laboratoire pierre aigrain
électronique et photonique quantiques

Building a Quantum Limited Amplifier from Josephson Junctions and Resonators

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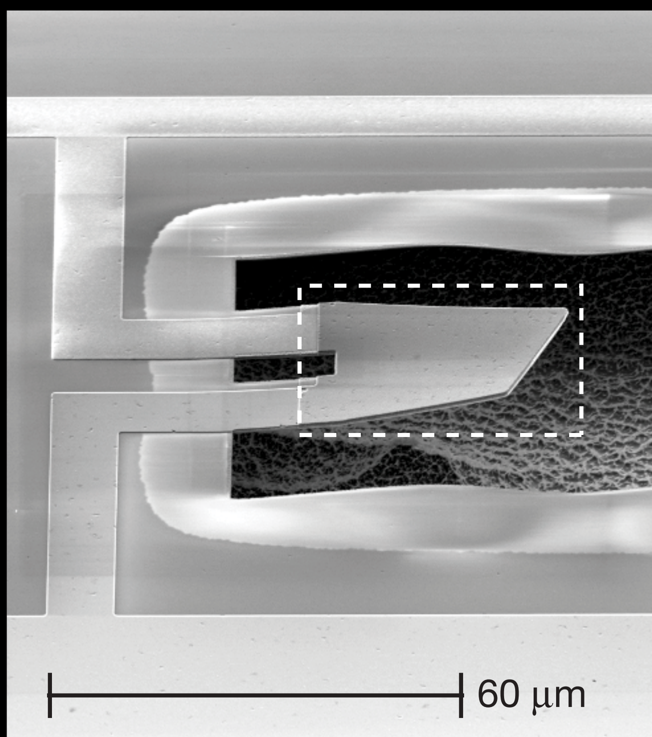
Emmanuel Flurin

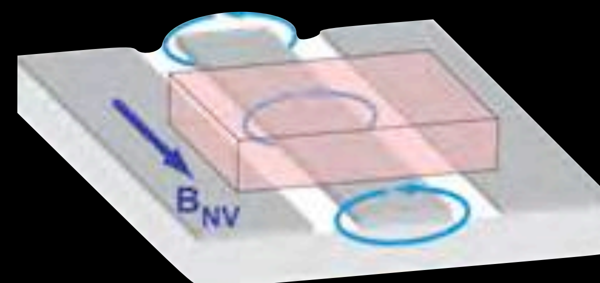

Philippe Campagne


Michel Devoret

Probing Quantum objects with microwave signals

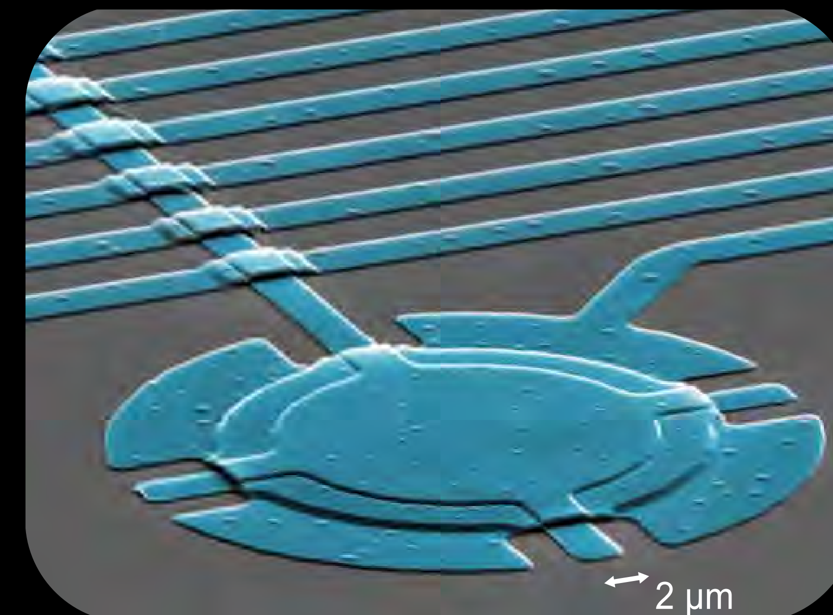
Santa Barbara 2010
see Andrew's talk at 3pm



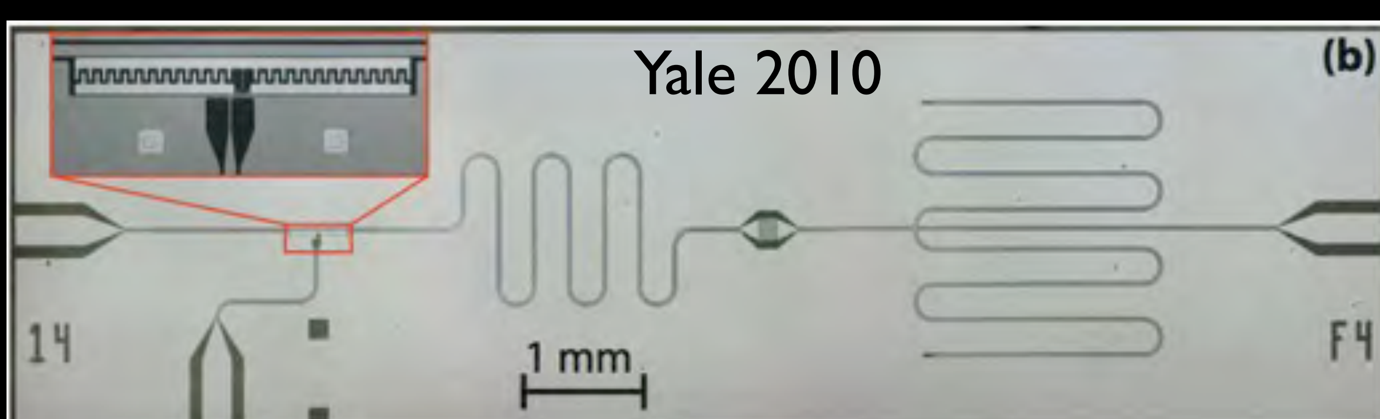


Saclay 2011

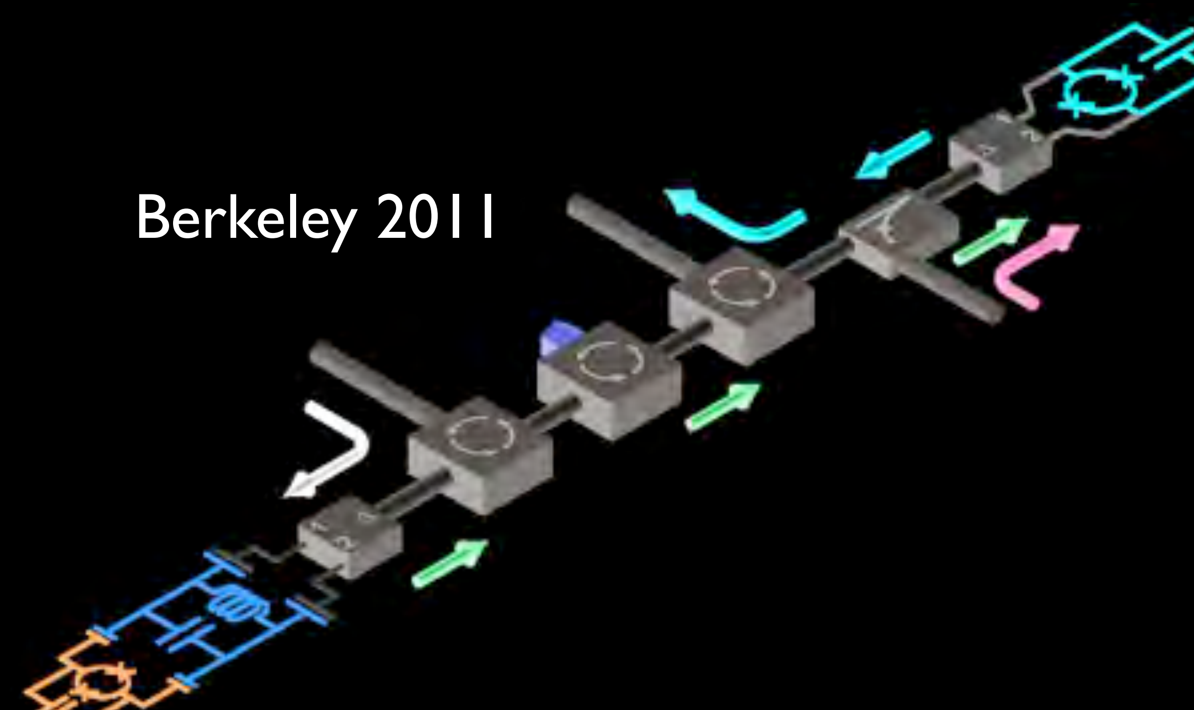
Boulder 2011



Yale 2010

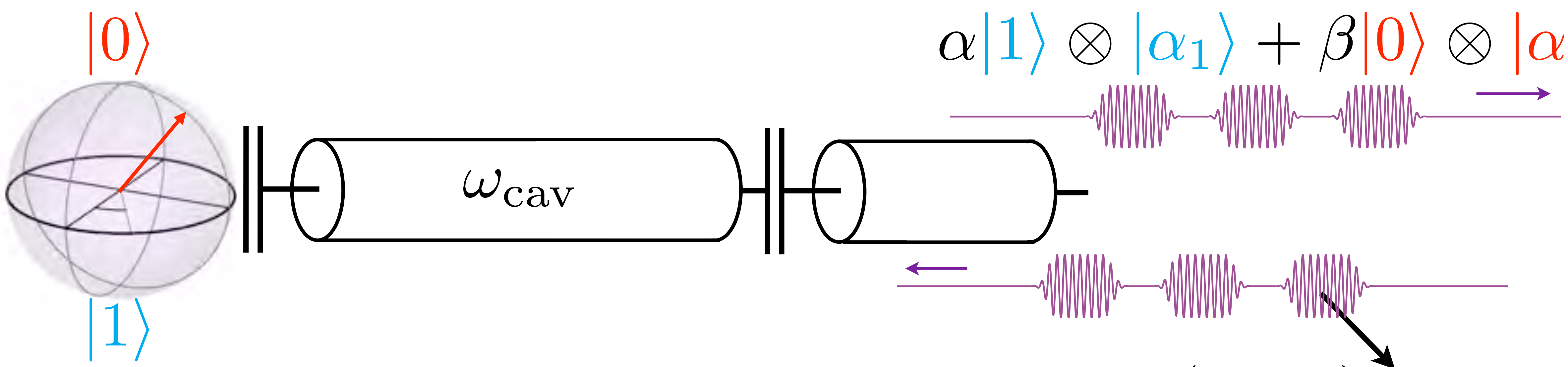


Berkeley 2011



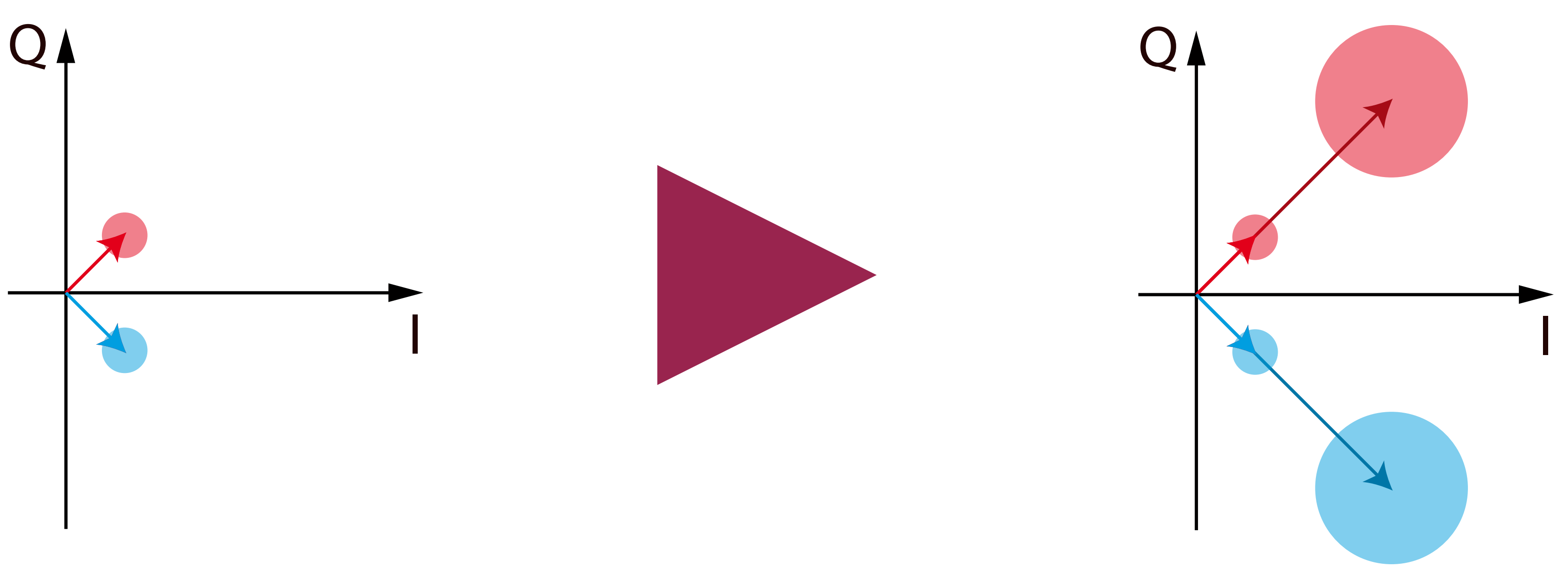
Example: measuring the state of a Qbit

$$\alpha|1\rangle \otimes |\alpha_1\rangle + \beta|0\rangle \otimes |\alpha_0\rangle$$



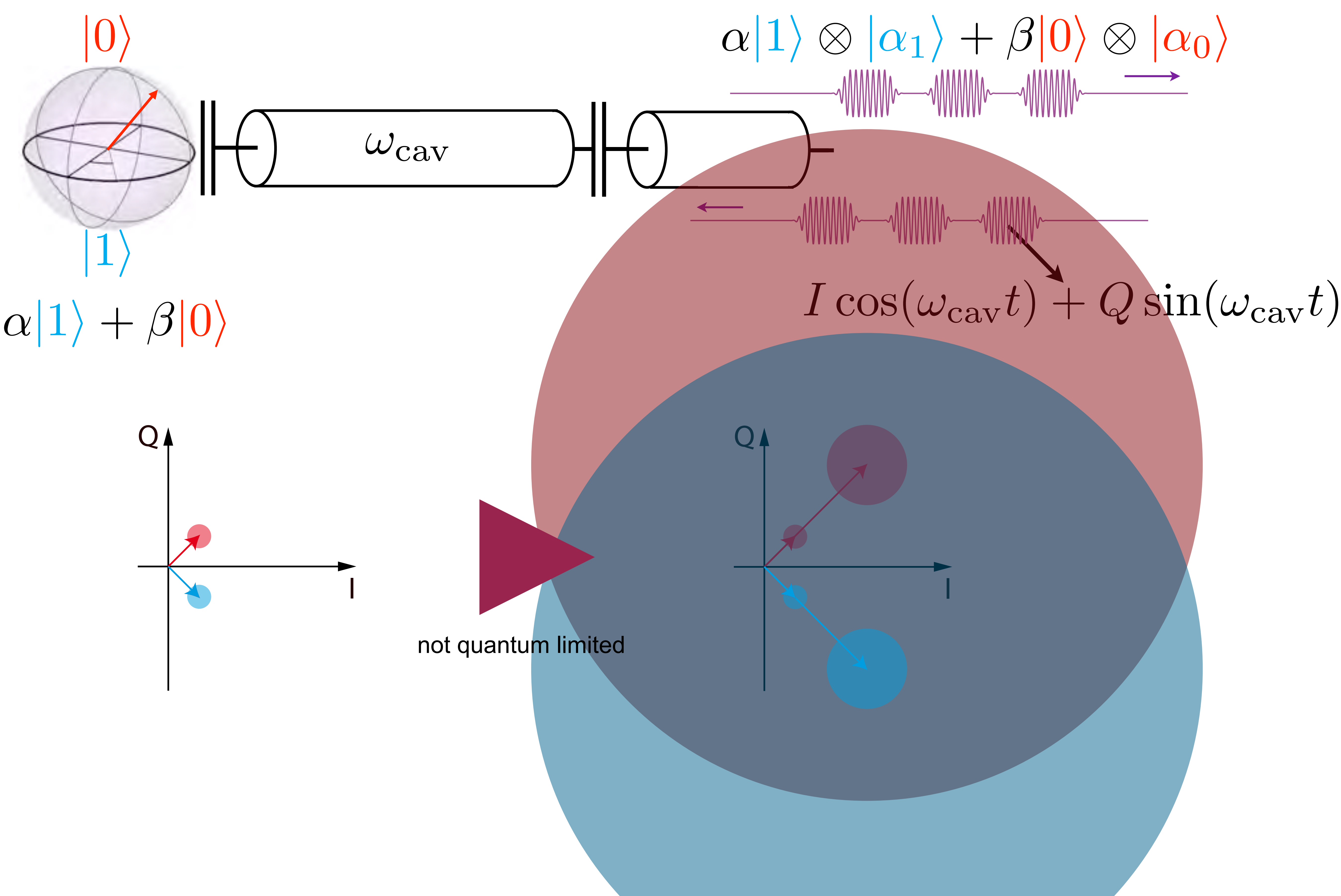
$$I\cos(\omega_{\rm cav}t) + Q\sin(\omega_{\rm cav}t)$$

$$\alpha|1\rangle + \beta|0\rangle$$



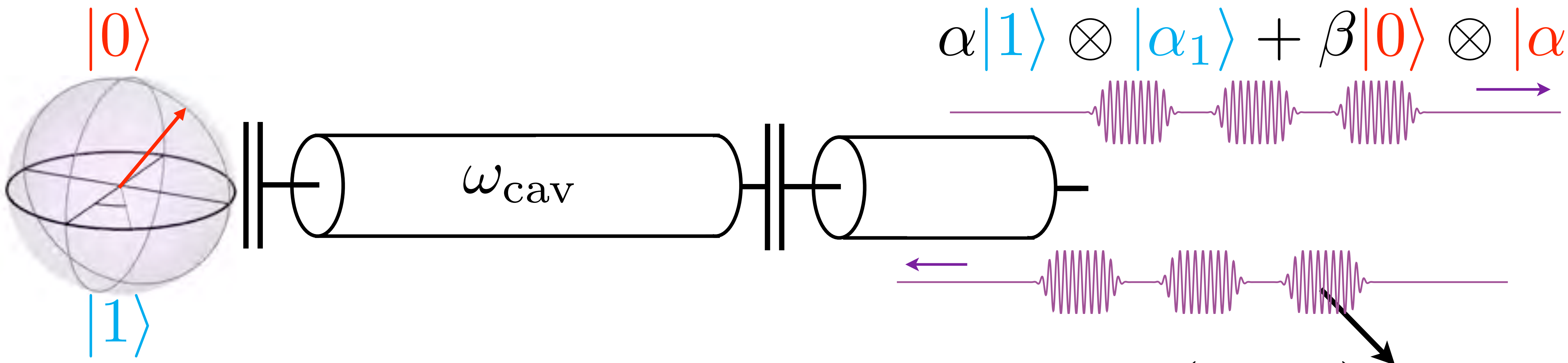
[see Lecture V]

Why do we need good amplifiers ?



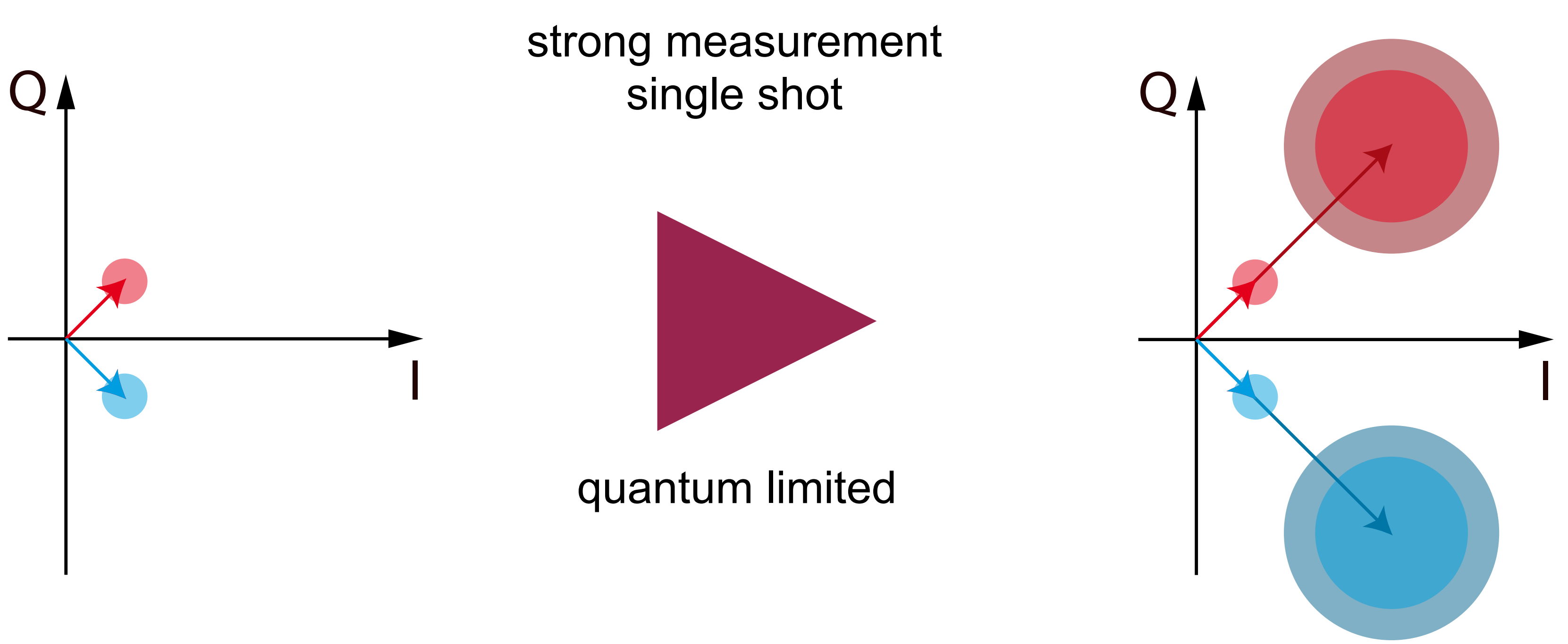
Why do we need good amplifiers ?

$$\alpha|1\rangle \otimes |\alpha_1\rangle + \beta|0\rangle \otimes |\alpha_0\rangle$$



$$\alpha|1\rangle + \beta|0\rangle$$

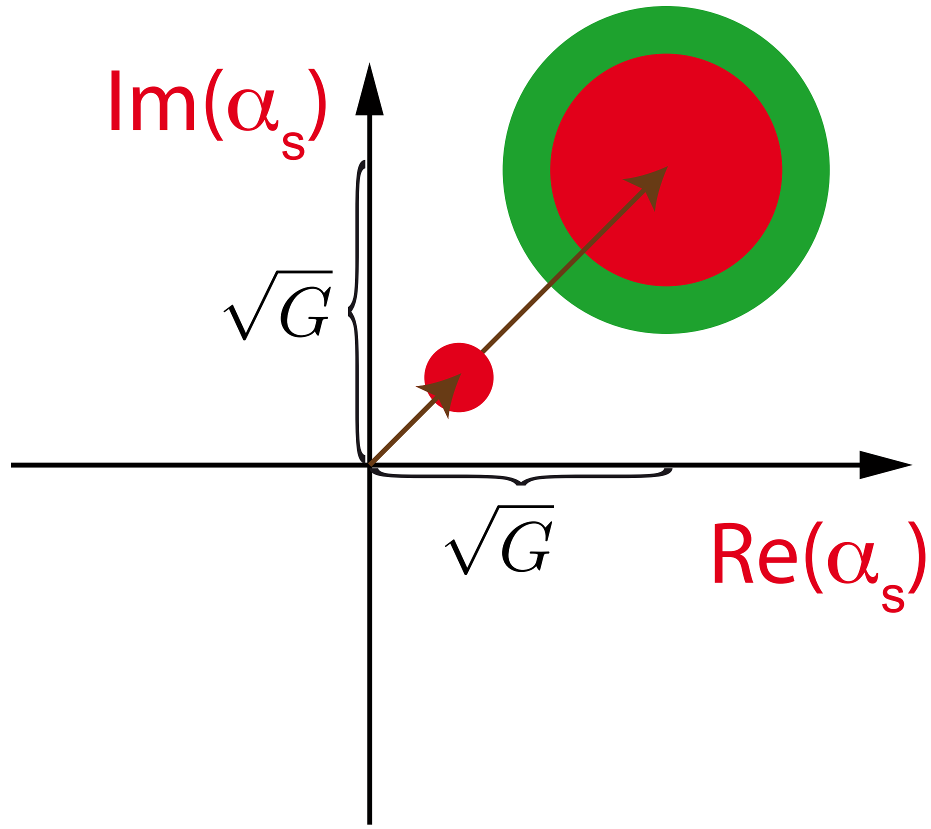
$$I\cos(\omega_{\mathrm{cav}}t) + Q\sin(\omega_{\mathrm{cav}}t)$$



