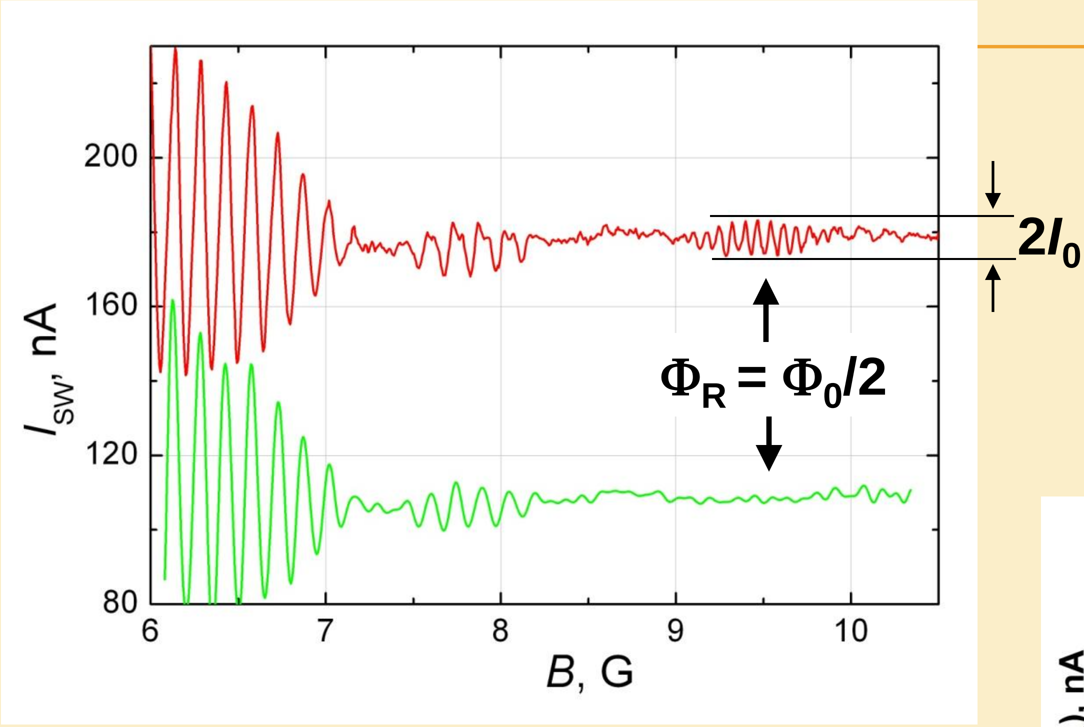


When Φ_R ~ (n+1/2) Φ_0, the effective Josephson energy of a rhombus is small, and the supercurrent of single Cooper pairs is blocked by quantum fluctuations.

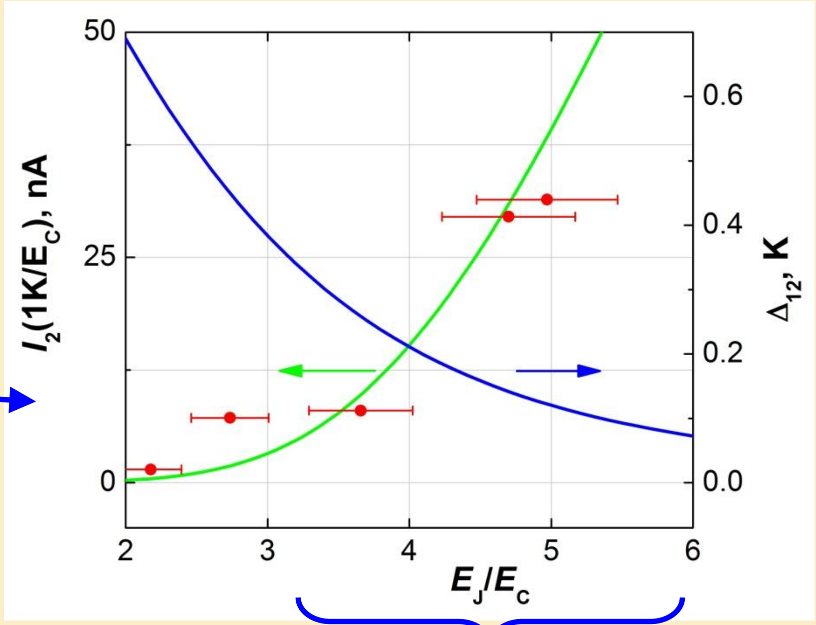
The oscillations of I_{SW} with the period $\Delta\Phi_L = \Phi_0/2$ are due to the correlated transport of pairs of Cooper pairs with charge 4e.