Goal: evolution of the quantum object directly given by the measurement outcome

Two kinds of linear amplifiers

phase preserving



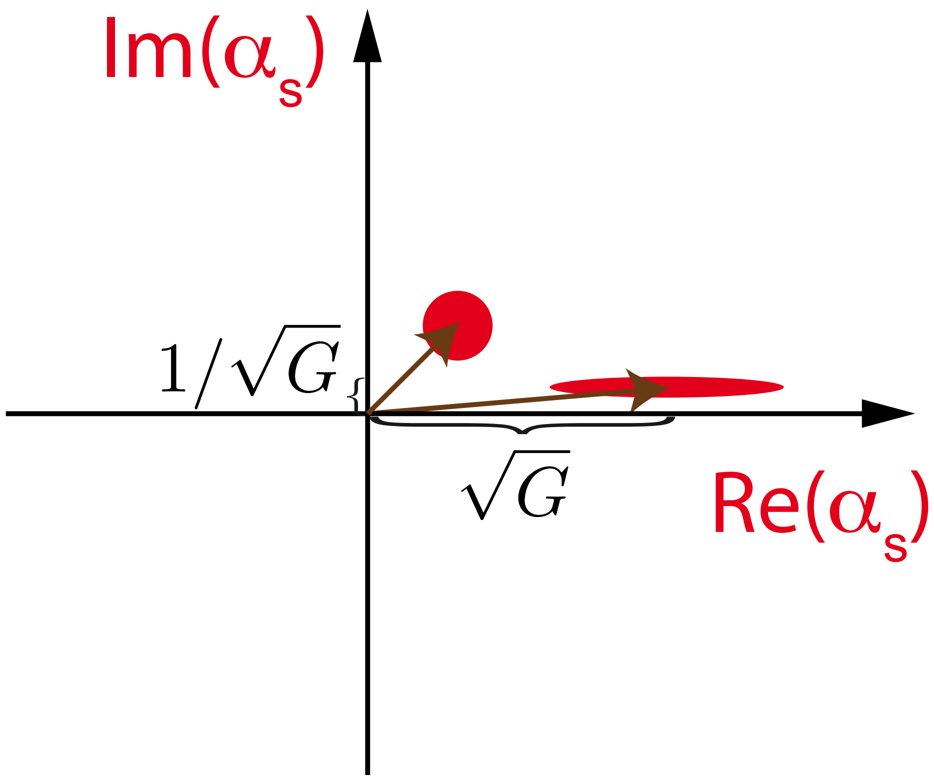
$$\hat{a}_{\rm out} = \sqrt{G}\hat{a}_{\rm in} + \hat{\mathcal{N}}$$

$$[\hat{a}_{\rm out}, \hat{a}^\dagger_{\rm out}] = 1 \Rightarrow [\hat{\mathcal{N}}^\dagger, \hat{\mathcal{N}}] = G - 1$$

$$\Delta\hat{\mathcal{N}}^2 = \frac{1}{2}\left\langle\left\{\hat{\mathcal{N}}, \hat{\mathcal{N}}^\dagger\right\}\right\rangle \geq \frac{G-1}{2}$$

[Caves, PRD (1982), Caltech HEMTs]

phase dependent



$$\hat{a}_{\rm out} = \frac{\sqrt{G}}{2}(\hat{a}_{\rm in} + \hat{a}^\dagger_{\rm in}) + \frac{1}{2\sqrt{G}}(\hat{a}_{\rm in} - \hat{a}^\dagger_{\rm in})$$

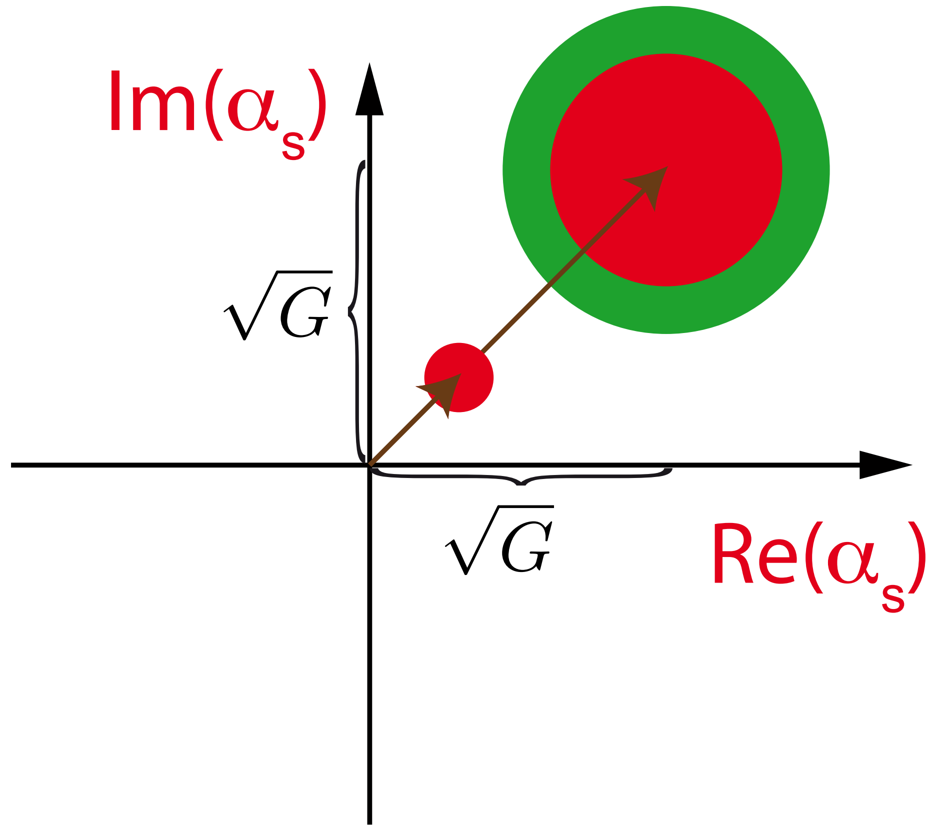
$$[\hat{a}_{\rm out}, \hat{a}^\dagger_{\rm out}] = 1$$

$$\Delta\hat{\mathcal{N}}^2 \geq 0$$

[Yurke et al., PRA (1989), Bell Labs]
[Castellanos-Beltran, Nat Phys. (2008), Boulder]
[Yamamoto et al., APL (2008), RIKEN]...

Two kinds of linear amplifiers

phase preserving



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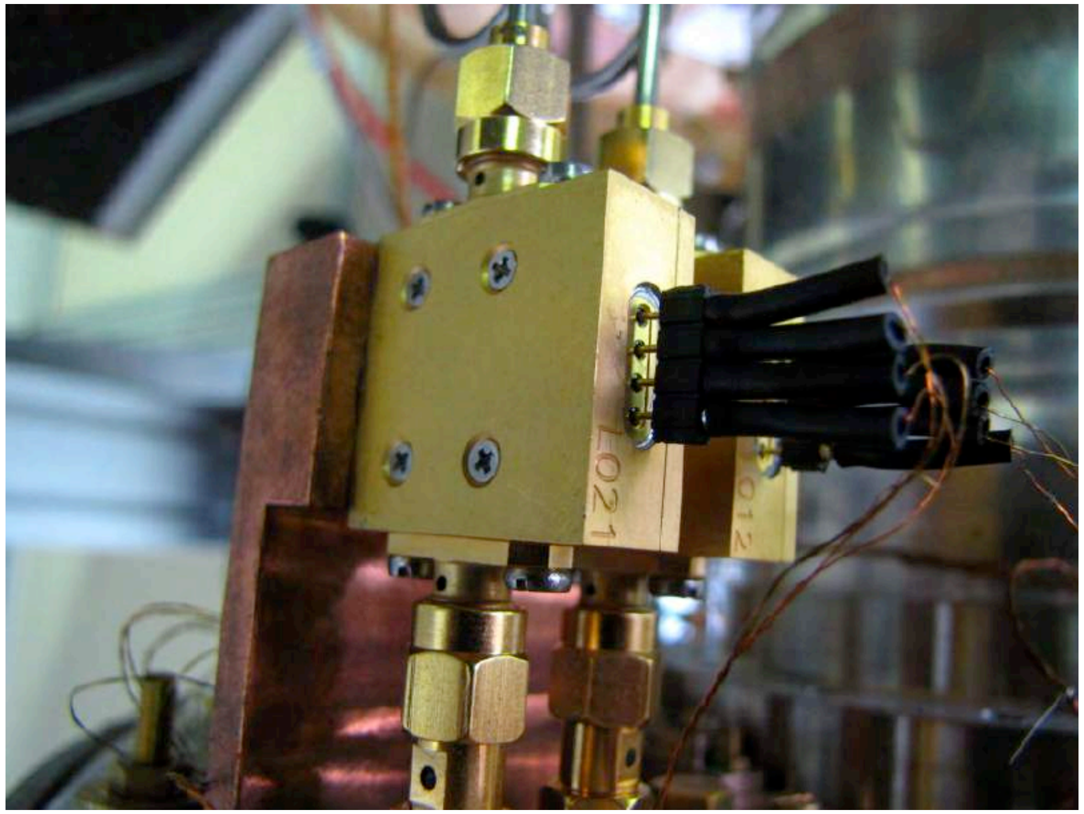
[Caves, PRD (1982), Caltech]

best commercial amplifiers

Caltech

LOW NOISE FACTORY



$$\Delta\hat{\mathcal{N}}^2 \approx 30\text{–}40\,\frac{(G-1)}{2}$$

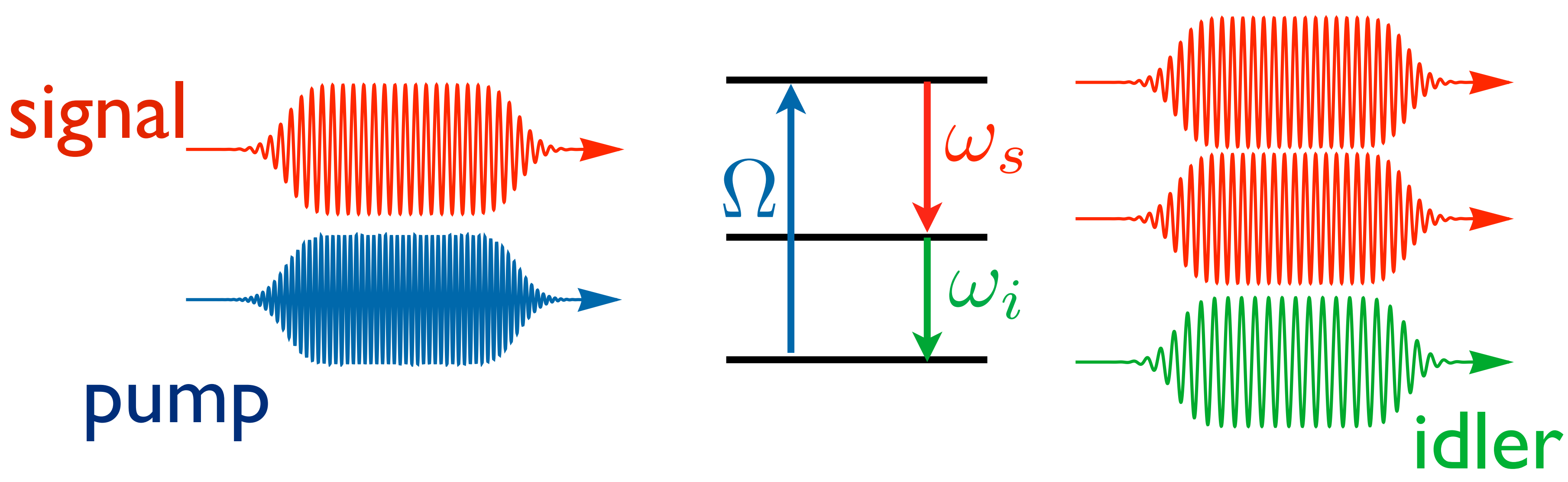
Core of the amplifier: 3-wave mixing

$$\hat{H}_{Mix} = \hbar g^{(3)} (a_S^\dagger a_I^\dagger a_P + a_S a_I a_P^\dagger)$$

[see Lecture III]

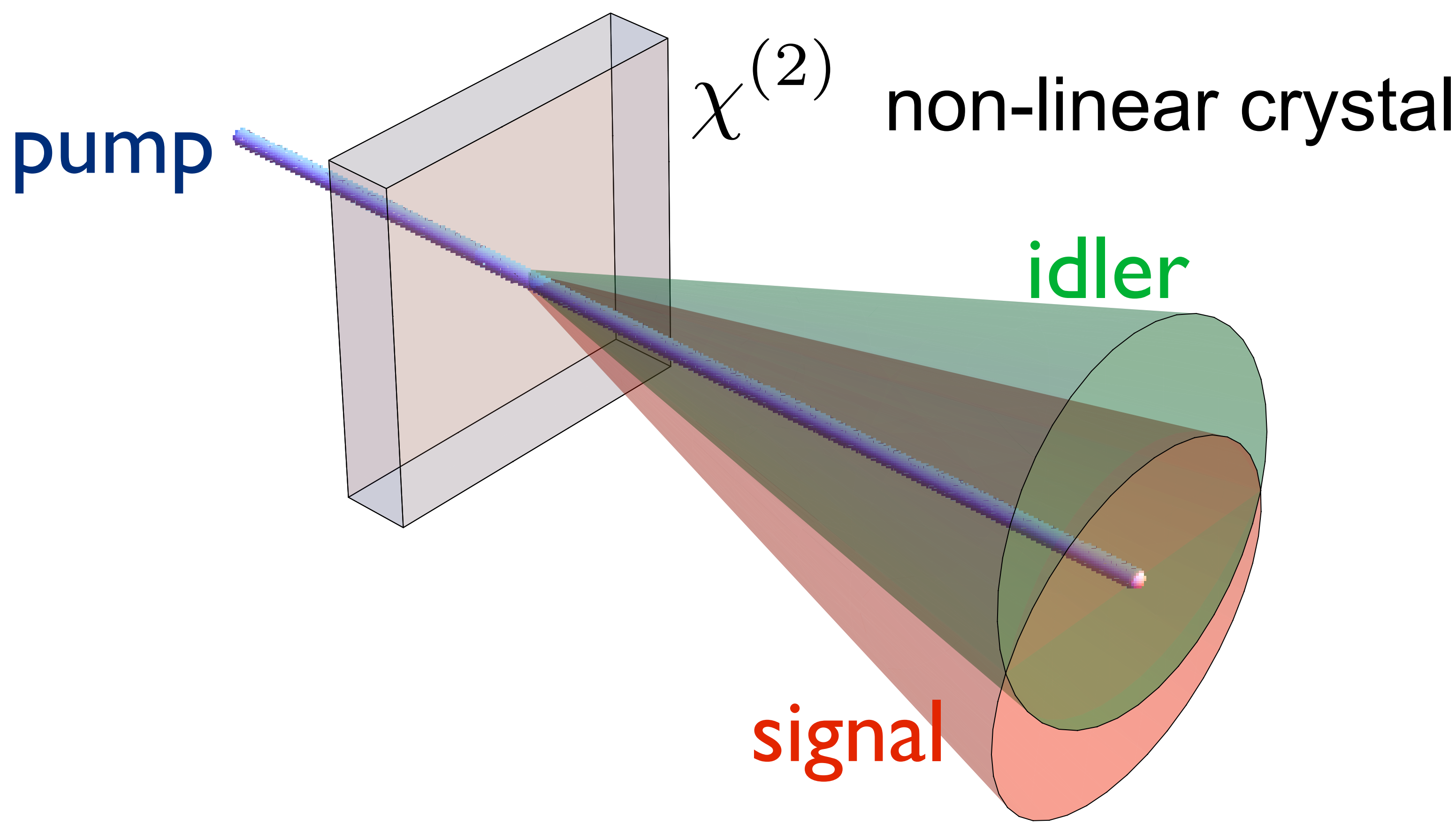
Basis of amplification
stimulated emission

$$\hat{H}_{Mix} |1_S, 0_I, \alpha_P\rangle = \alpha_P \hbar g^{(3)} |2_S, 1_I, \alpha_P\rangle$$

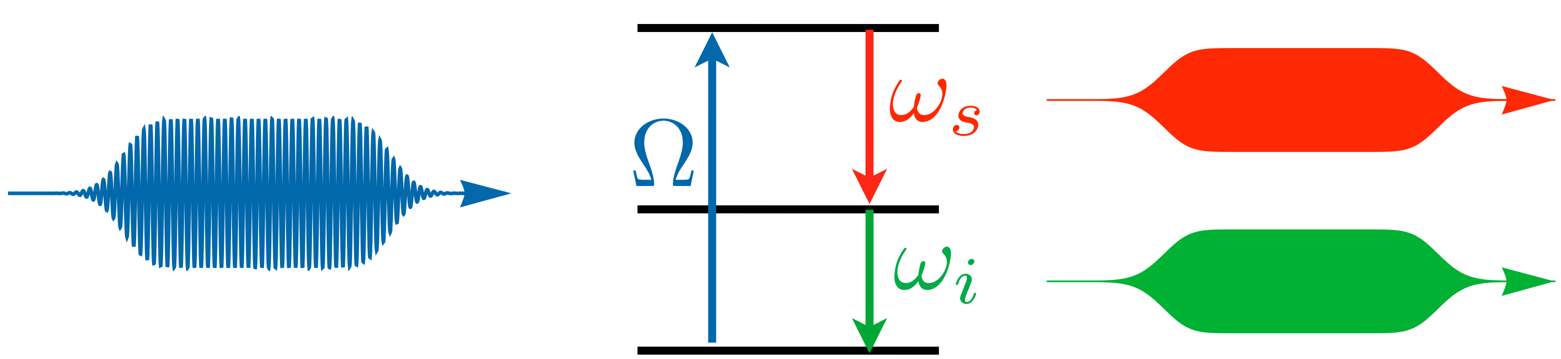


Implementation in optics: non-linear crystal

$$\hat{H}_{Mix} = \hbar g^{(3)} (a_S^\dagger a_I^\dagger a_P + a_S a_I a_P^\dagger)$$



$$\hat{H}_{Mix} |0_S, 0_I, \alpha_P\rangle = \alpha_P \hbar g |1_S, 1_I, \alpha_P\rangle$$

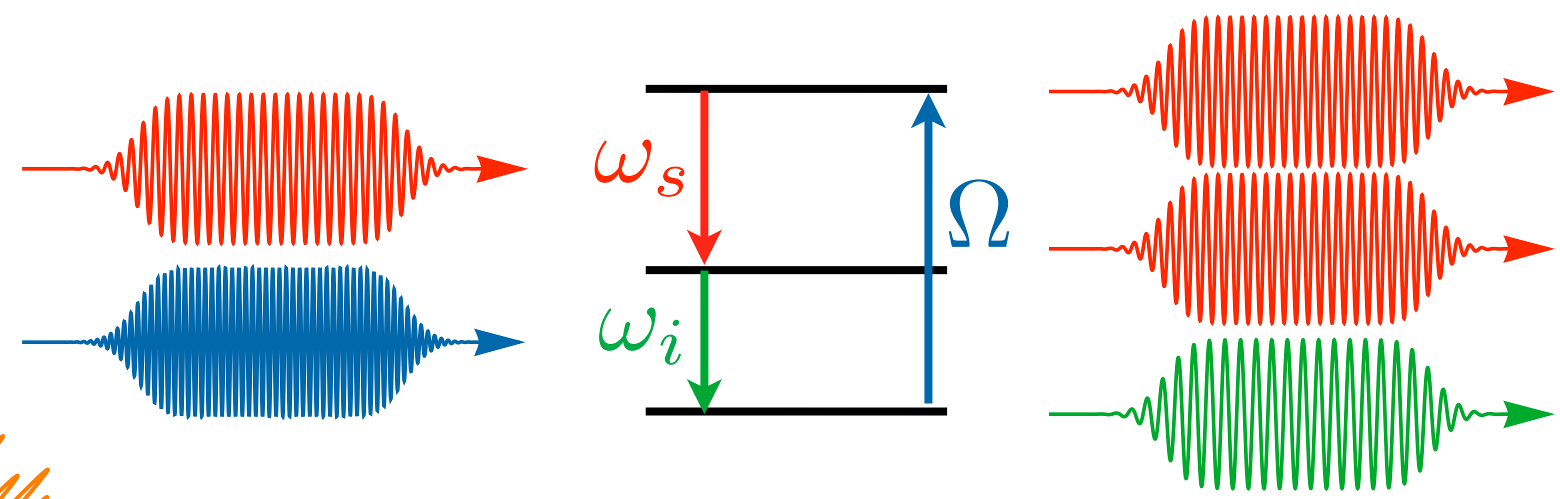


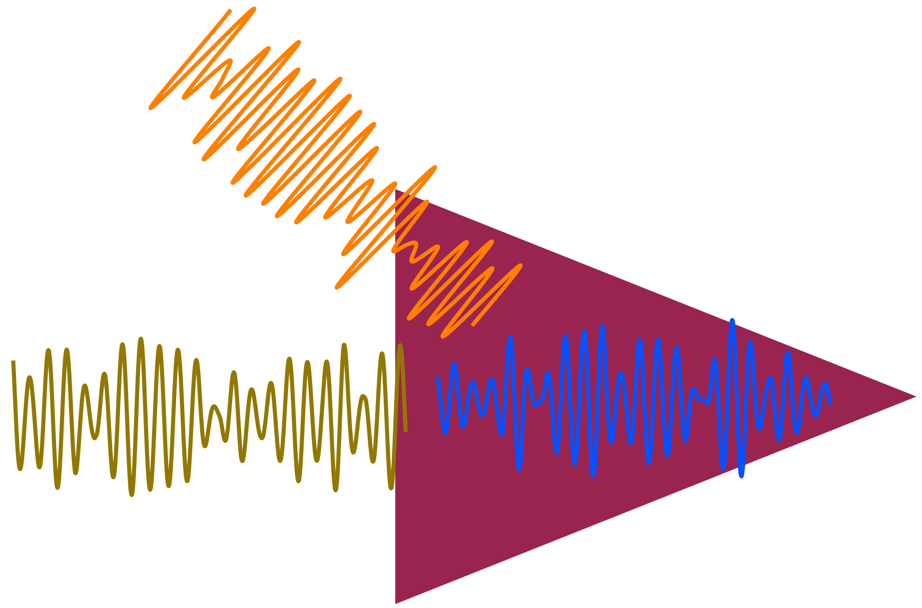
spontaneous parametric down-conversion

How to reach the quantum limit for microwaves ?

$$\Delta\hat{\mathcal{N}}^2 \approx \frac{G-1}{2}$$

Need to minimize the number of information channels to 3

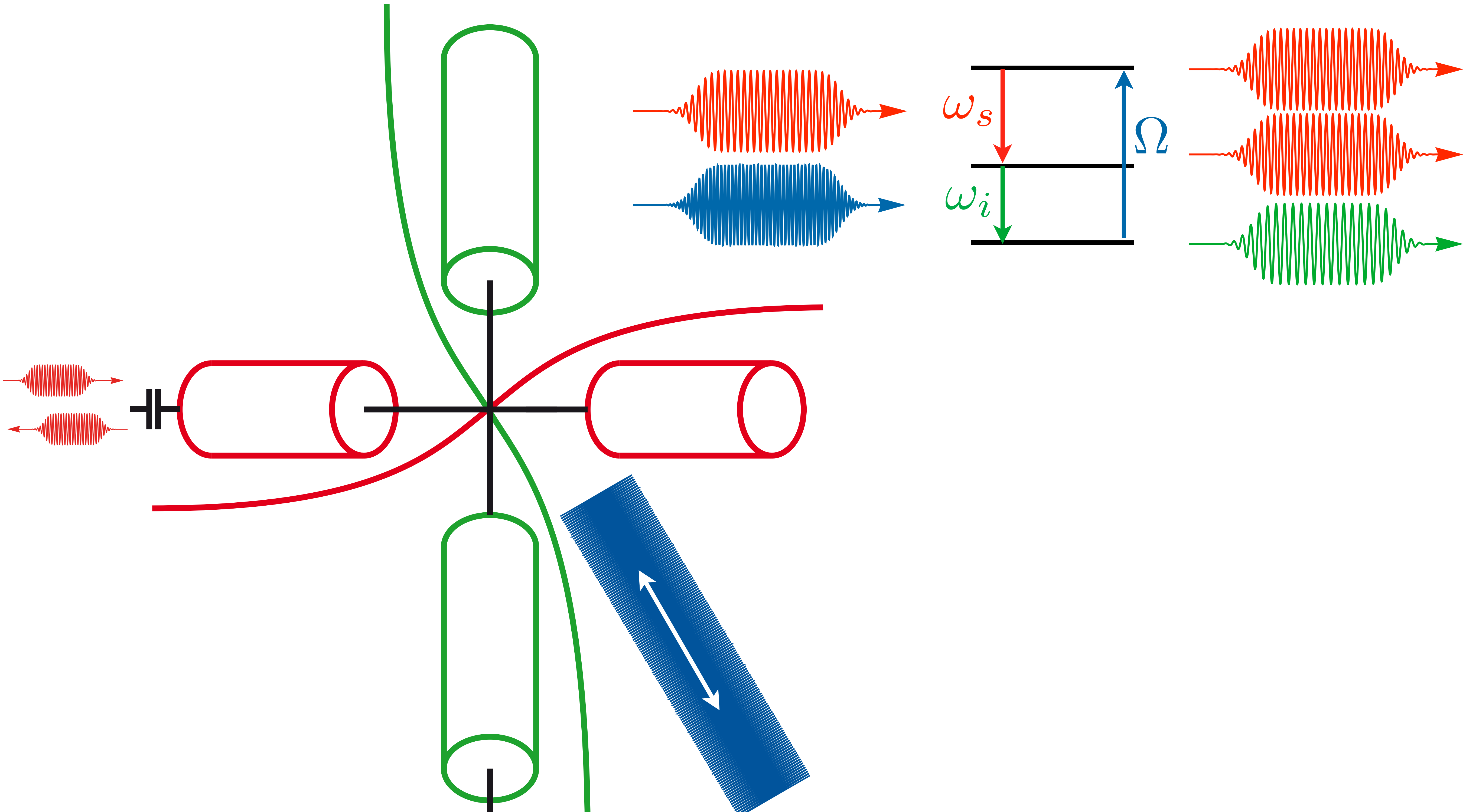


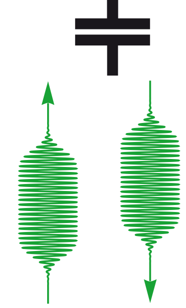


superconducting circuits ⇨ no dissipation
collective degree of freedom

GHz signals ⇨ no thermal photons at dilution fridge temperatures

proper filtering ⇨ no external electromagnetic noise

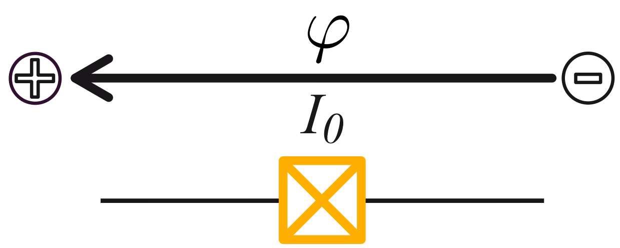




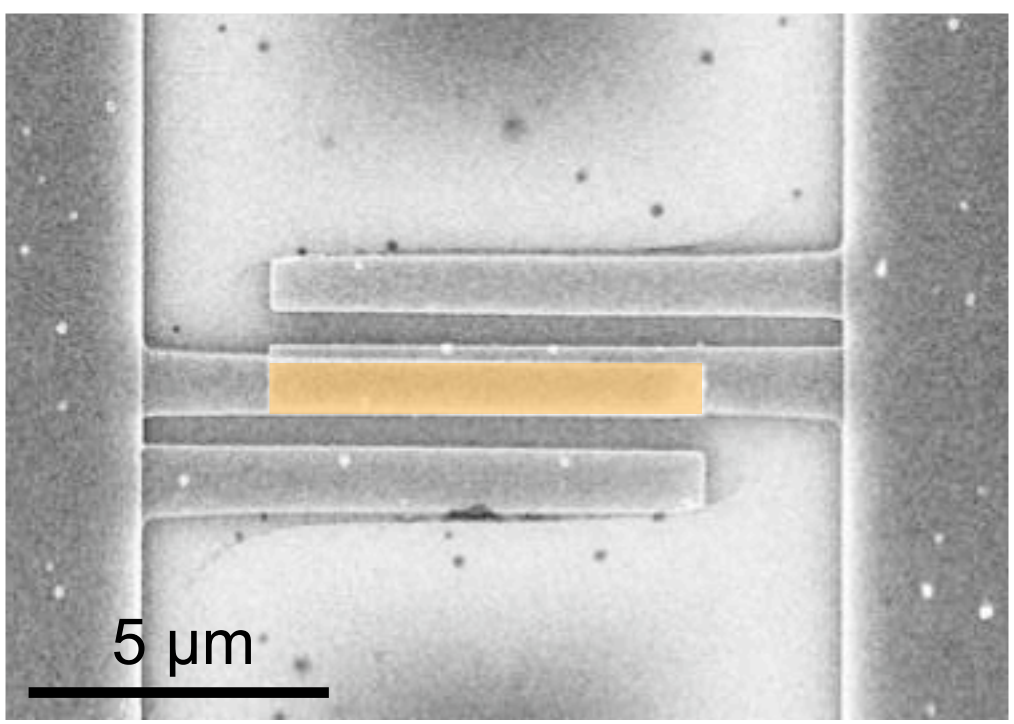
3-wave mixing term needed

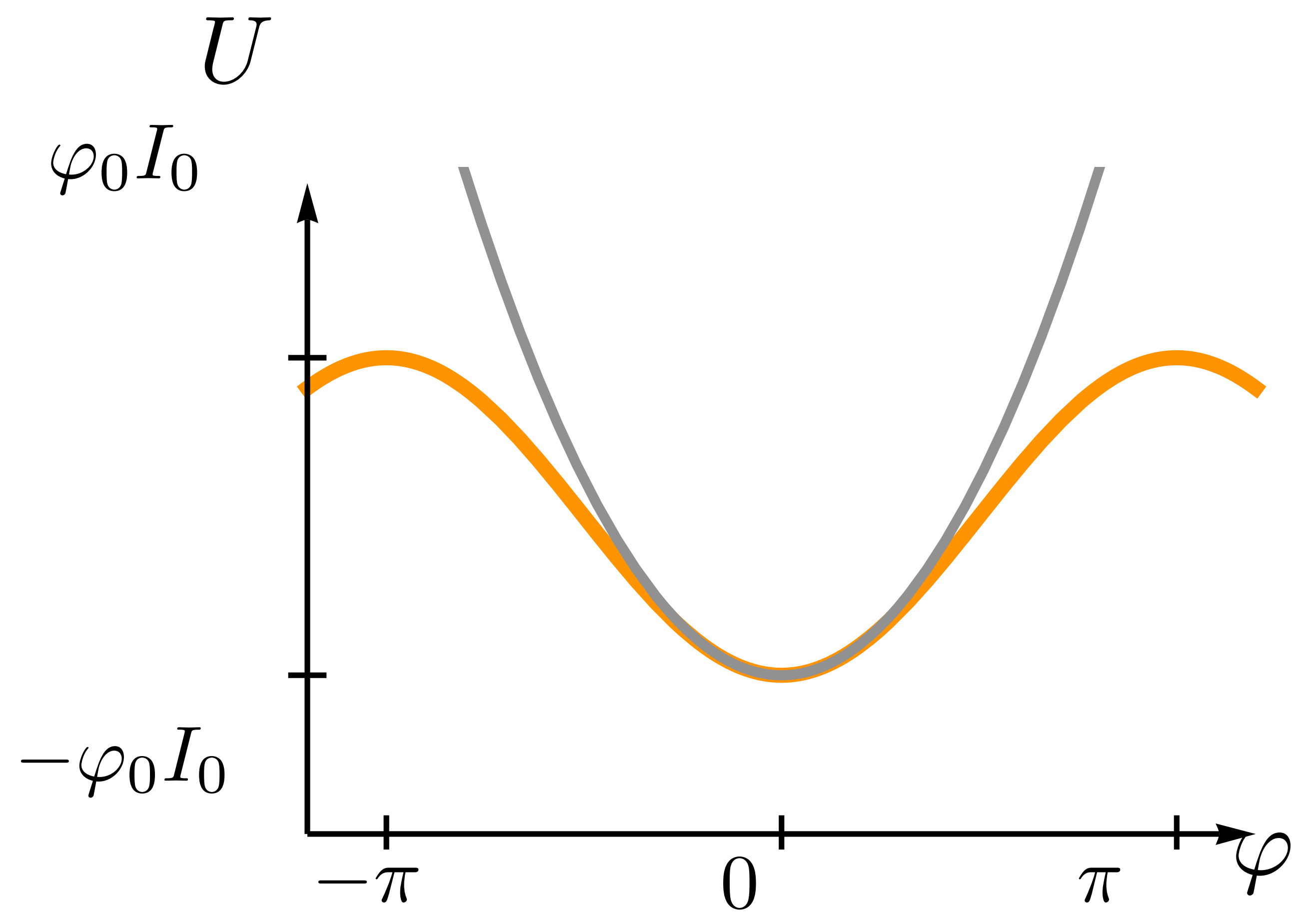
$$\hat{H}_{Mix} = \hbar g^{(3)}\left(a_S^{\dagger} a_I^{\dagger} a_P + a_S a_I a_P^{\dagger}\right)$$

Non linear element: Josephson junction

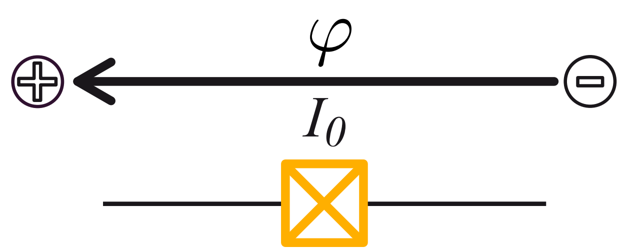


$$U = -\varphi_0 I_0 \cos(\varphi)$$

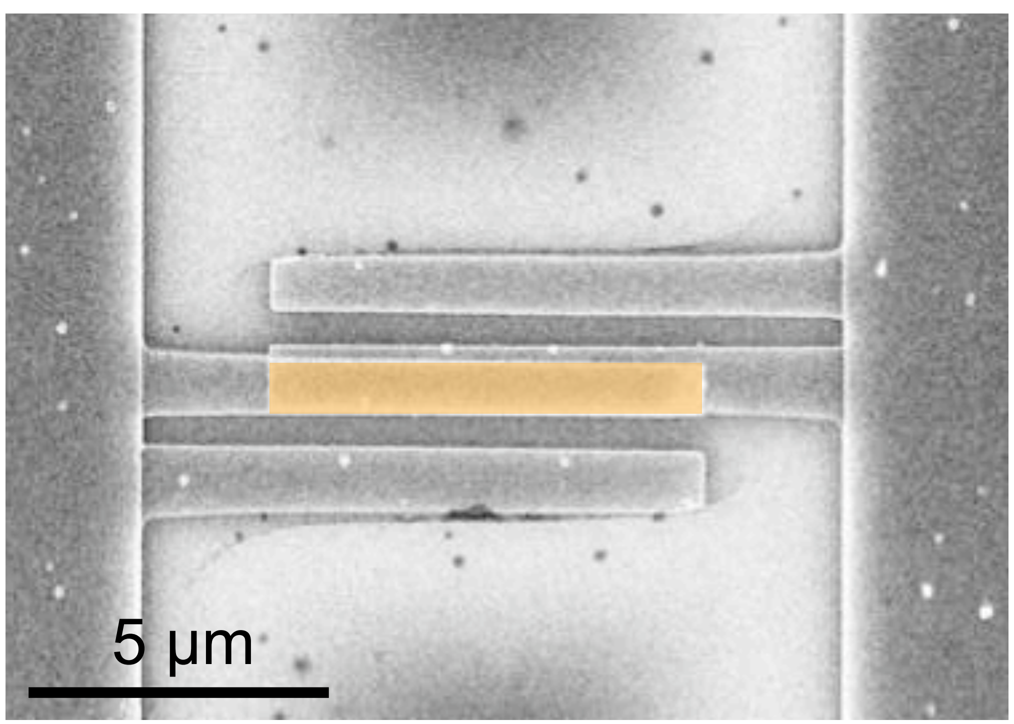


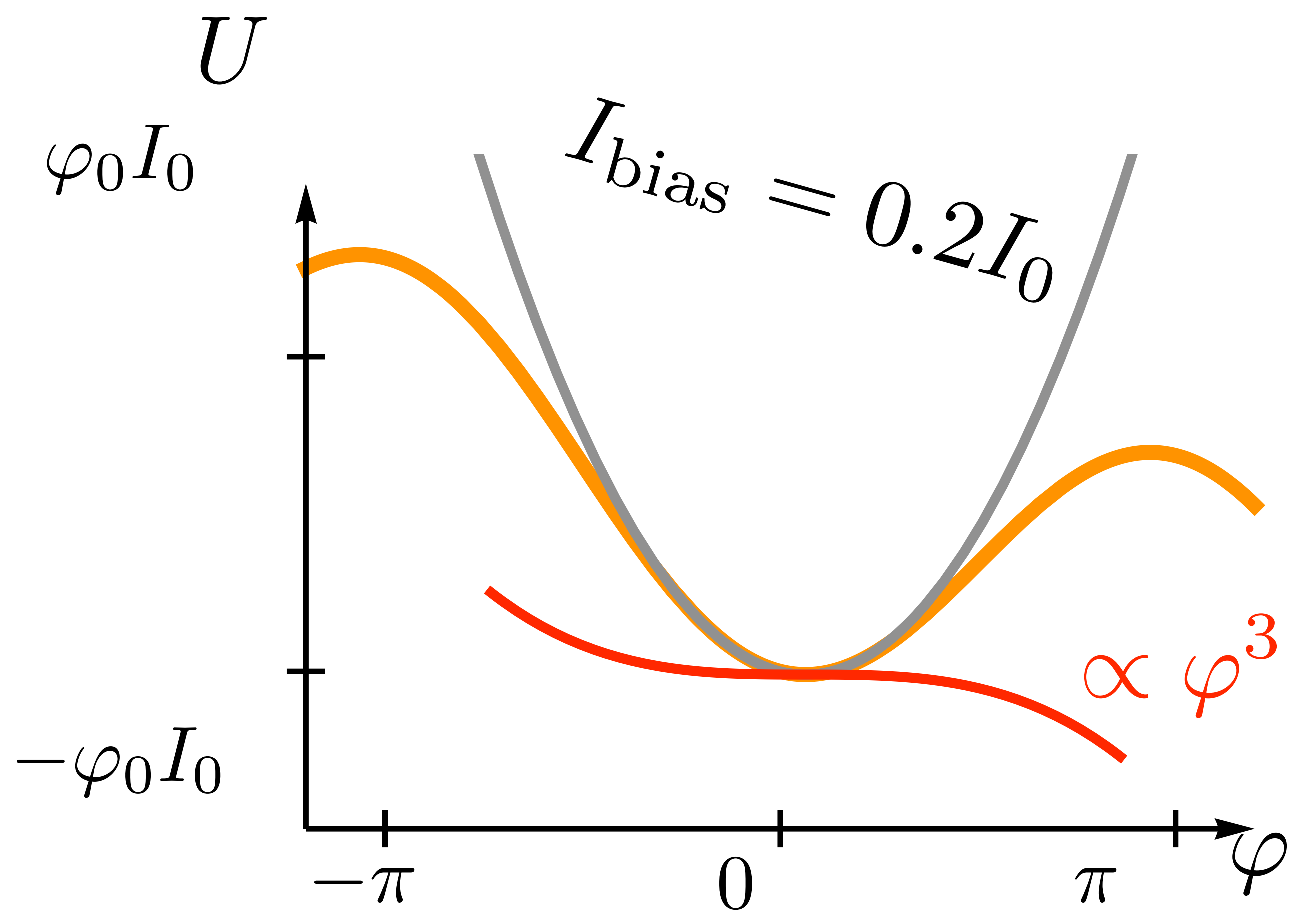


Non linear element: Josephson junction

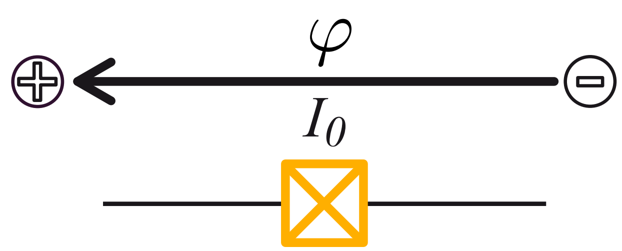


$$U = -\varphi_0 I_0 \cos(\varphi) - I_{\mathrm{bias}} \varphi_0 \varphi$$





Non linear element: Josephson junction



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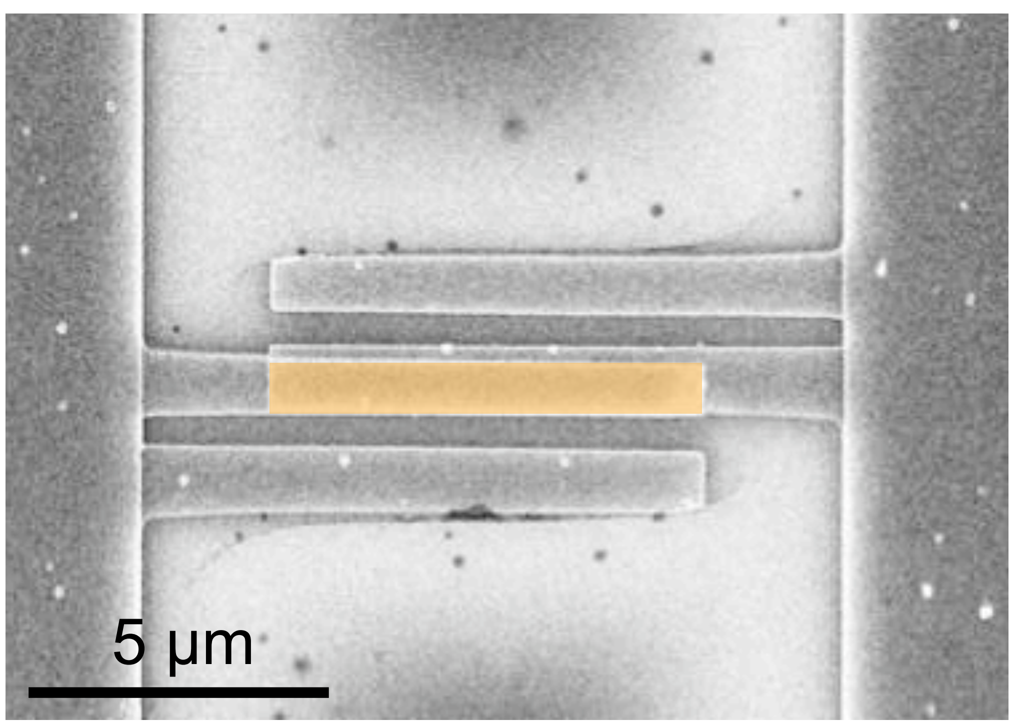
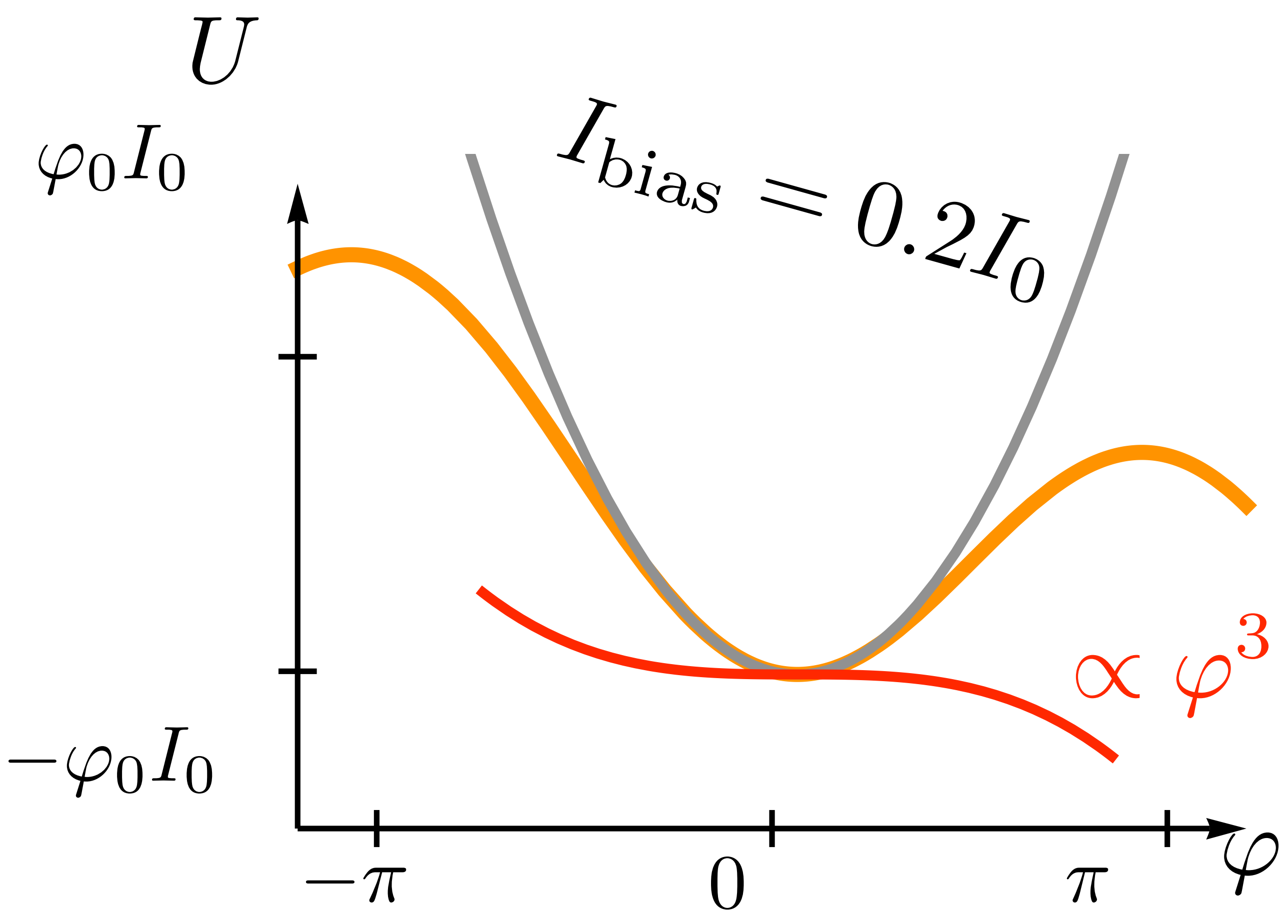