The first harmonic (the un-attenuated effect of Φ_L) is suppressed in the *N*=4 chain well beyond the linear order. ☺

COMPARISON WITH THE THEORY



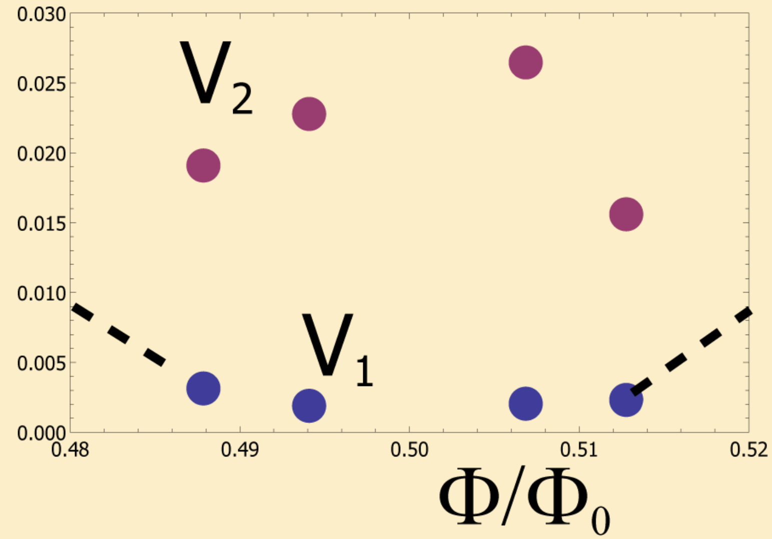
The amplitude I_2 of the second harmonic is in good agreement with our numerical simulations