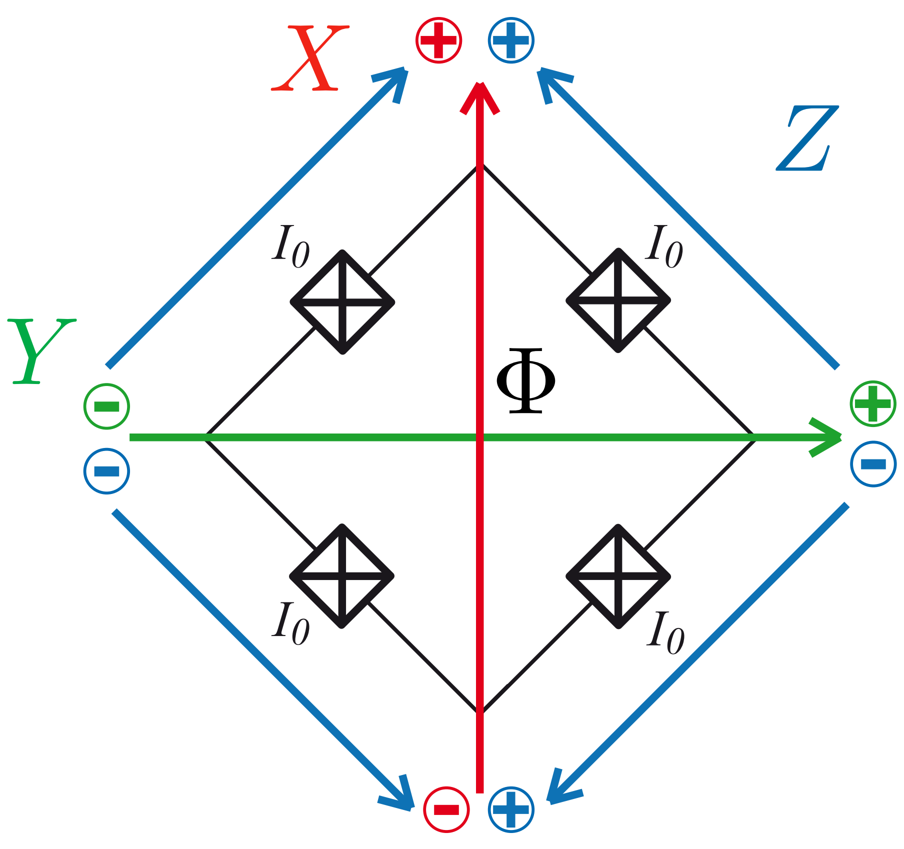
to get $a_S^\dagger a_I^\dagger a_P$, we use $\varphi\varphi\varphi$

need to decompose $\varphi^3 \mapsto \varphi\varphi\varphi$

Josephson Parametric Converter (JPC)

spatial decomposition using a ring

$$U = \alpha XYZ + \mu(X^2 + Y^2 + Z^2) + O(\ldots^4)$$



symmetry forbids undesired terms

$\cancel{XY}$ $\cancel{X^3}$ $\cancel{XY^2}$

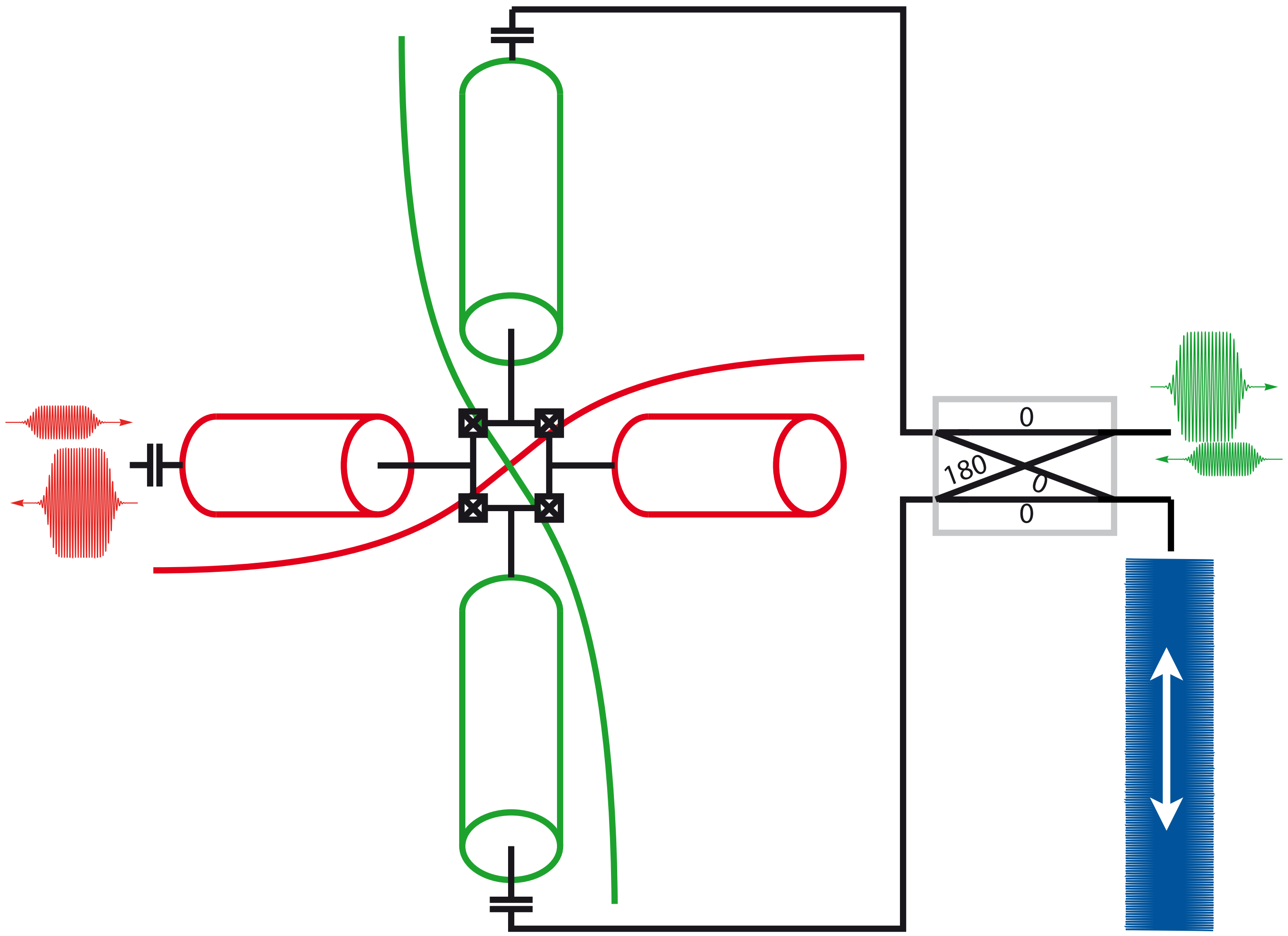
magnetic flux provides current bias

$$\Phi \stackrel{\sim}{\Leftrightarrow} I_{\mathrm{bias}}$$

but phase slips possible !

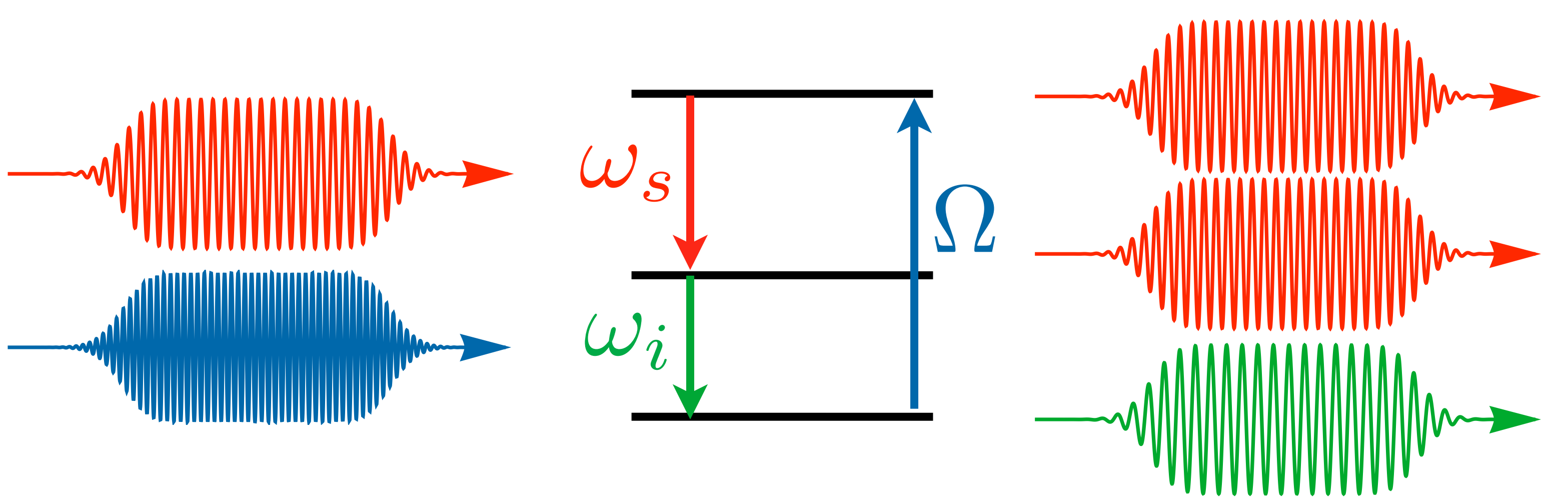
[see Lecture III]

Josephson Parametric Converter (JPC)



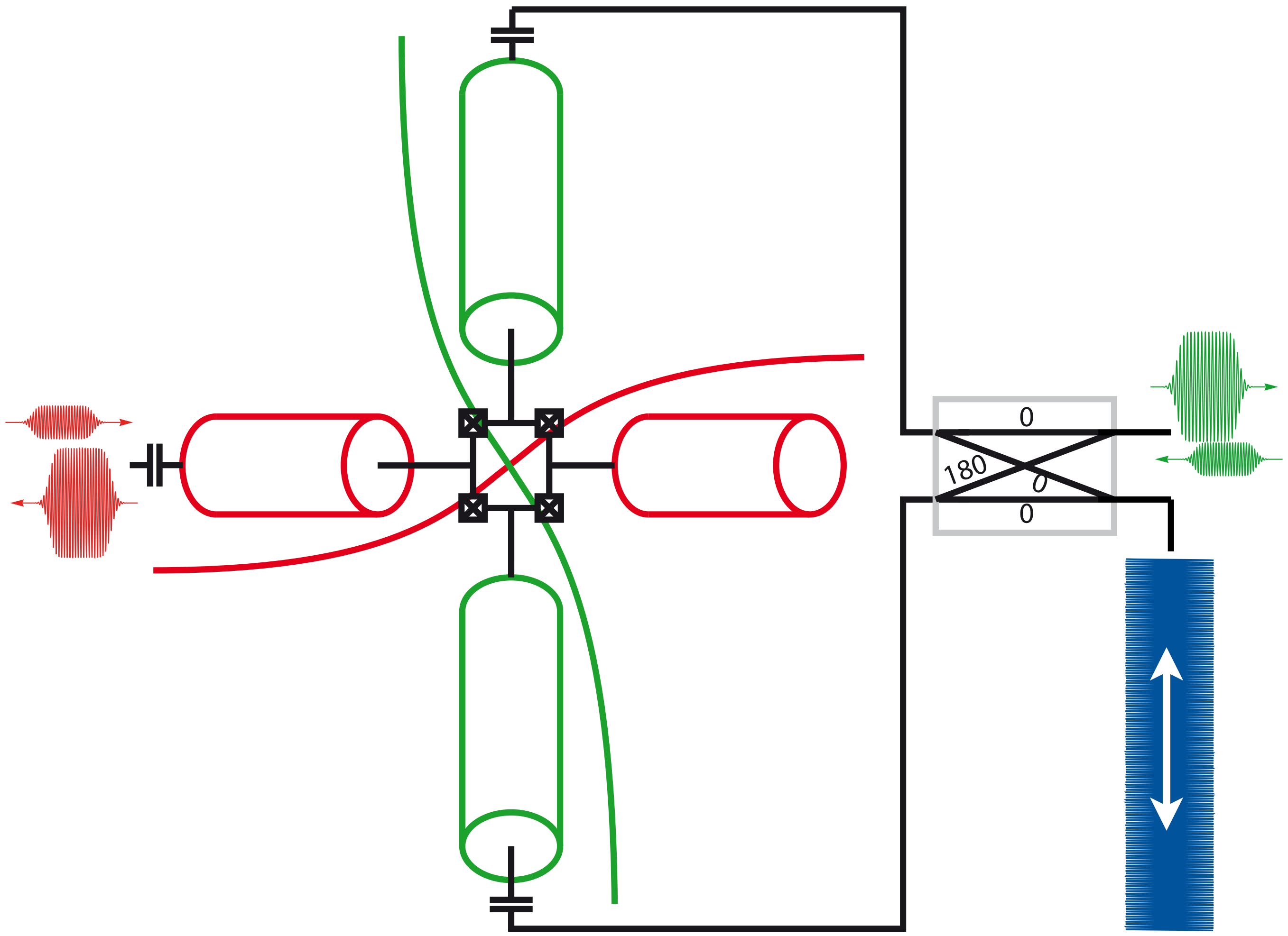
$$H \approx \alpha XYZ + \mu(X^2 + Y^2 + Z^2)$$

$$a_s + a_s^\dagger \qquad a_i + a_i^\dagger \qquad \mathcal{A}_p \cos(\Omega t + \varphi_p)$$



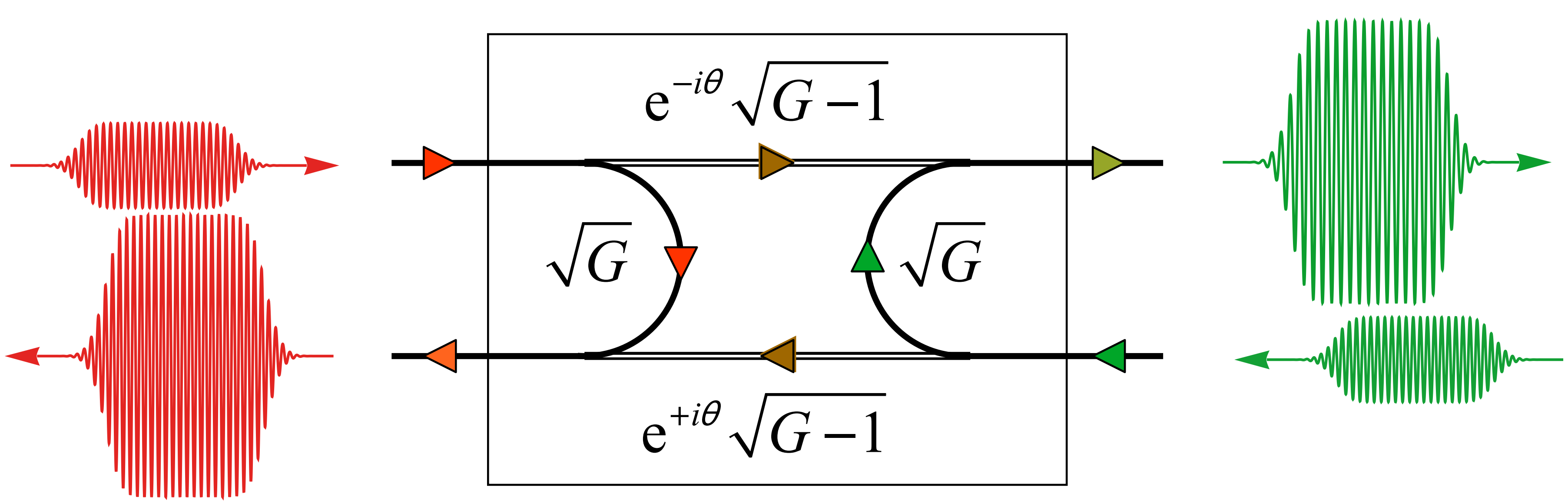
[Bergeal et al., Nat. Phys. (2010)]

Josephson Parametric Converter (JPC)



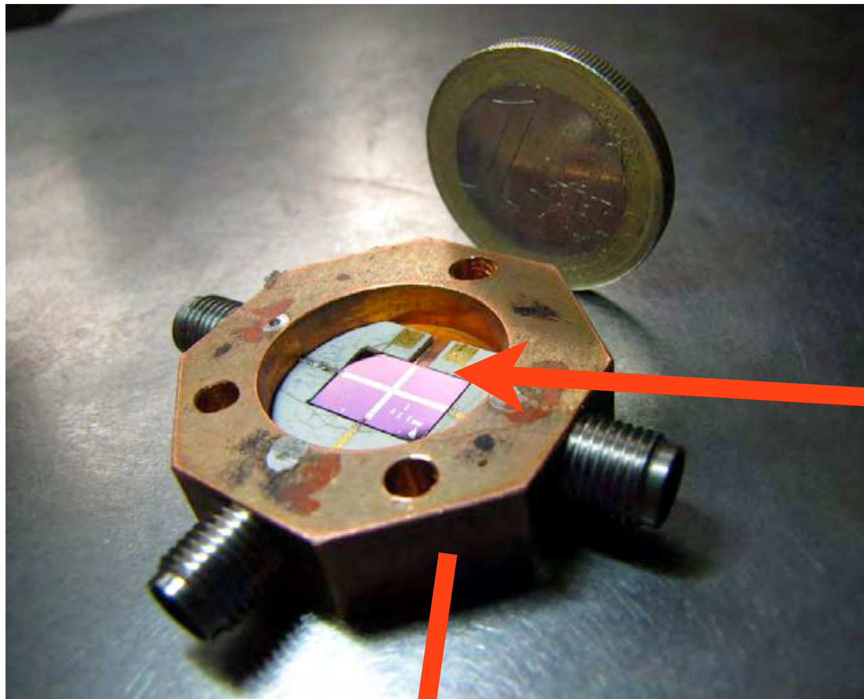
$$G = \left(\frac{1+\rho^2}{1-\rho^2} \right)^2$$

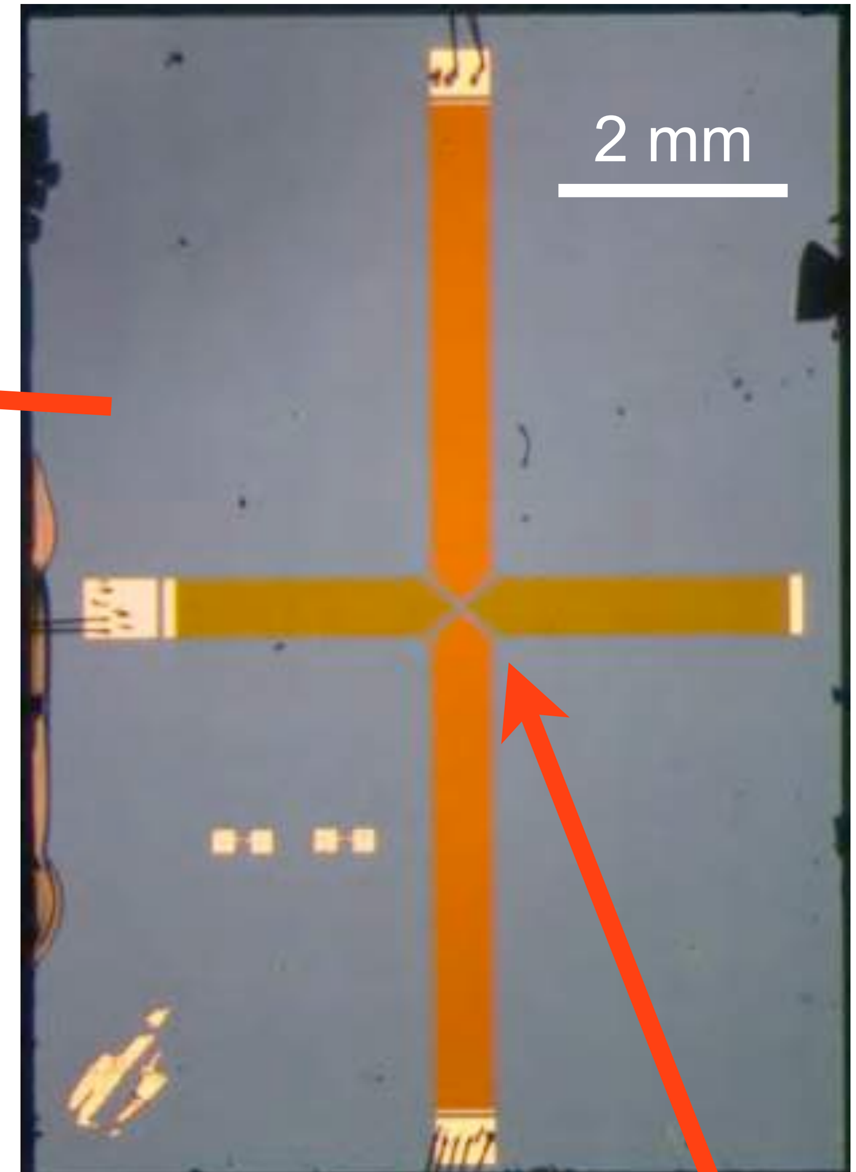
$$\rho = \frac{\sqrt{2}}{4} \frac{I_p}{I_0} \sqrt{p_s Q_s p_i Q_i}$$

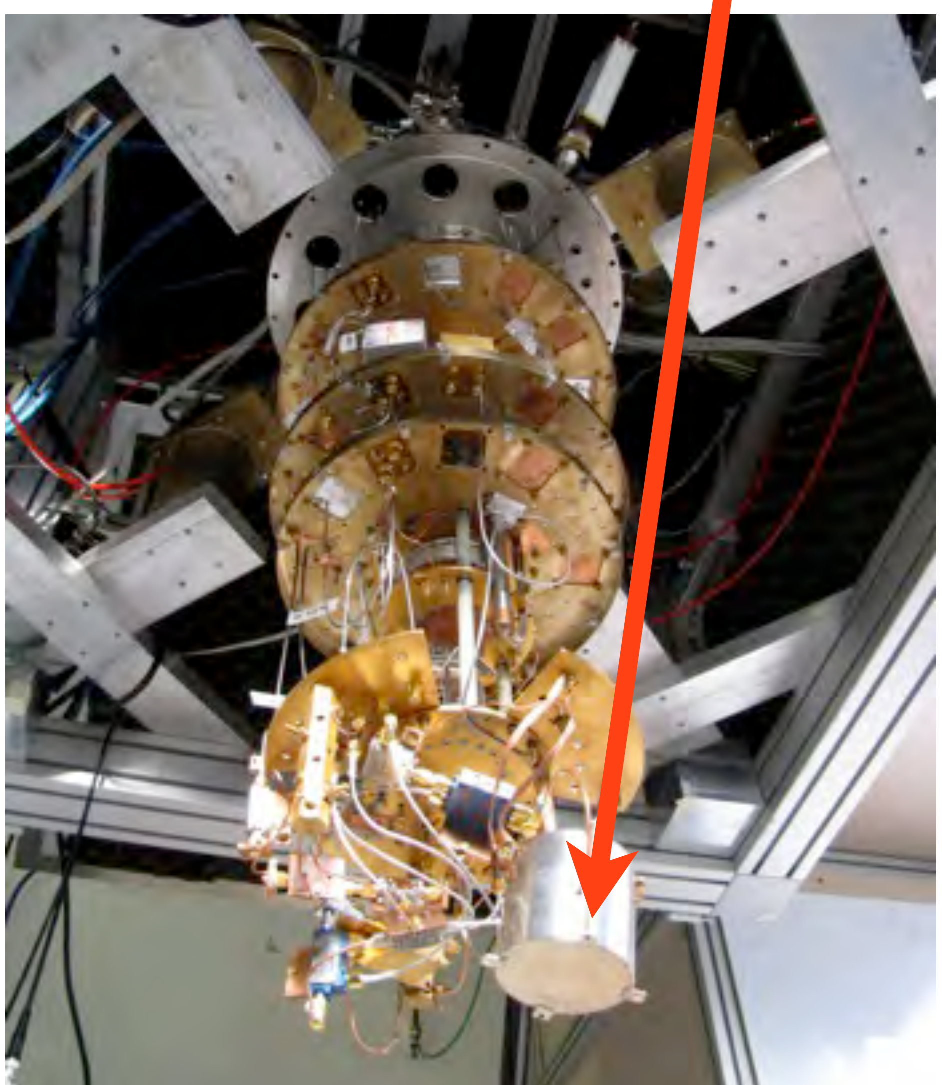


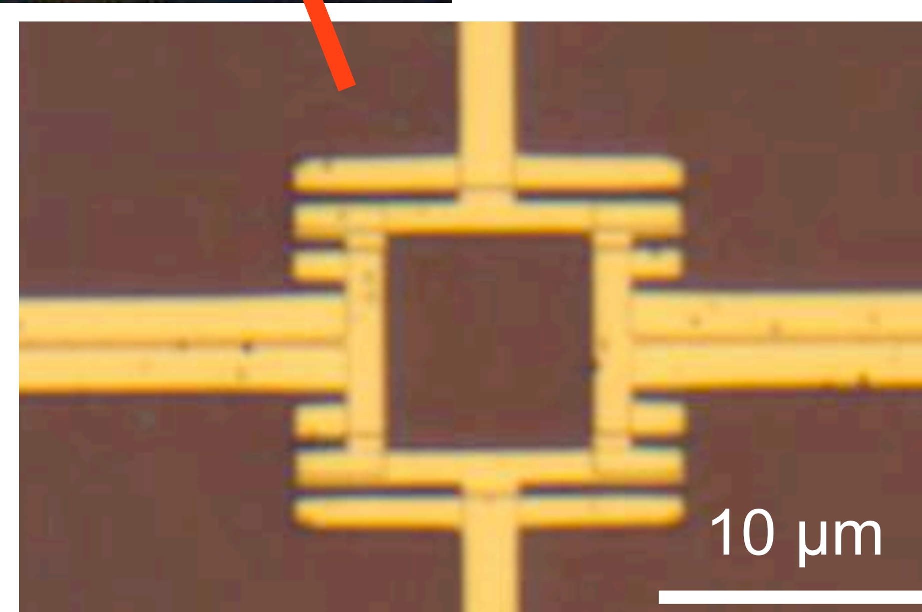
[Bergeal et al., Nat. Phys. (2010), Lecture III]

Realization



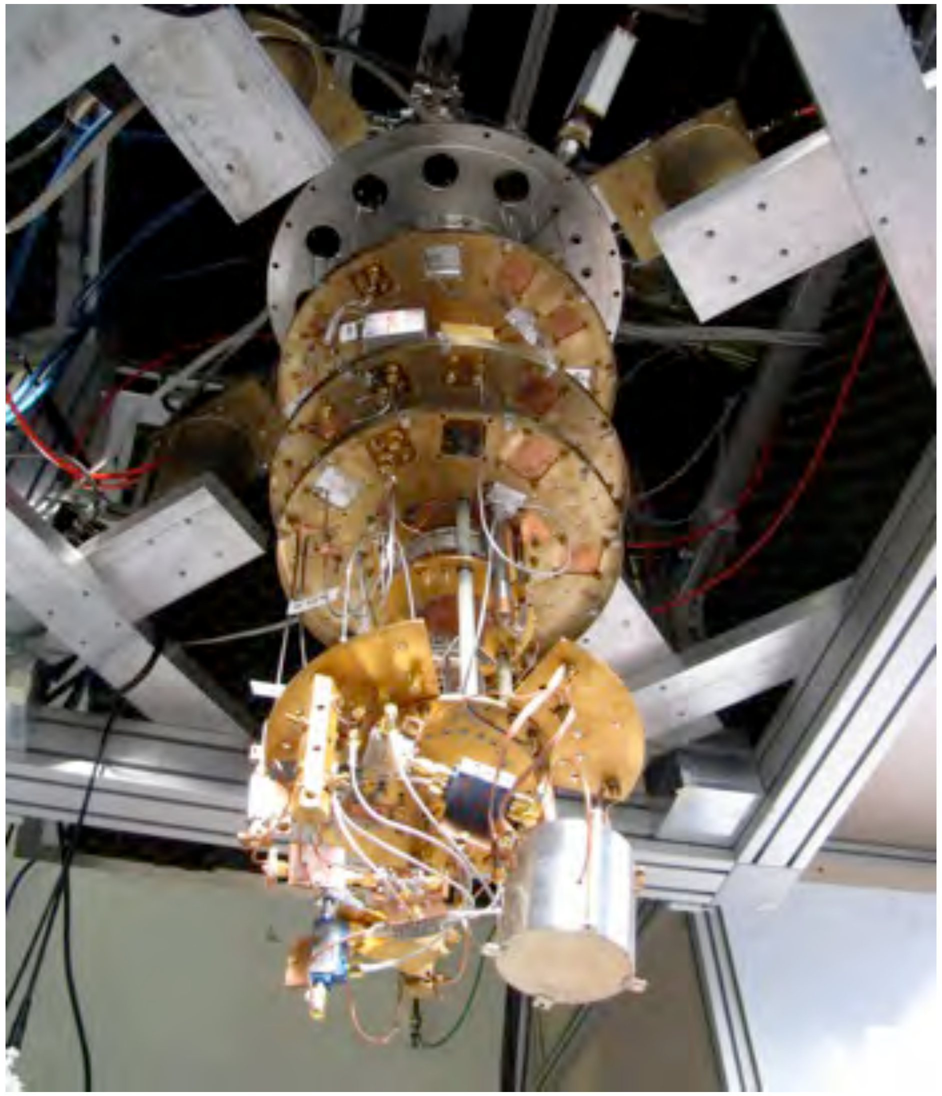


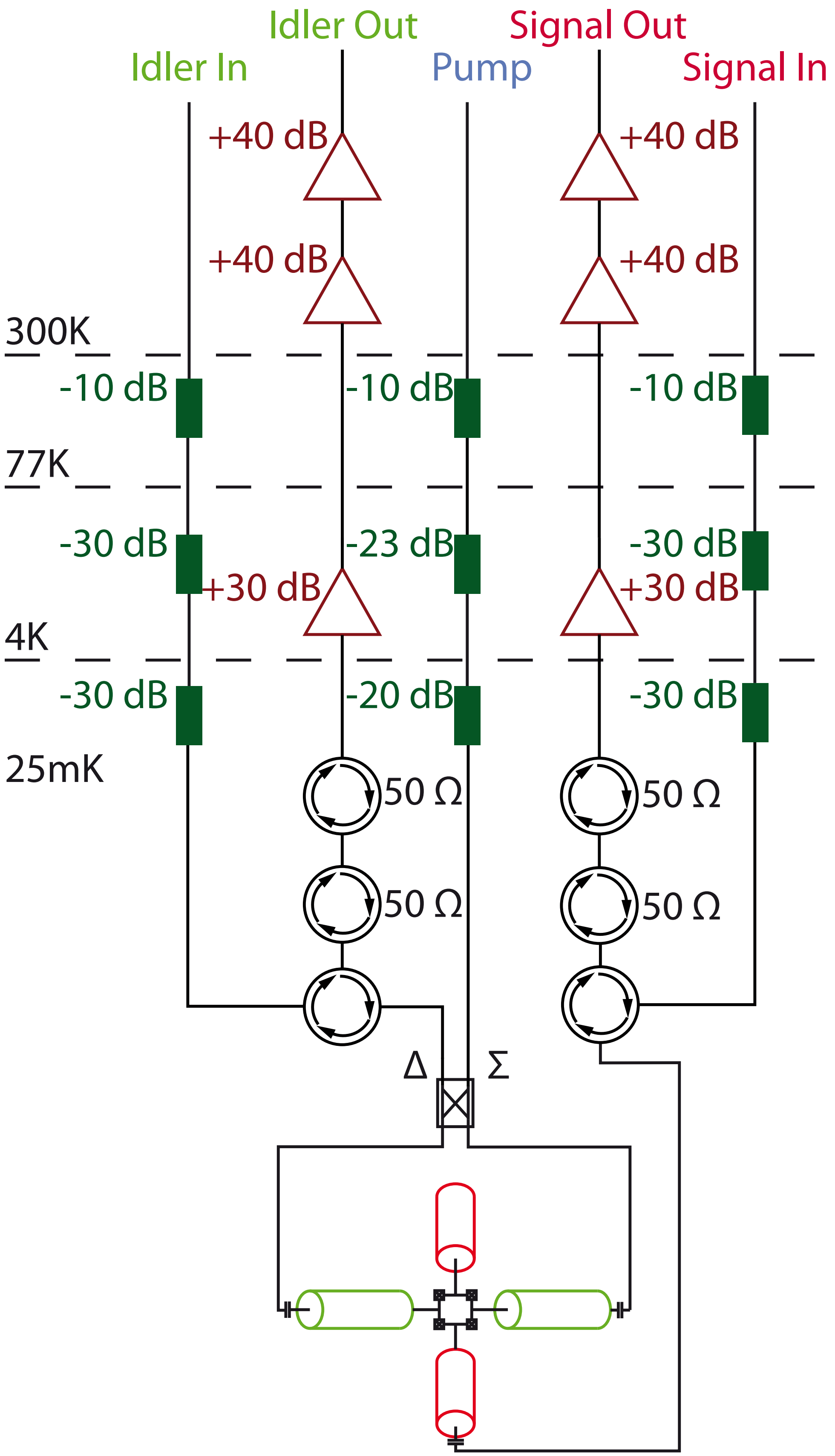




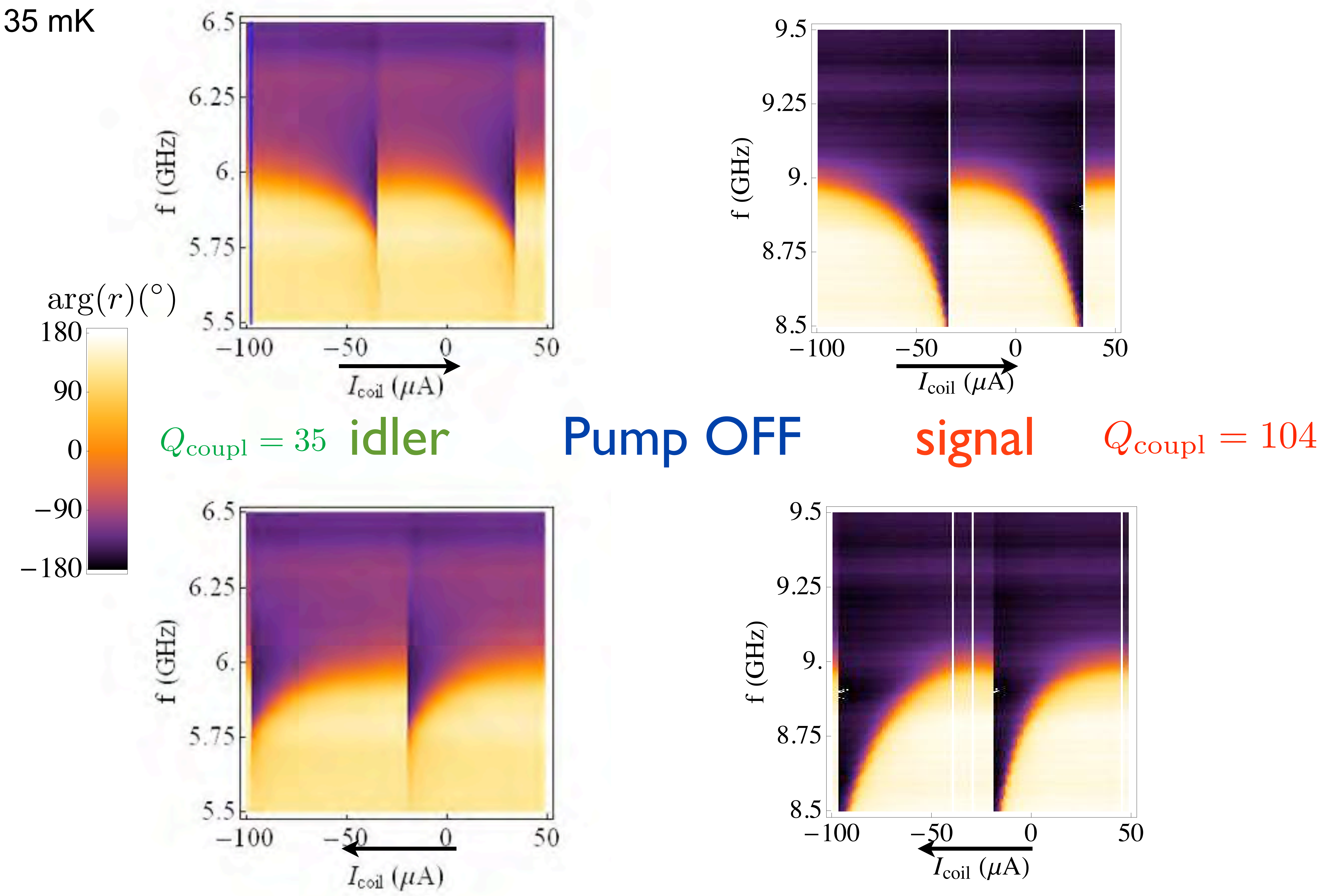
$I_0 \approx 5\ \mu A$

Cabling of the dilution fridge



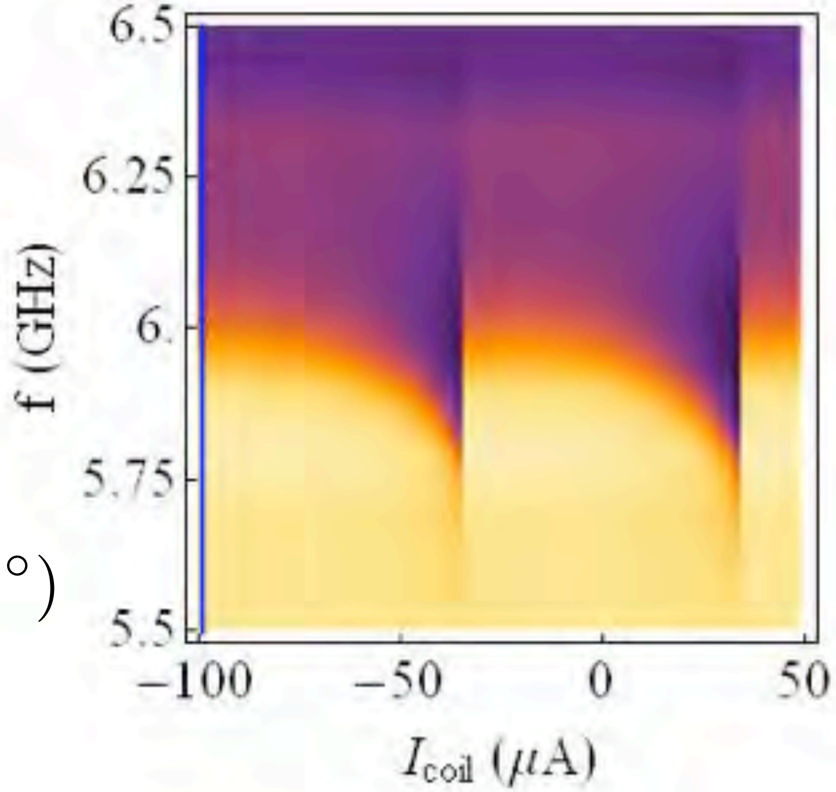


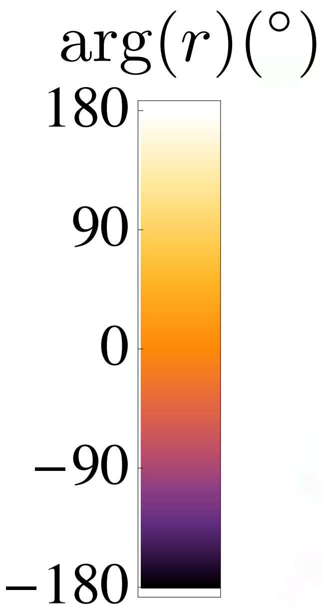
Resonance frequency as a function of field

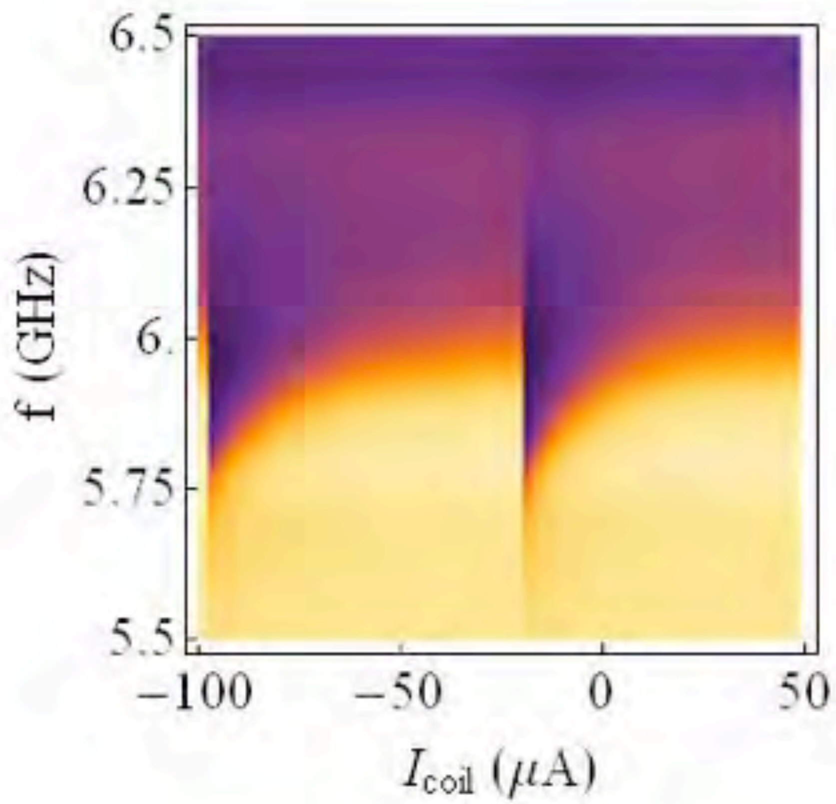


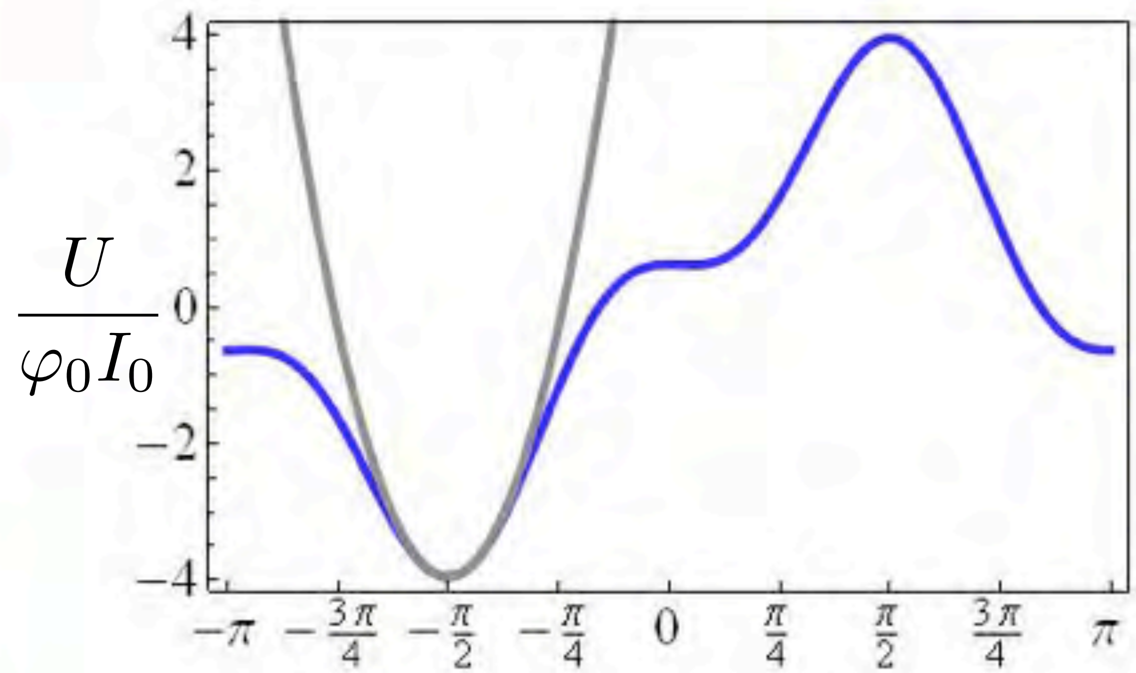
Resonance frequency as a function of field

35 mK



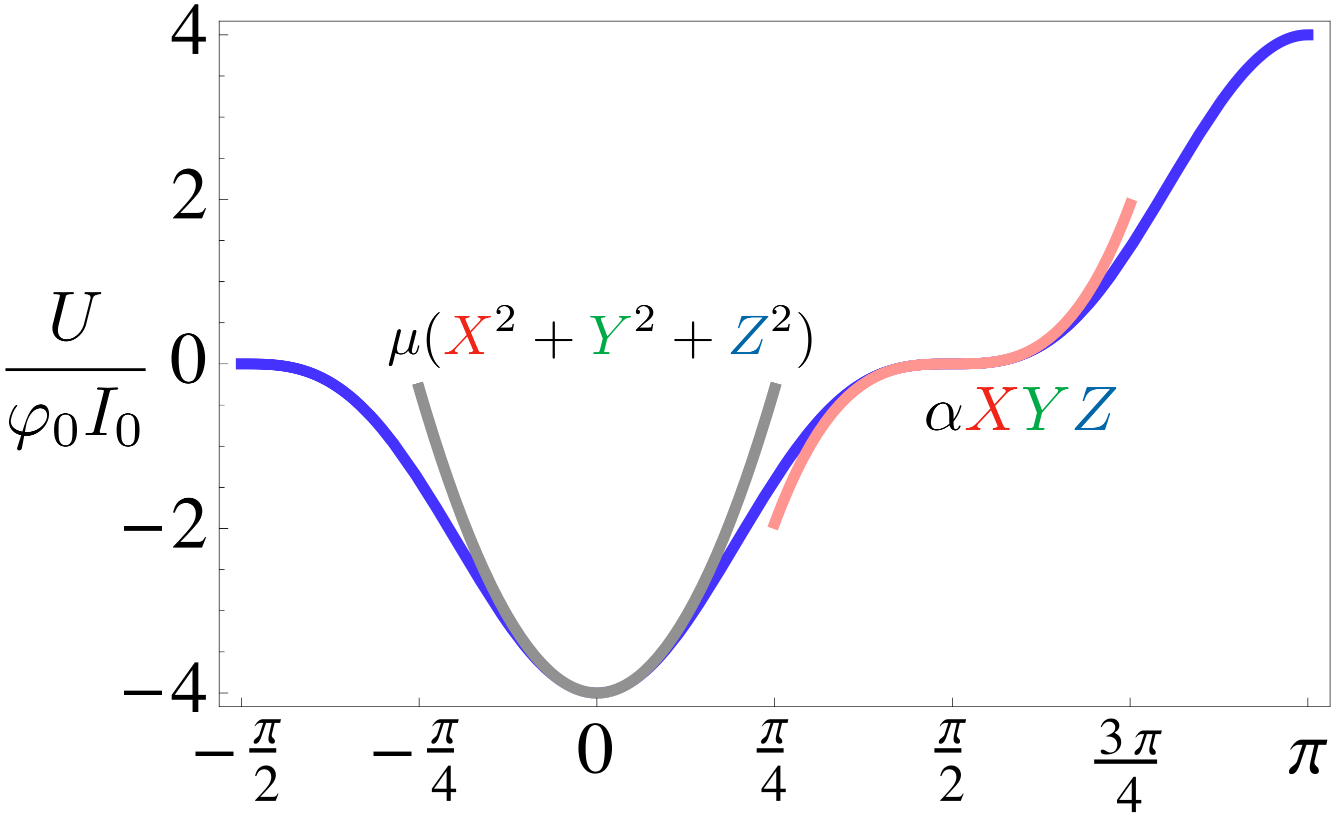






$$\frac{X}{2\varphi_0} = \frac{Y}{2\varphi_0} = \frac{Z}{2\varphi_0}$$

3-wave mixing with the Josephson ring



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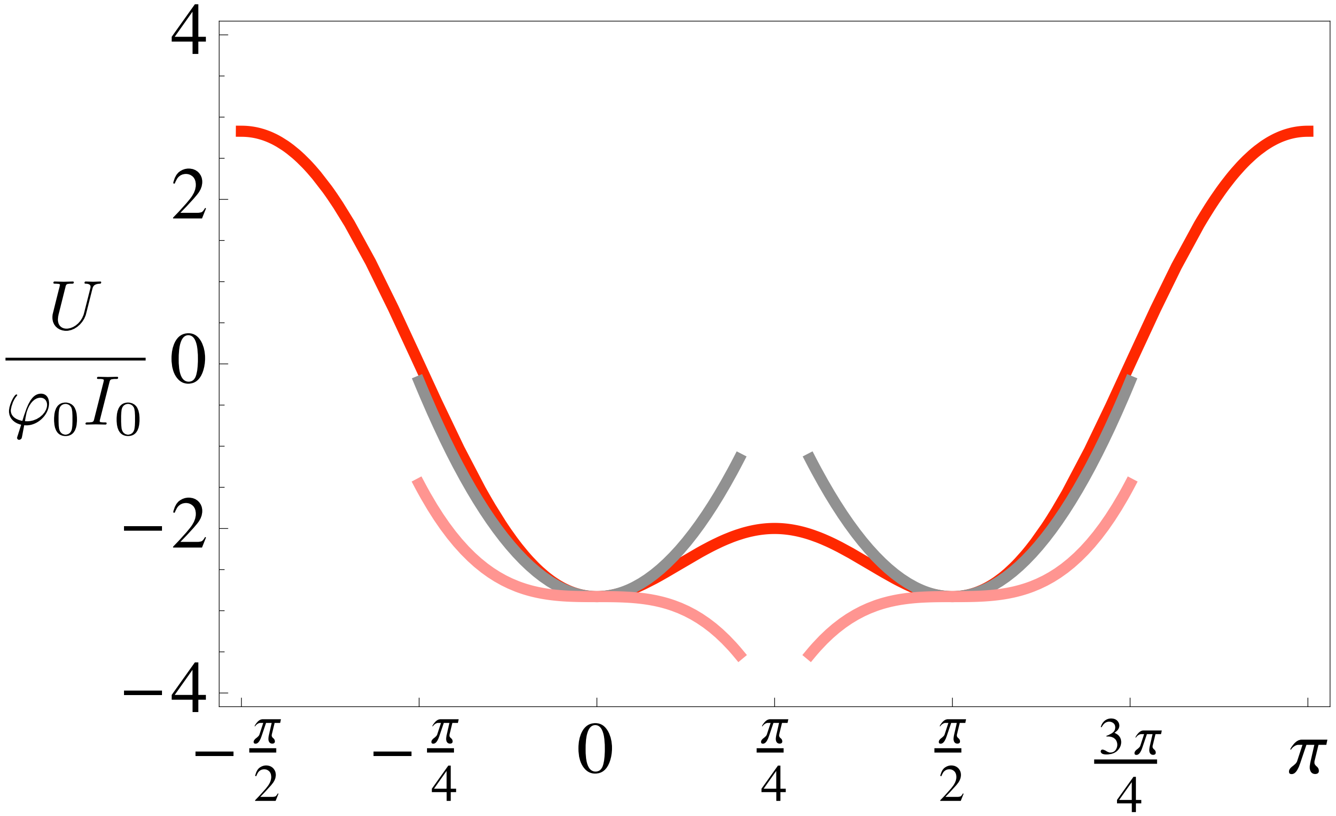