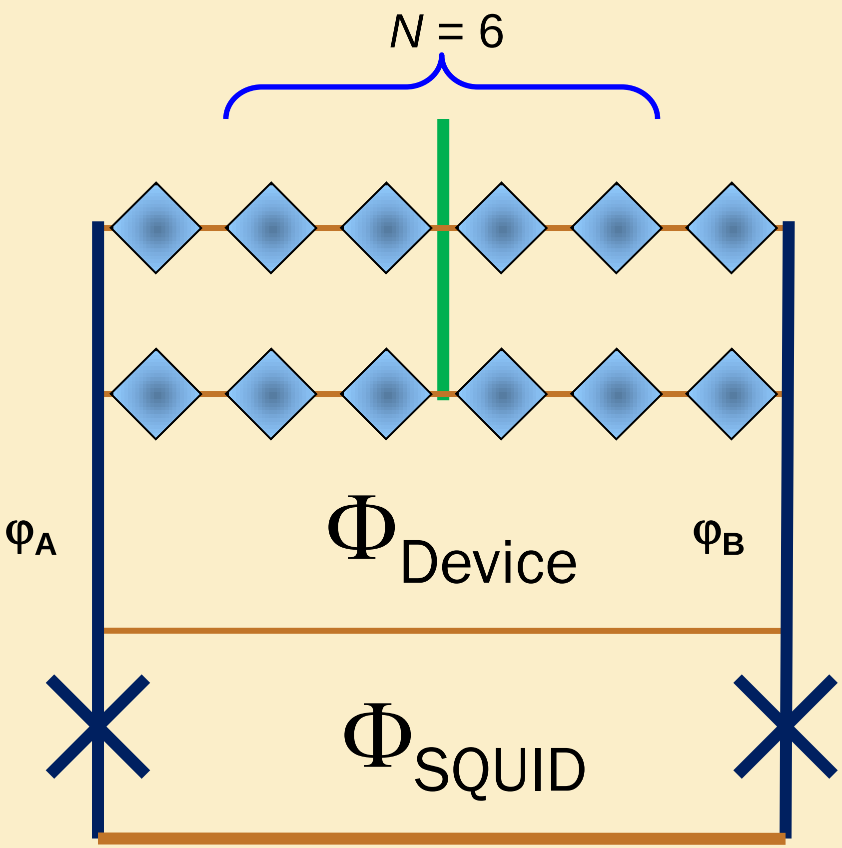


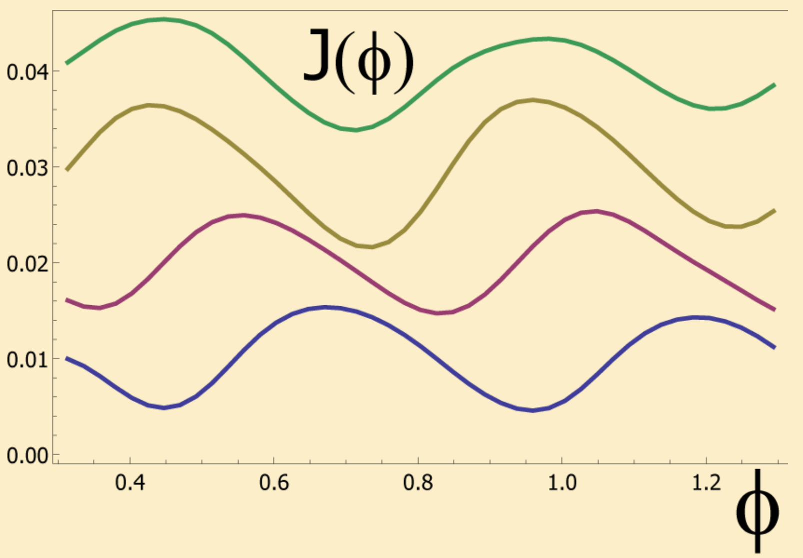
The experiment confirms that the rhombi fluctuate between their two classical states