$$\Phi = 0$$

best non-linearity
unstable

$$H \approx \alpha XYZ + \mu(X^2 + Y^2 + Z^2)$$

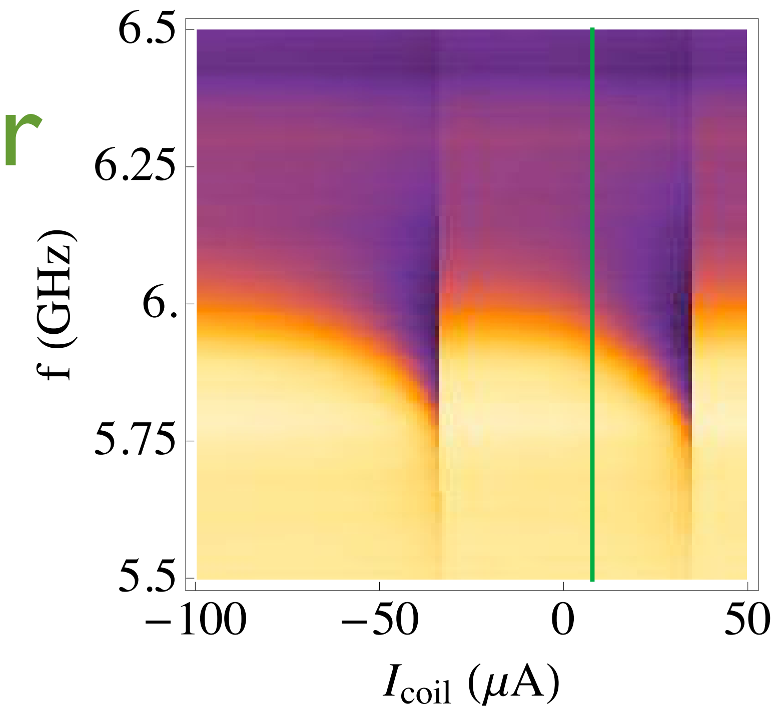
$$\Phi = \Phi_0/2$$

average non-linearity
stable

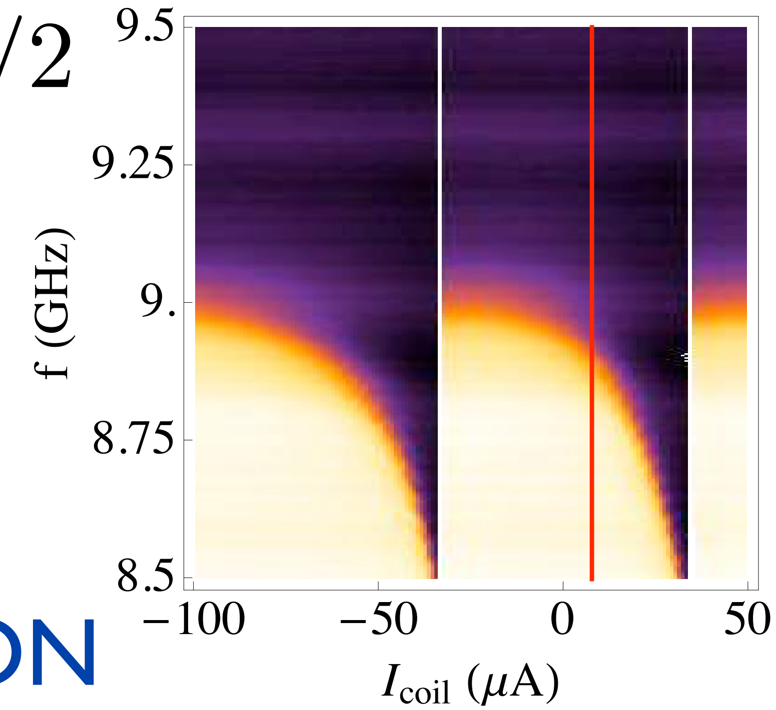
Gain as a function of pump power

35 mK

idler

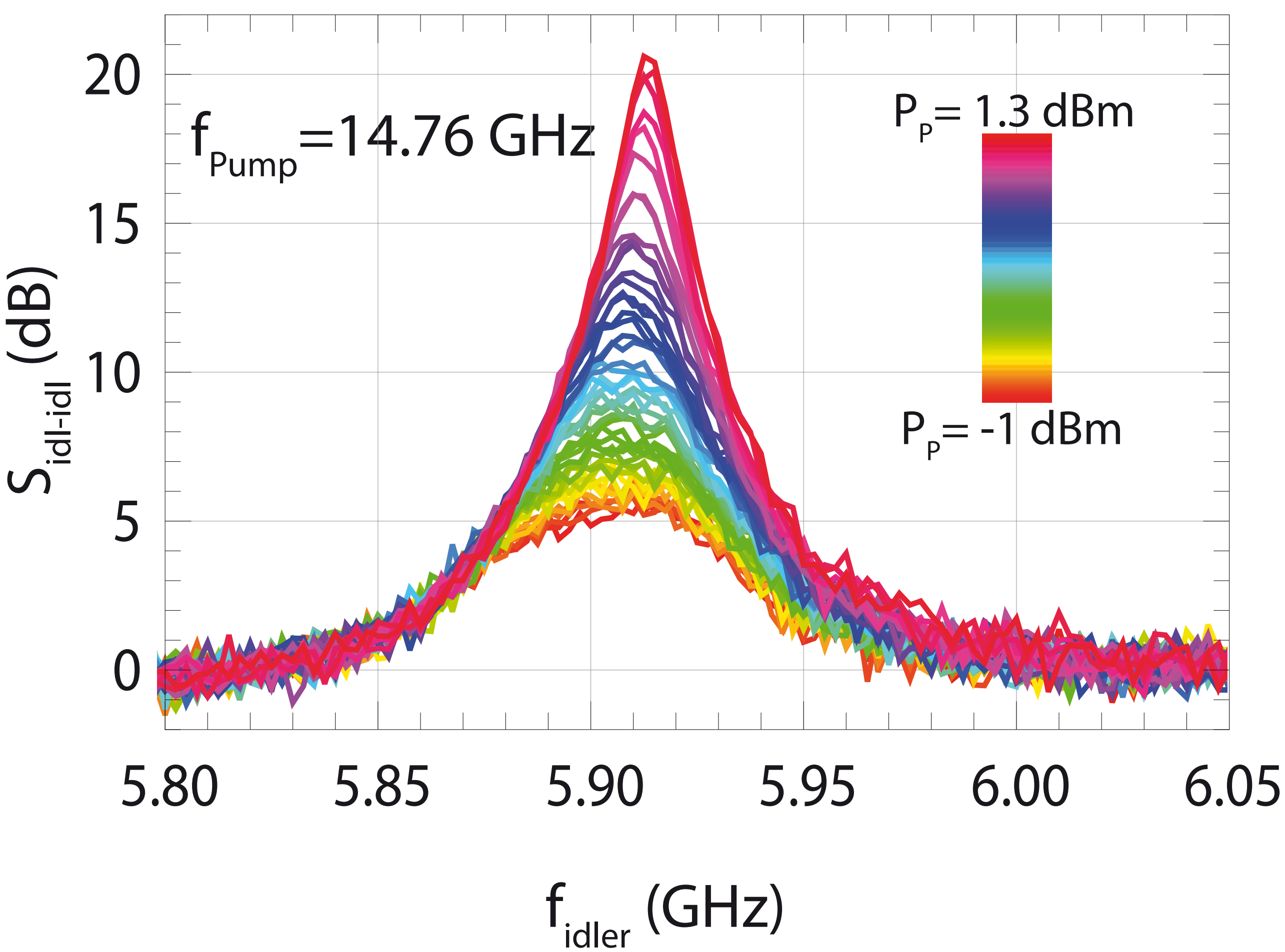


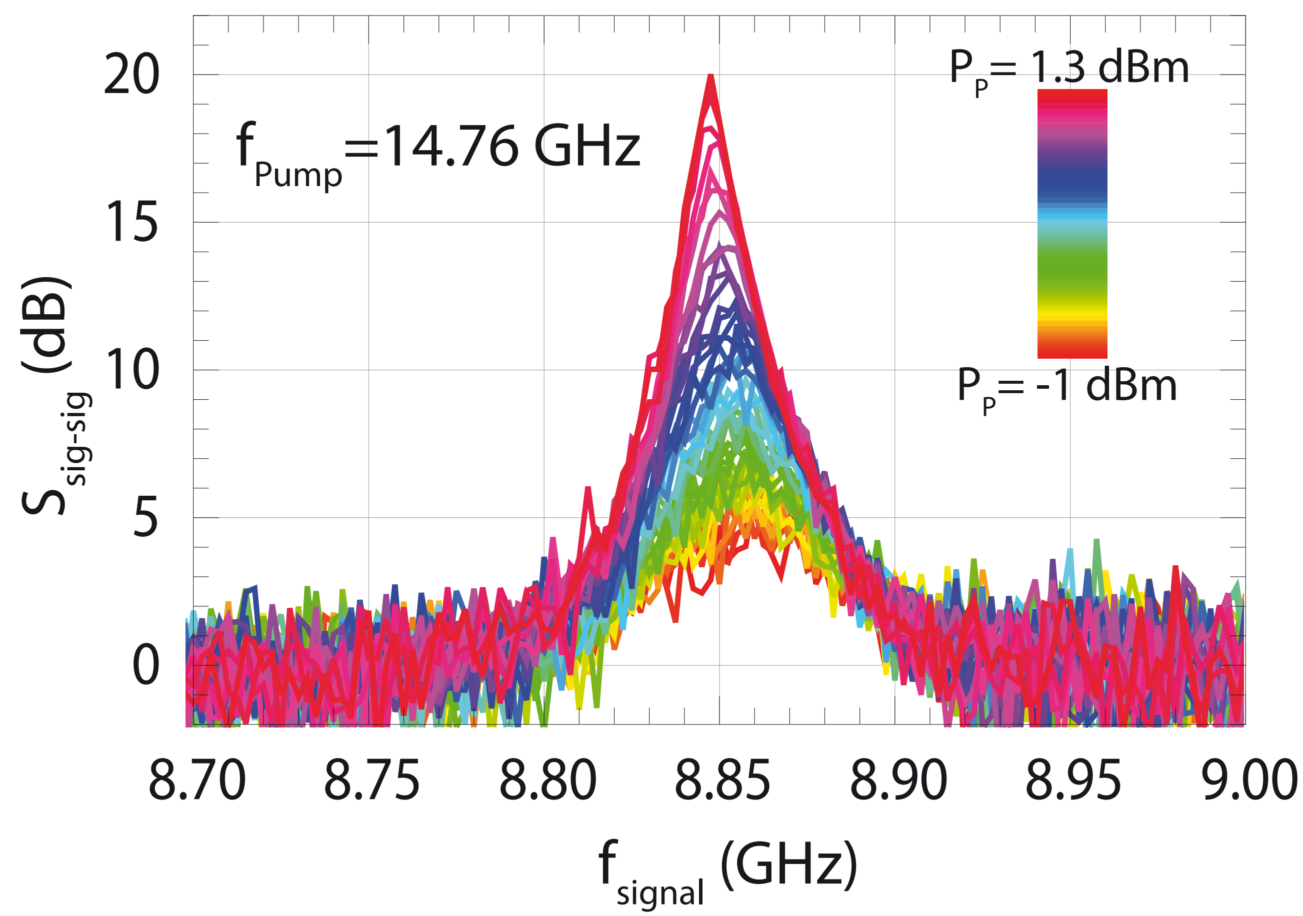
$\Phi = \Phi_0/2$



signal

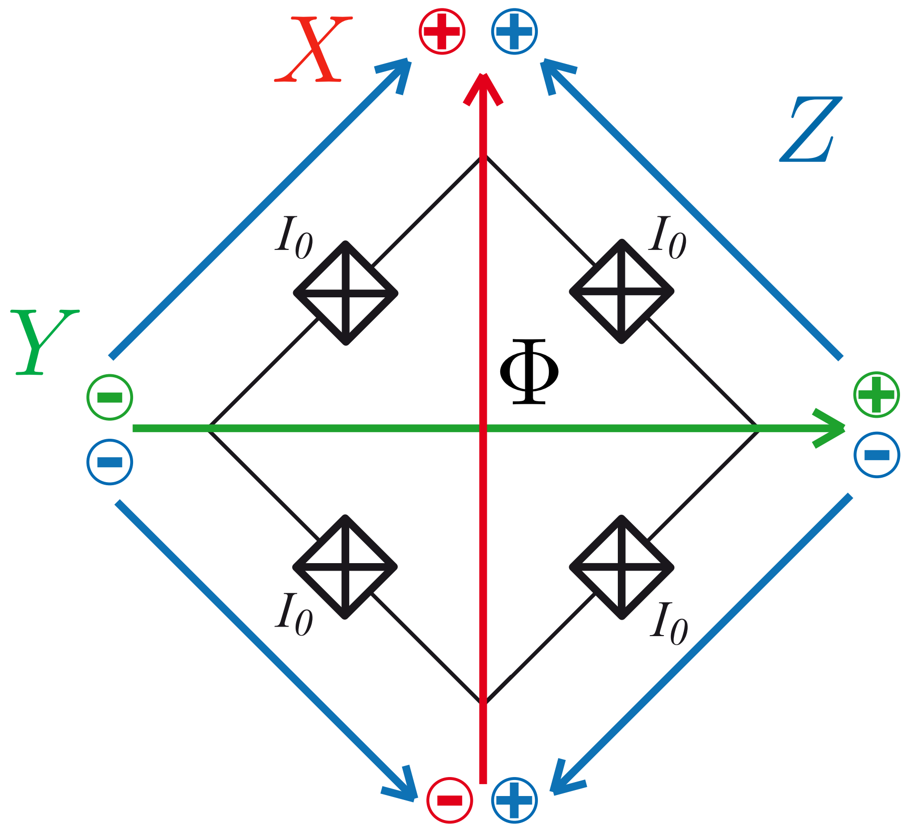
Pump ON





$\Delta f \approx 20$ MHz for $G = 100$

How to improve the JPC ?



magnetic flux provides current bias

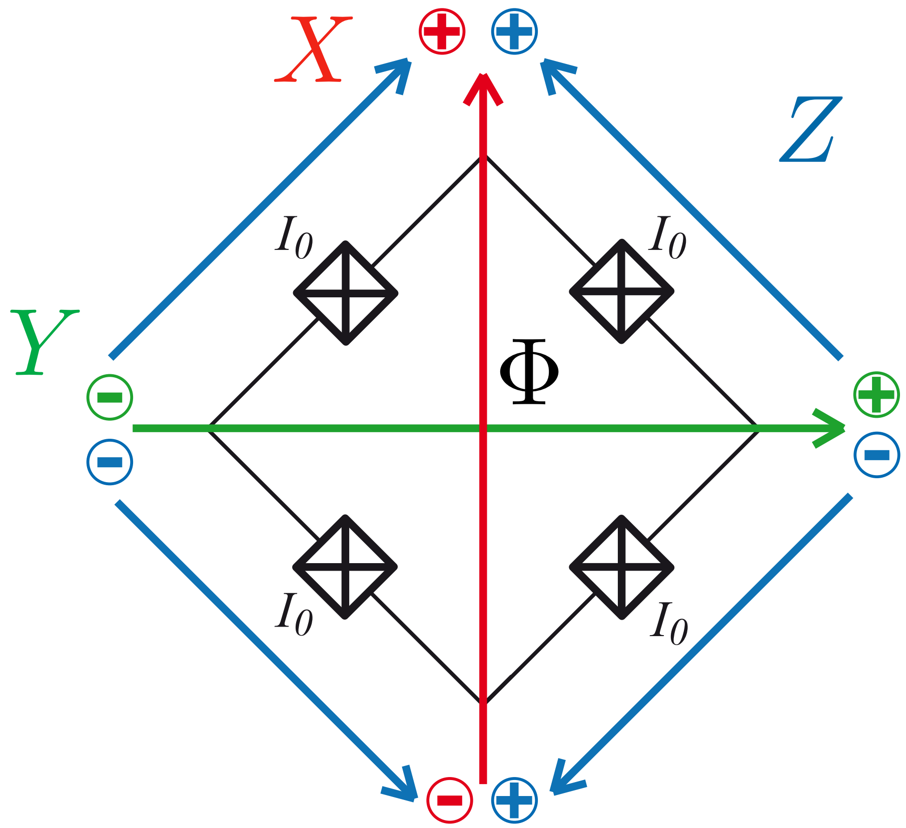
$$\Phi \overset{\sim}{\Leftrightarrow} I_{\text{bias}}$$

phase slips possible !

frequency tunability with the flux ⇒ can**not** be tuned if stability required

robustness of the amplifier ⇒ requires stability

How to improve the JPC ?



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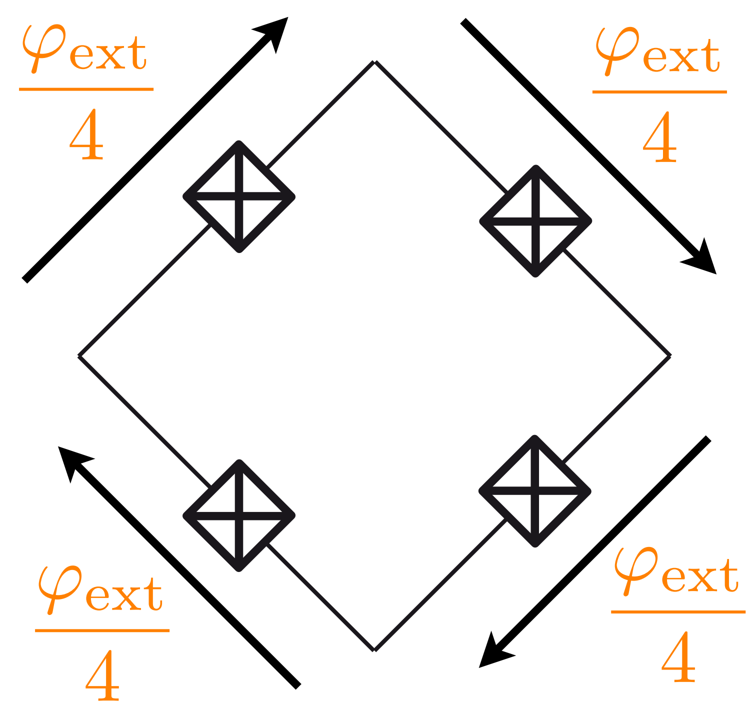
phase slips possible !

ideally, $\vec{B}_{\text{ext}}$ $\odot$ $\updownarrow$ φ_{ext}

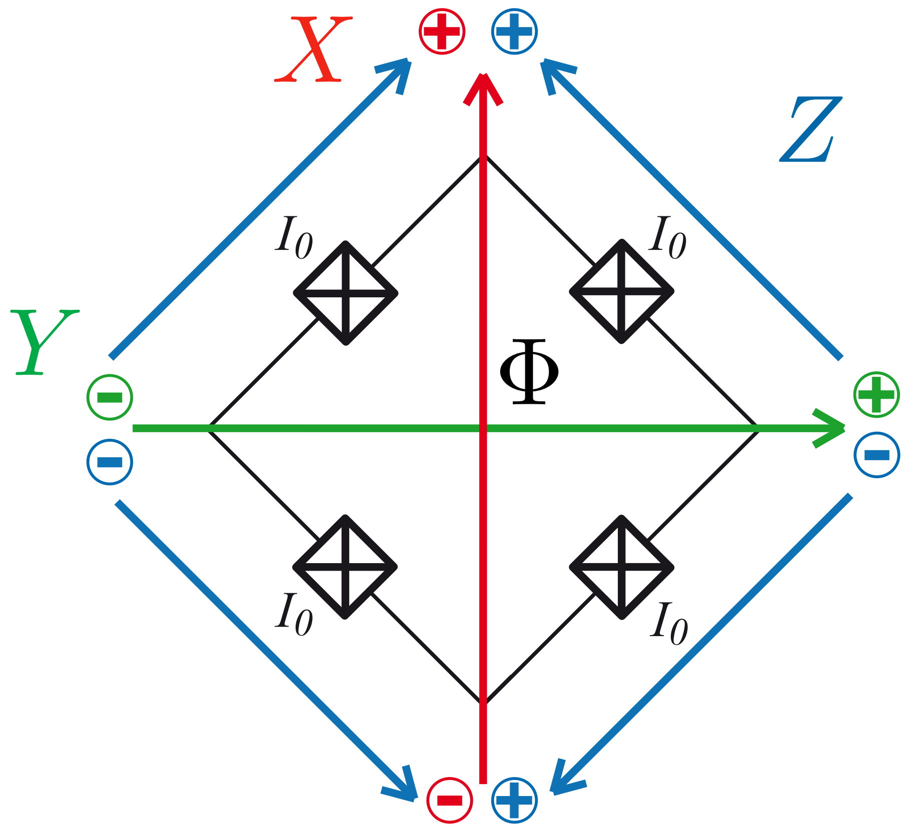


but phase slip because

$$L_J = \frac{\varphi_0}{I_0 \cos(\varphi_{\text{ext}}/4)}$$

goes negative when $\varphi_{\text{ext}}/4 > \frac{\pi}{2}$

How to improve the JPC ?



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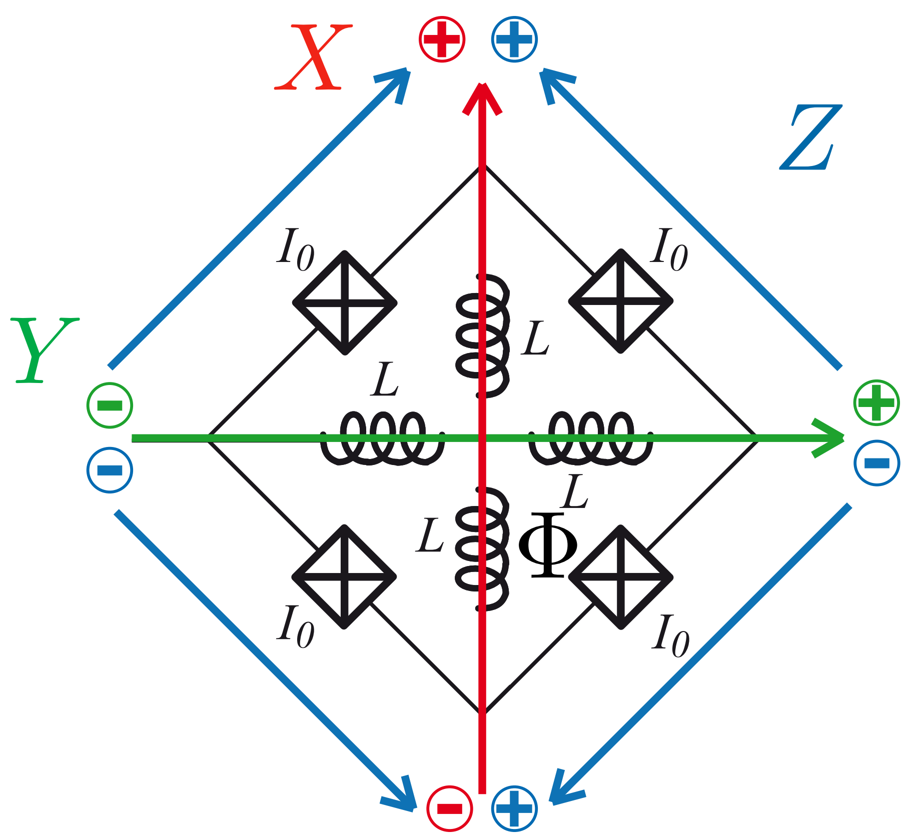
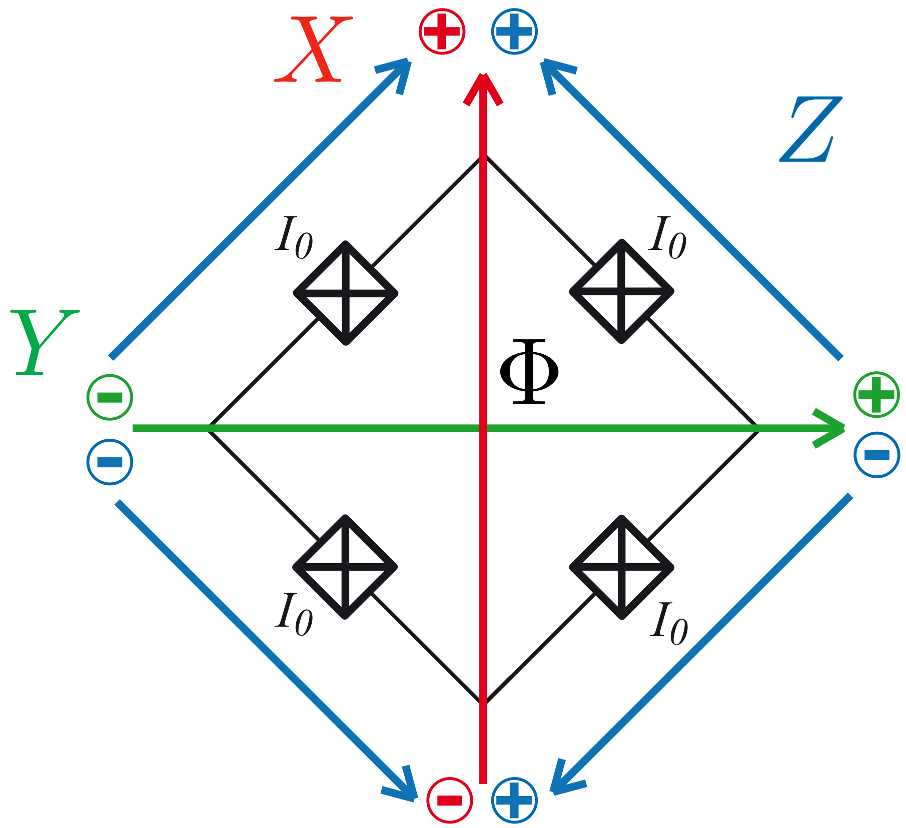


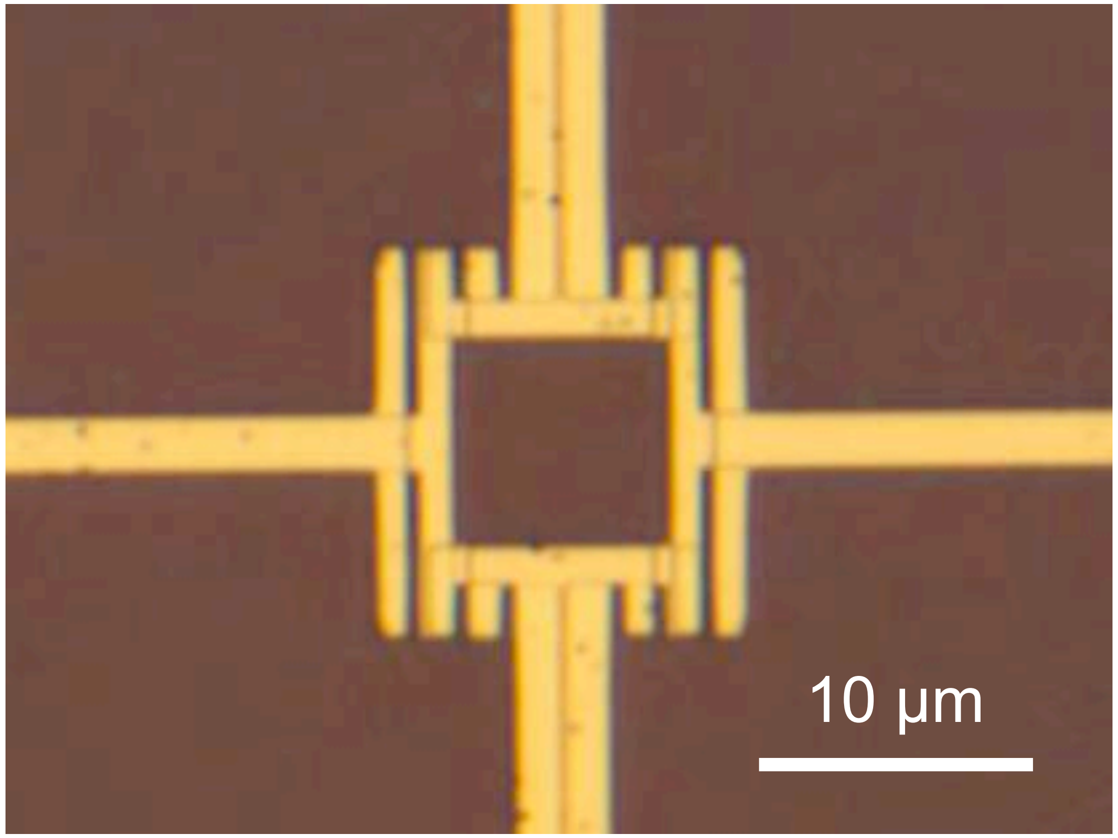
Solution: add inductances

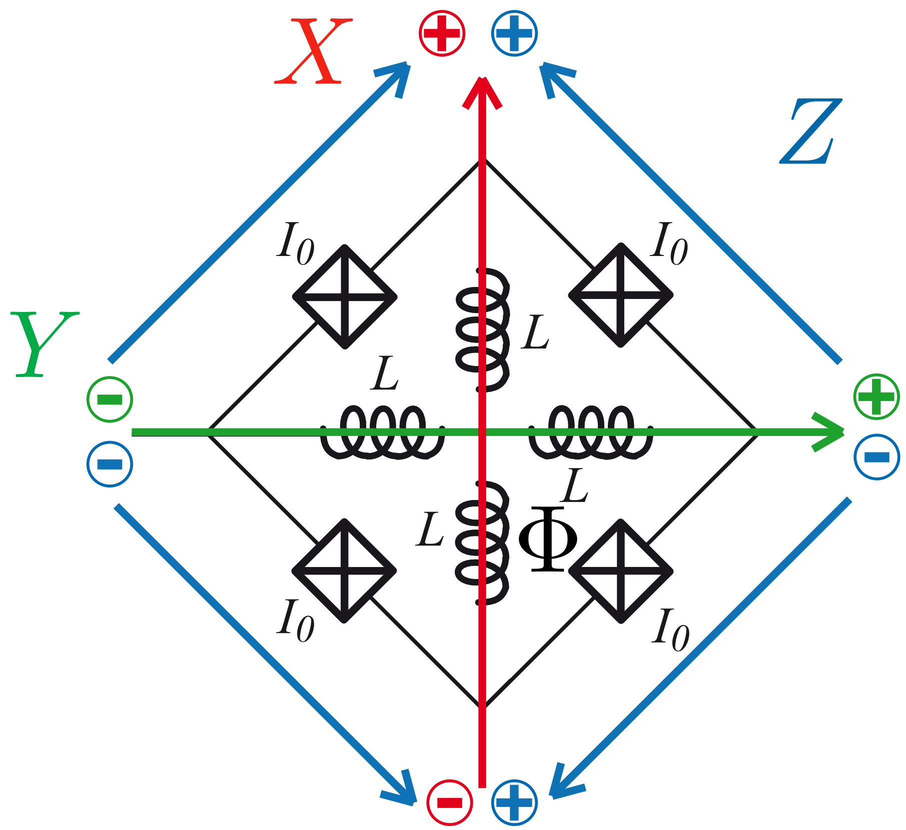
$$U \longmapsto U + \frac{E_L}{4}\left(2X^2 + 2Y^2 + Z^2\right)$$

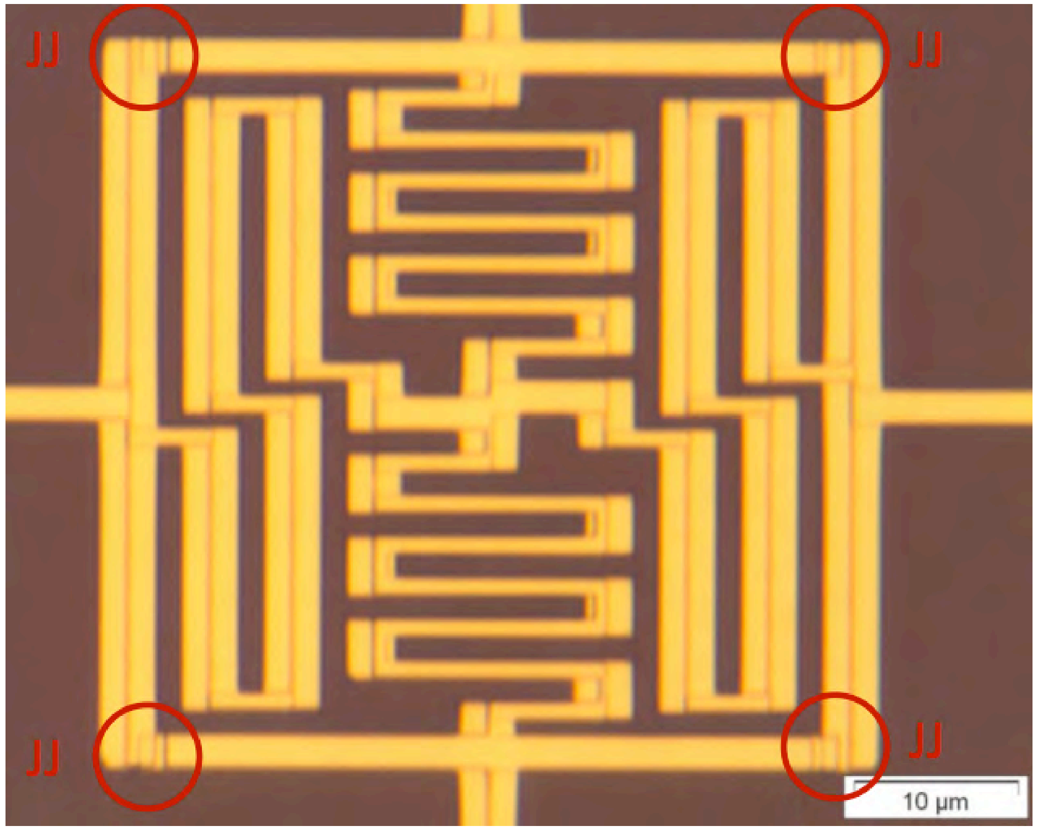
no phase slip if $L_J = \dfrac{\varphi_0}{I_0} > \dfrac{12}{5}L$

New generation



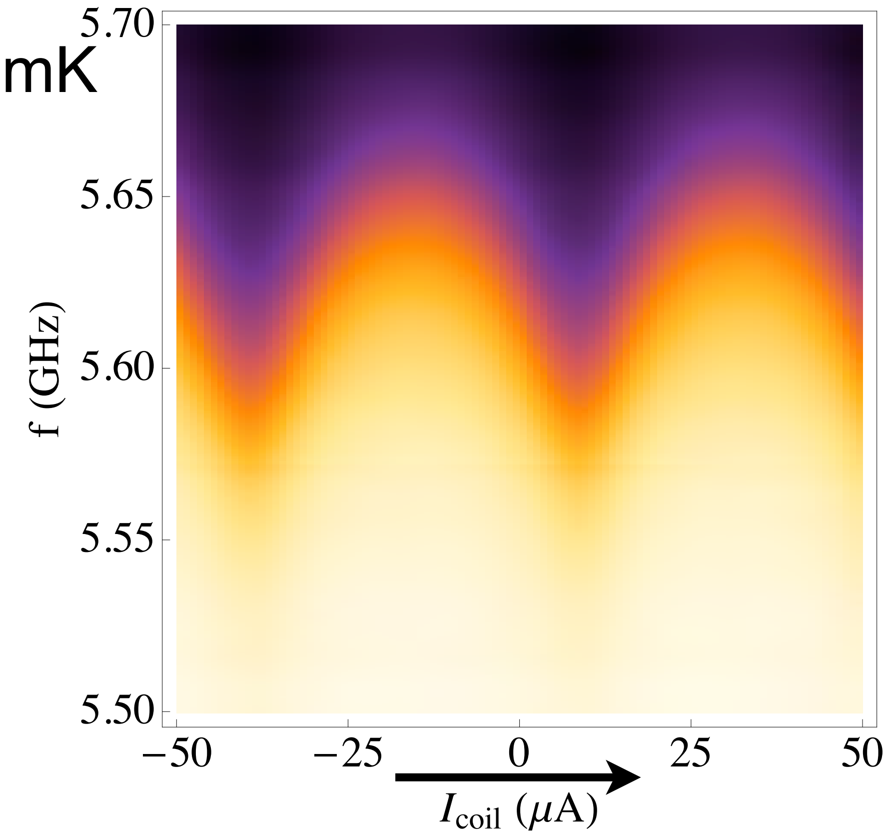


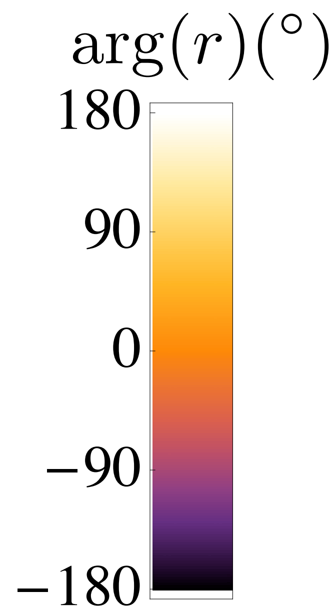


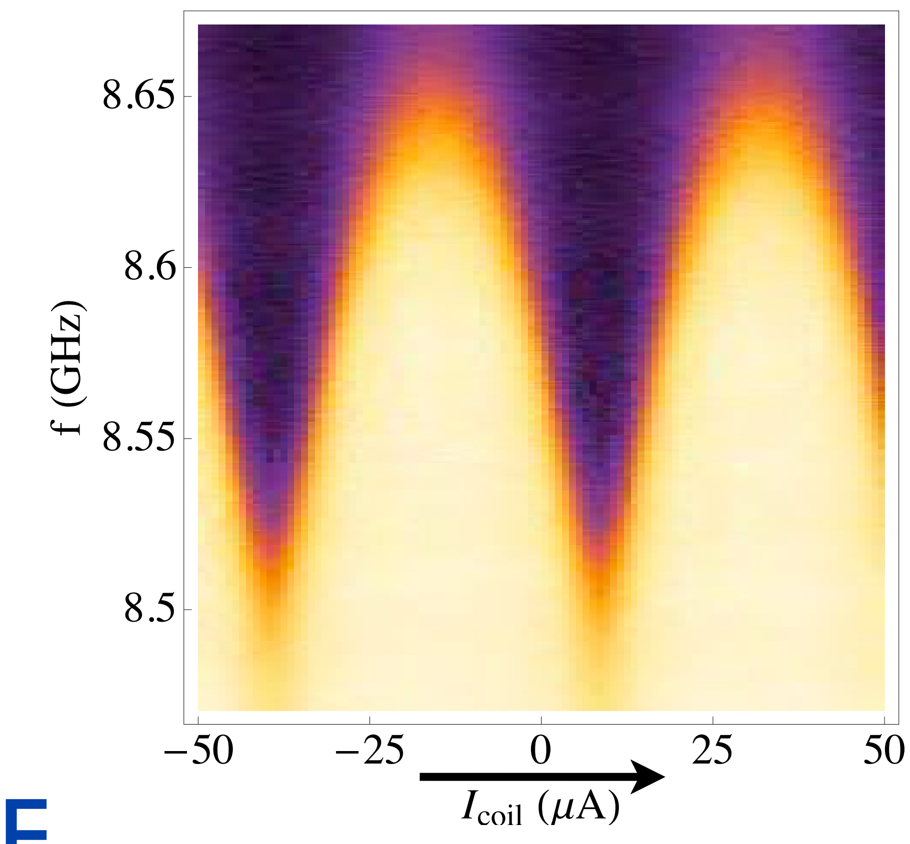


Resonance frequency as a function of field

35 mK



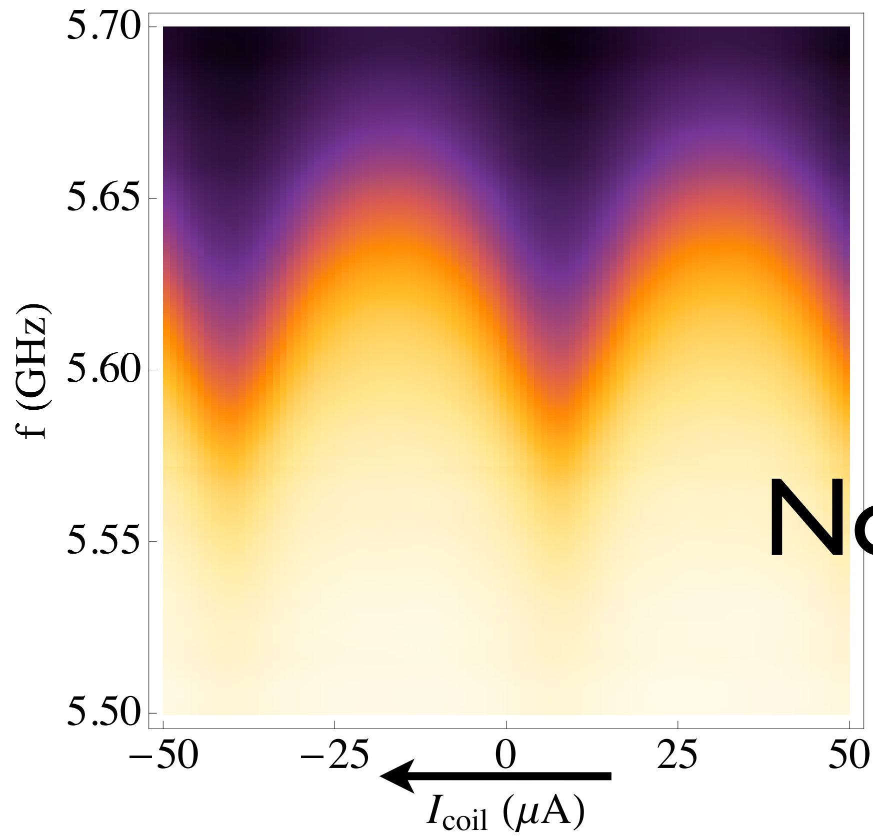


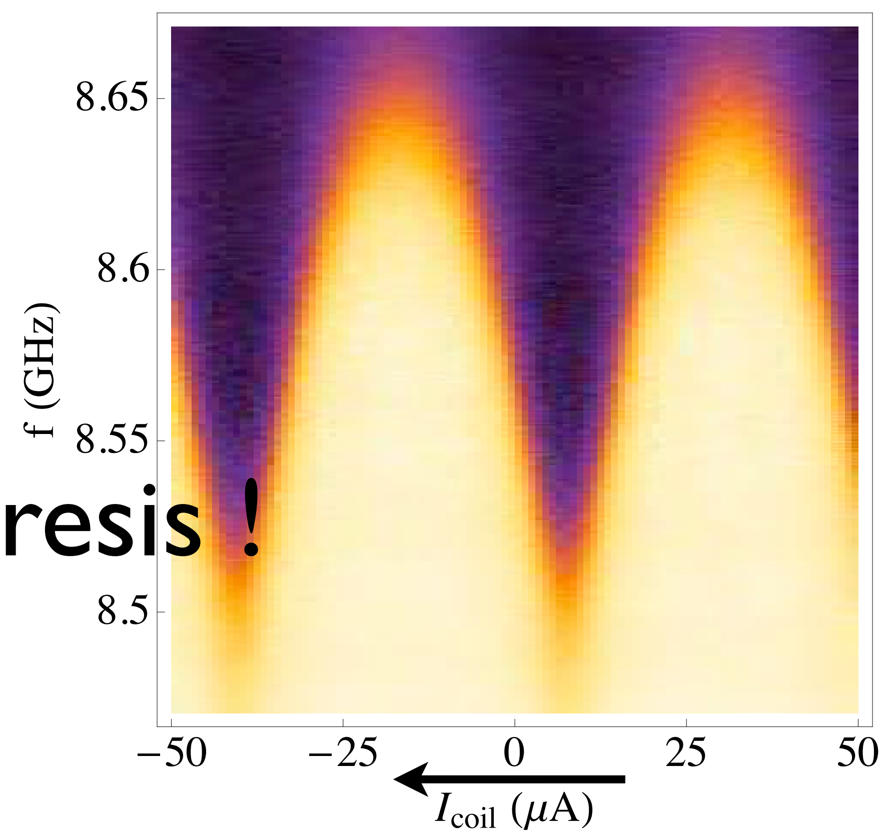


$Q_{coupl} = 132$ idler

Pump OFF

signal $Q_{coupl} = 220$

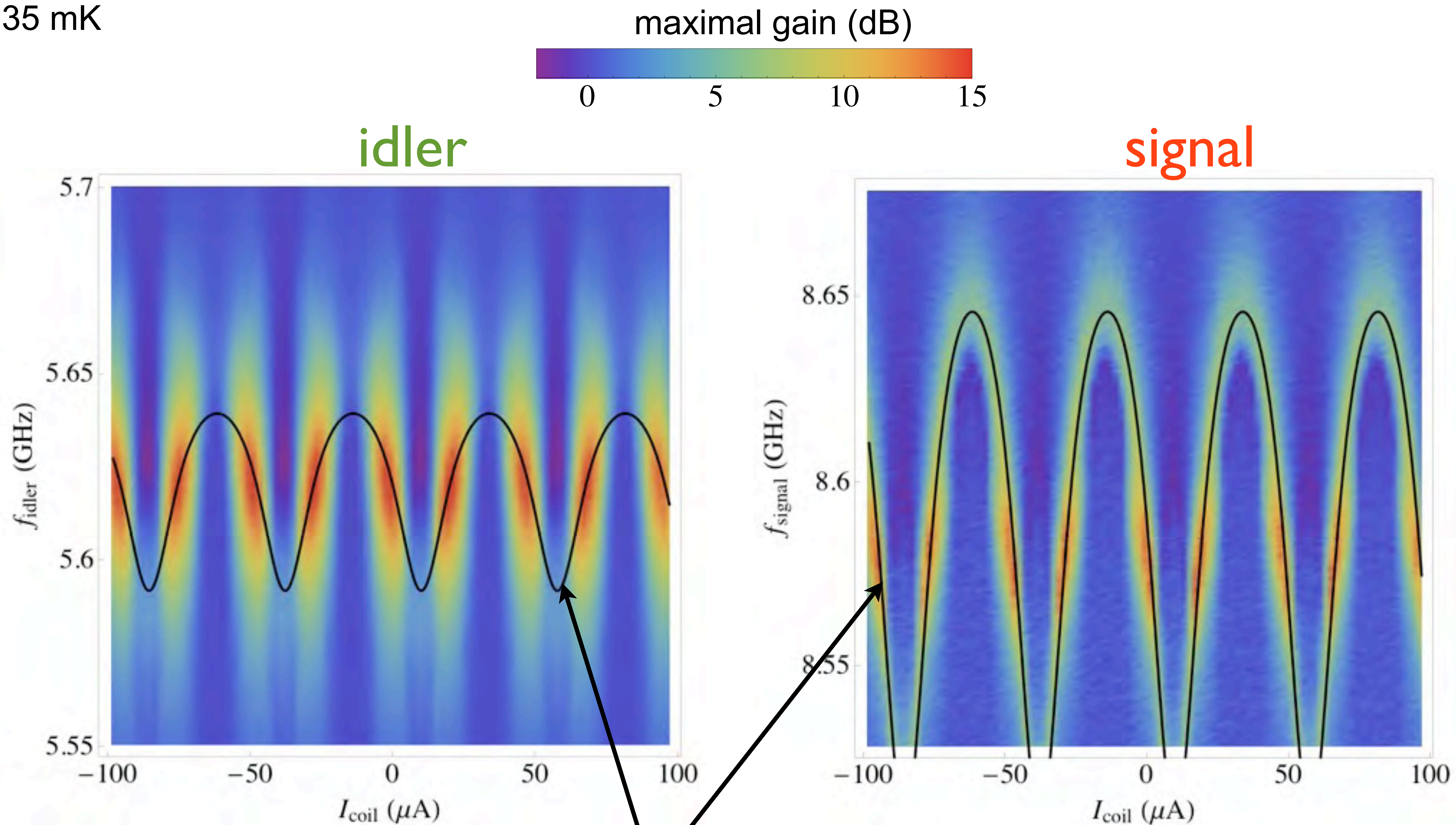




No more hysteresis !