"Sweet spot":
relatively large Δ_{12} and I_2

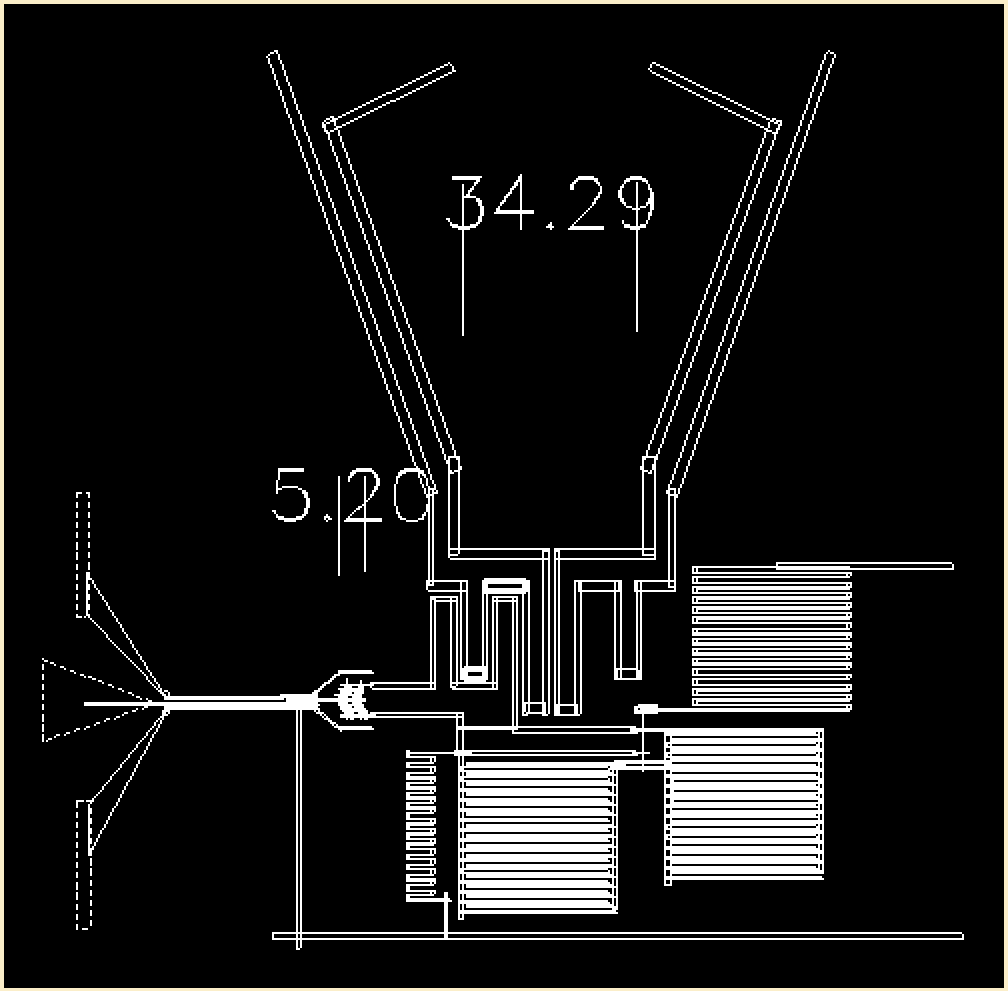
DIRECT MEASUREMENT OF CURRENT-PHASE DEVICE CHARACTERISTICS

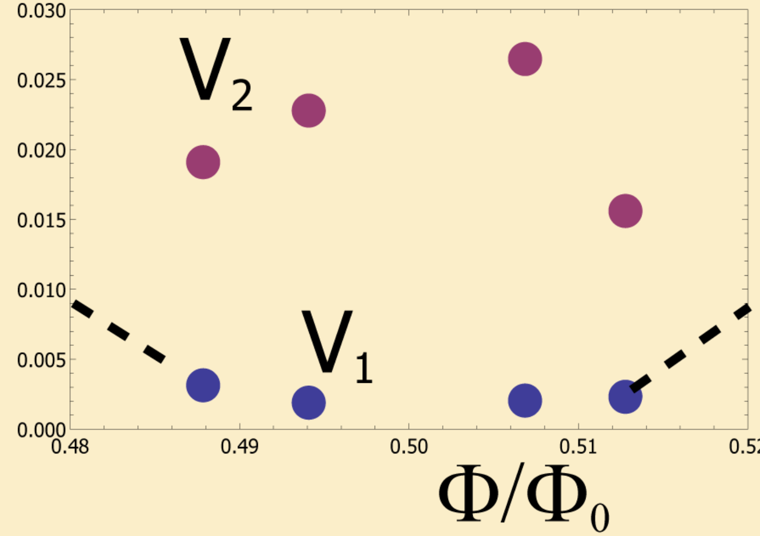


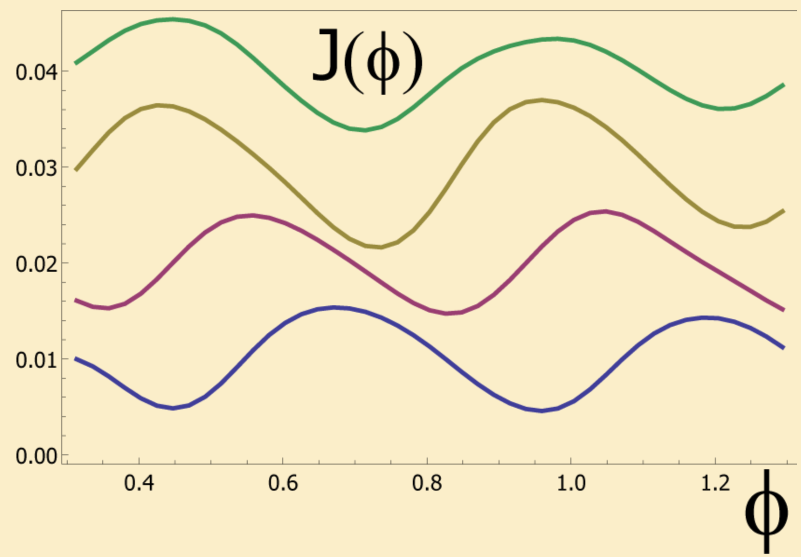




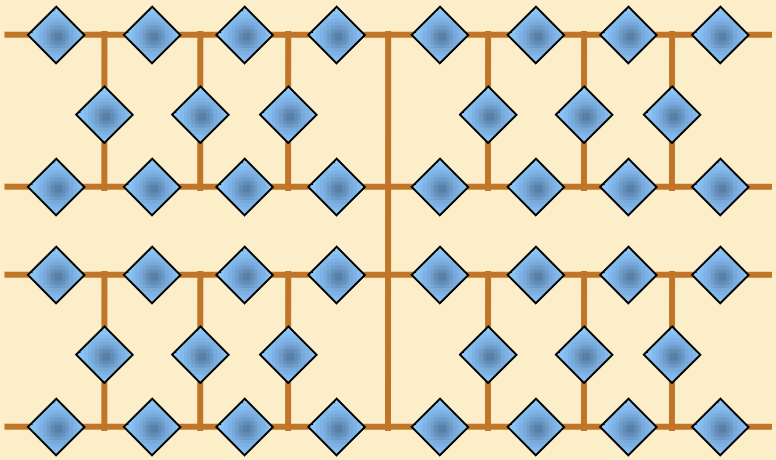
DIRECT MEASUREMENT OF CURRENT-PHASE DEVICE CHARACTERISTICS



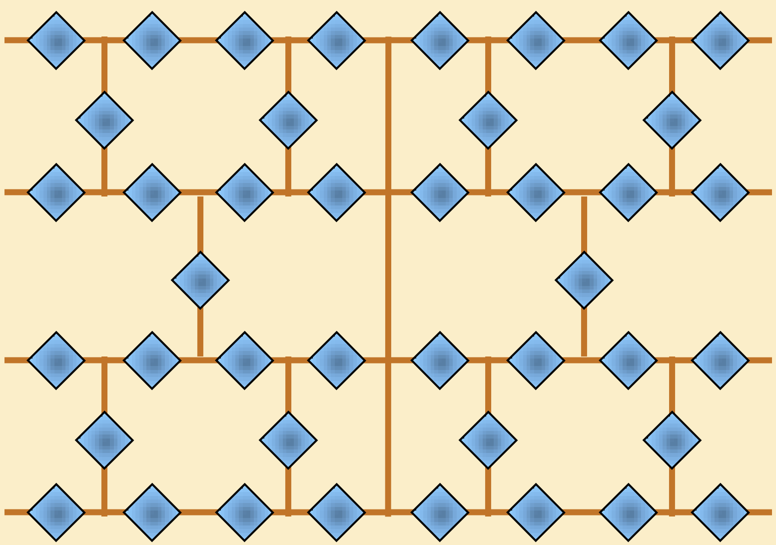




ALTERNATATIVE DESIGNS TO CONSIDER



Double chain

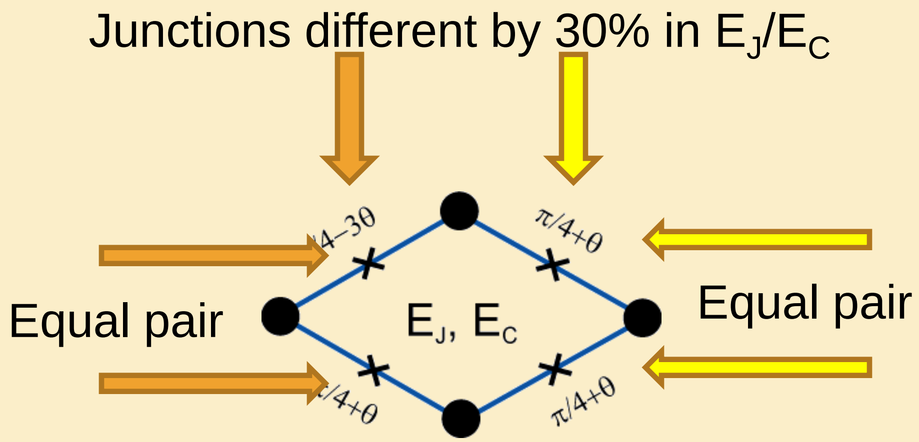


Part of the hexagonal array.

EFFECT OF RANDOM STATIC CHARGES ON ARRAY PROPERTIES.

Preliminary result:

1. Change by 10-30% of each rhombus effective capacitance.
2. Effects gets smaller for assymetric rhombi:



RELAXATION AND DECAY RATES OF REALISTIC HIERARCHICAL STRUCTURES

Theory (+simulations):
Optimal regime $E_J \approx$ 6-8 E_C
K=3 hierarchy (N=4)

$$\Gamma_2^{hier} = \Gamma_2 \left(\gamma \frac{\delta\Phi}{\Phi_0} \frac{E_J}{r} \right)^{N-1} \approx \Gamma_2 \left(10 \frac{\delta\Phi}{\Phi_0} \right)^{N-1}$$

$$\Gamma_2^{hier} = \Gamma_2 \left(\gamma' \frac{\delta E_J}{r} \right)^{N-1} \approx \Gamma_2 \left(\frac{\delta E_J}{E_J} \right)^{N-1}$$

Contributions from
- flux (area) variations between the loops

- Josephson junction variations in the same loop

CONCLUSIONS

- Parallel chains of approximately π-periodic discrete Josephson elements should provide ‘topological’ protection from the noise: decoupling in higher orders or suppressed linear order.
- Problem of soft phase fluctuations in long chains can be solved by hierarchical construction
- Experimental realization shows appearance of π-periodicity which magnitude is in (rough) agreement with theoretical predictions and suppression of 2π-periodicity.