Gain as a function of magnetic field

35 mK

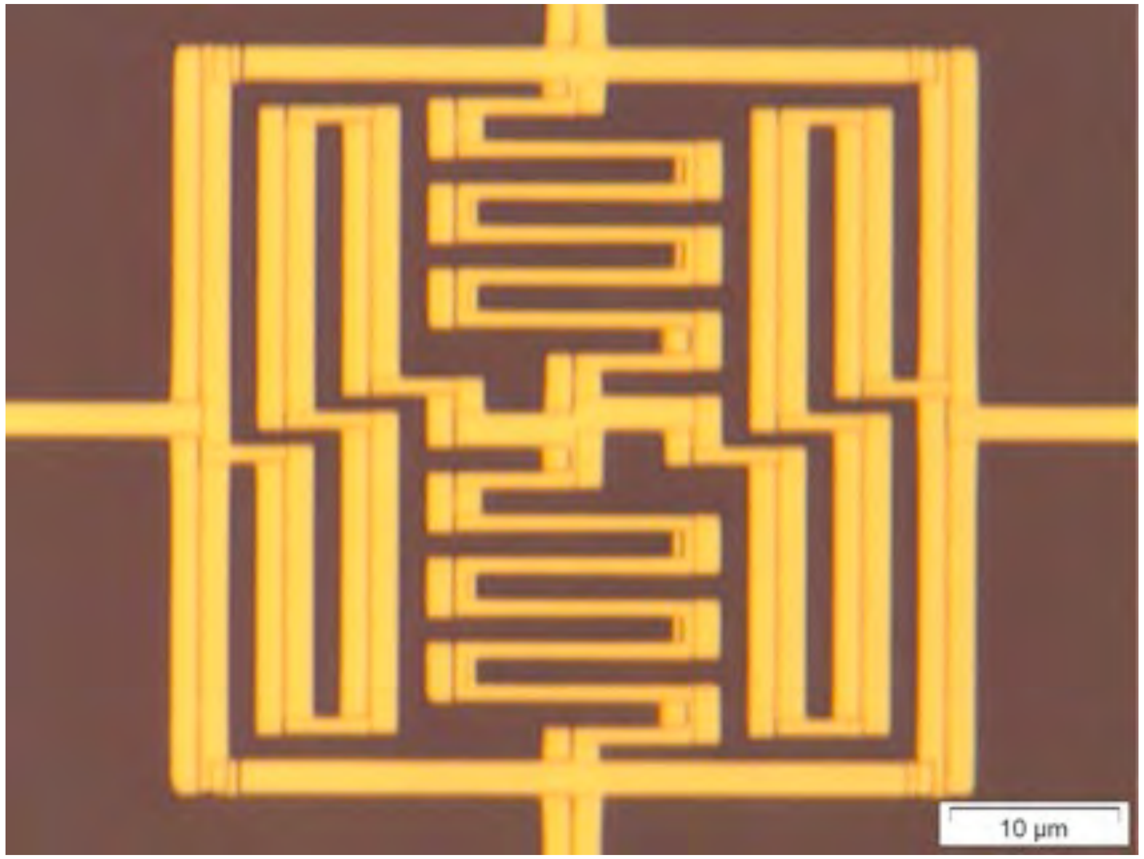


resonance @ P_{pump}=0

tunability !

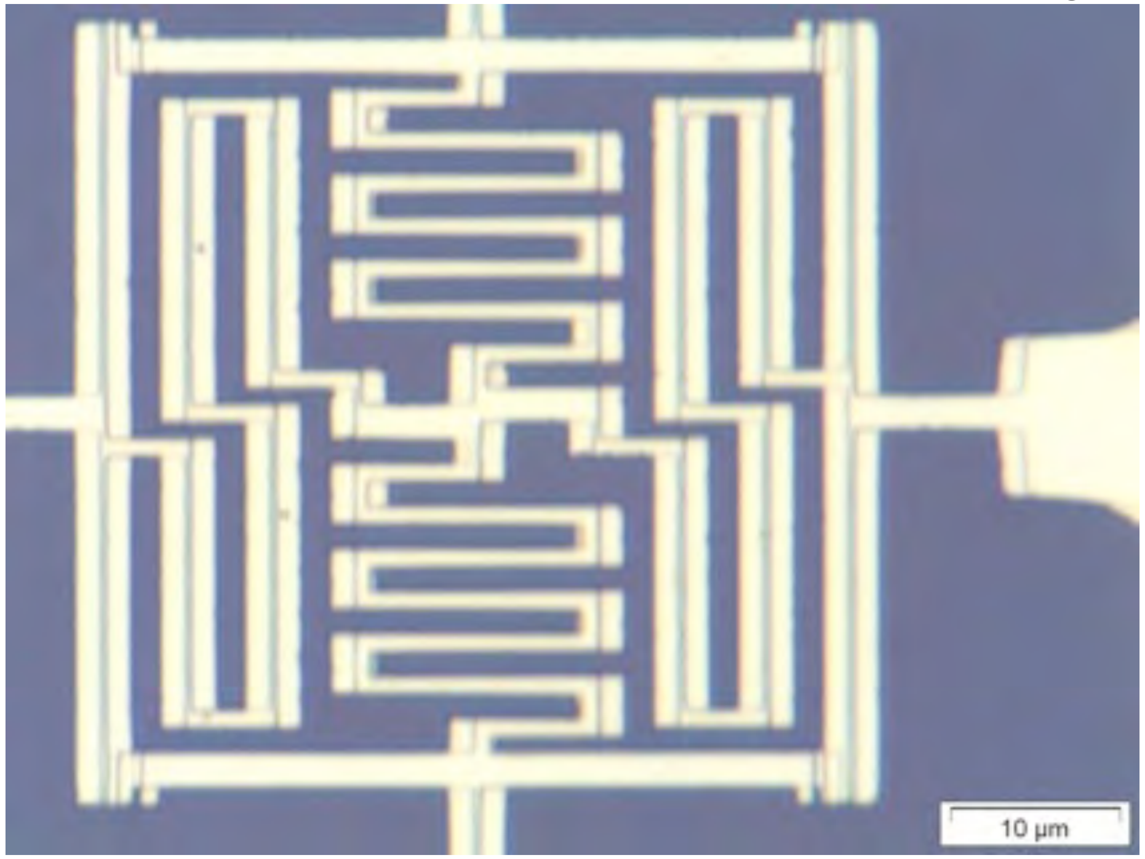
$$\frac{\mathrm{d}\omega_S}{\mathrm{d}\Phi} = \Phi_0 \frac{{\omega_S}^2}{2Z_0} \frac{I_0 L}{\varphi_0} L$$

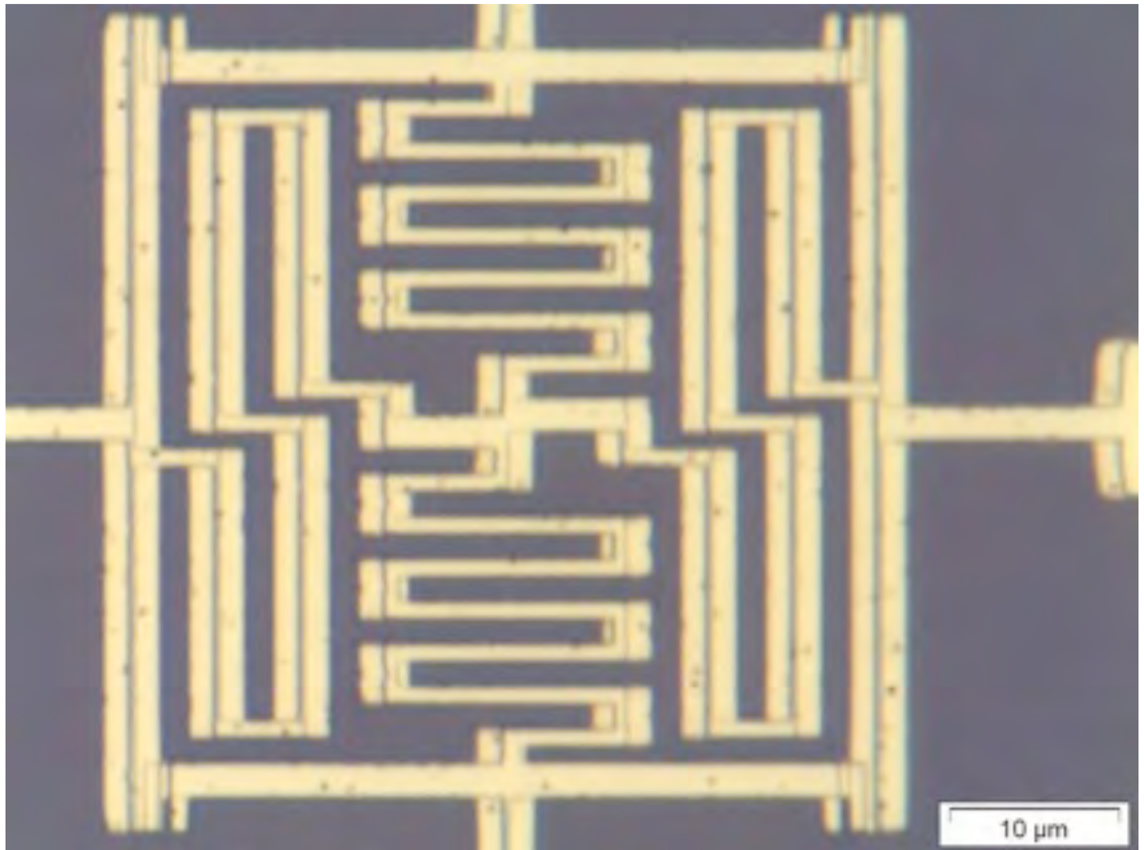
Varying the critical current



small I_0

medium I_0



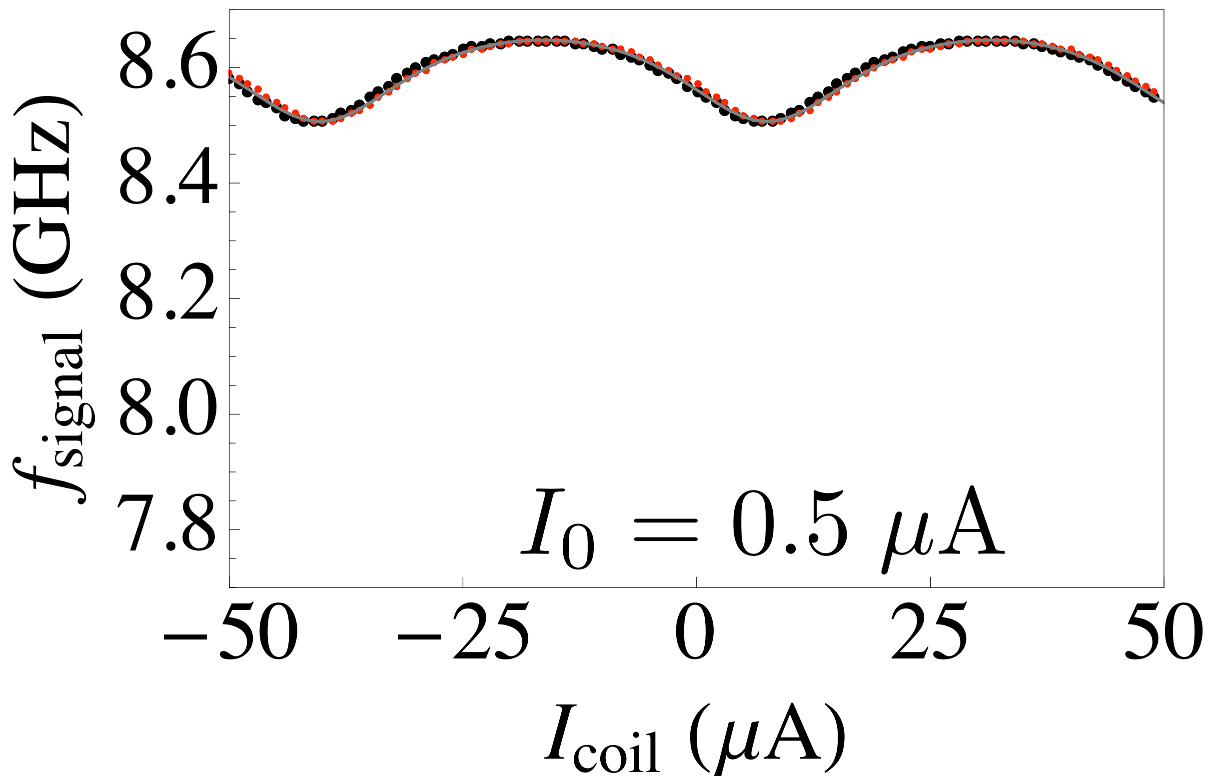


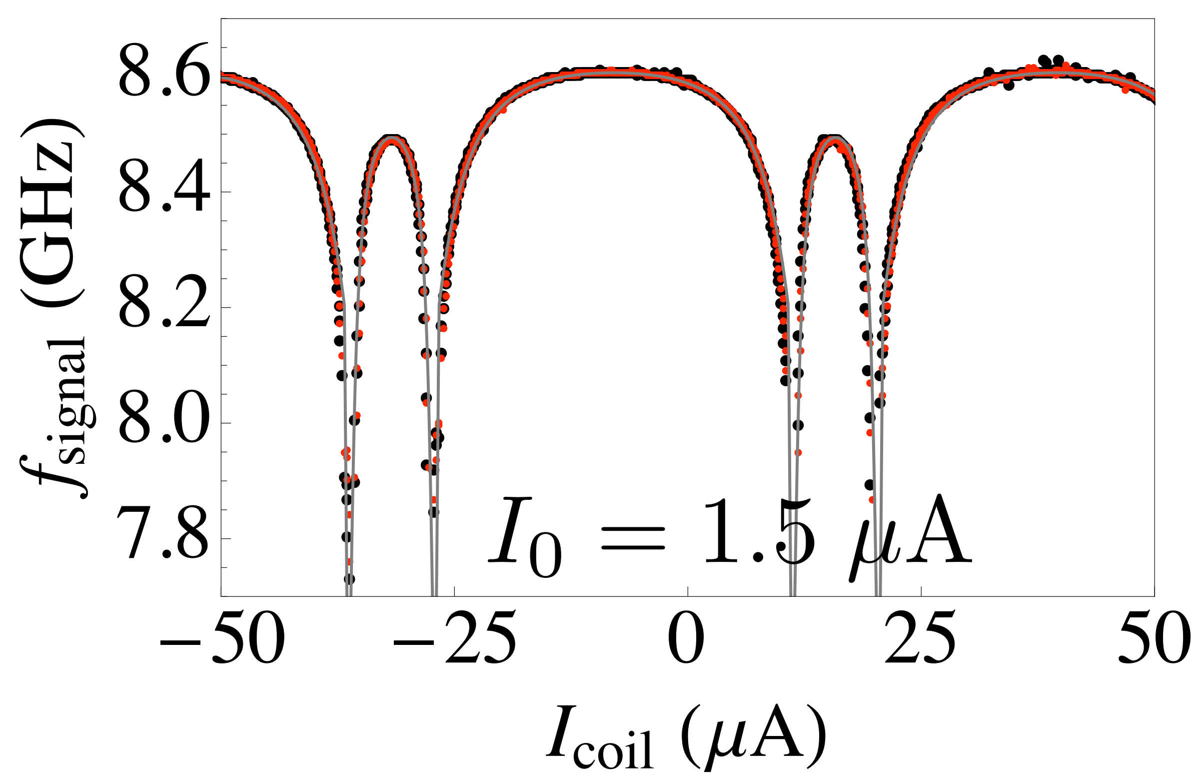
large I_0

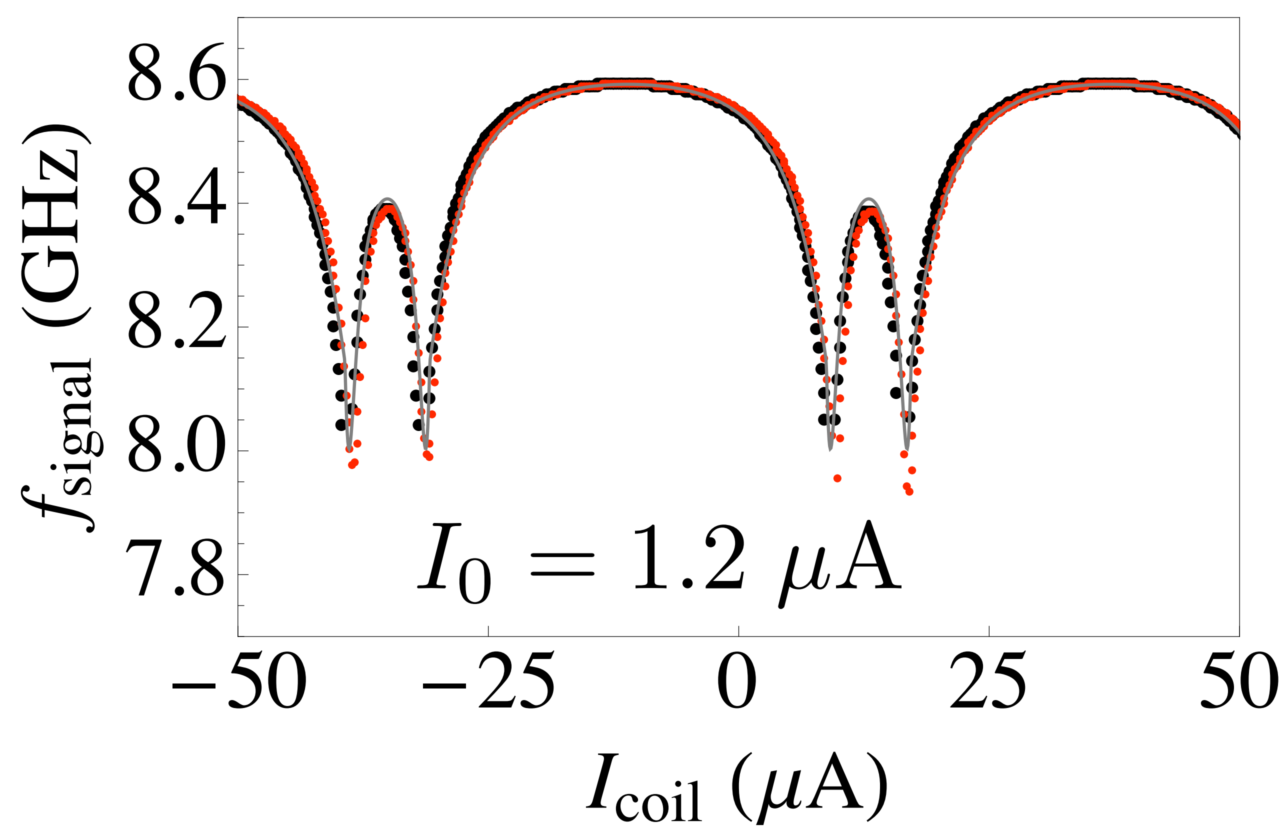
Resonance frequency as a function of field

● meas[t] $I_{\text{coil}} \searrow$ ● meas[t] $I_{\text{coil}} \nearrow$

—— theory with $L = 0.06 - 0.07$ nH
$L_{\text{series}} = 0.07 - 0.08$ nH
still OK with $\pm 20\%$





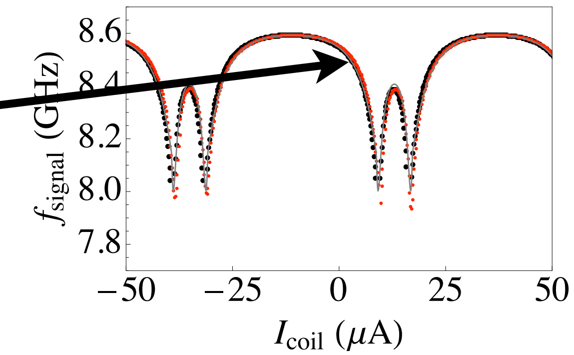


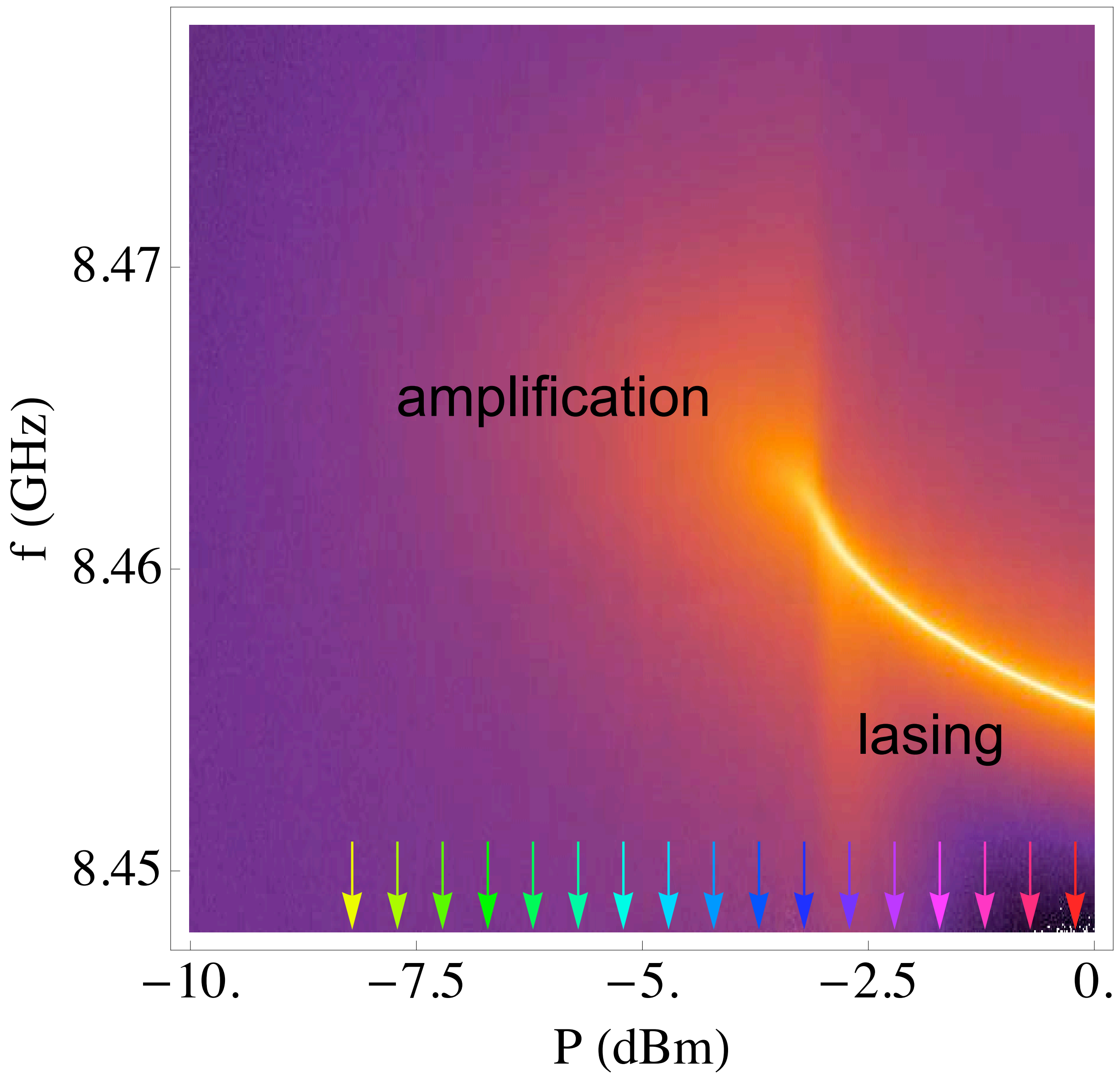
tunability ↗ with I_0 ↗

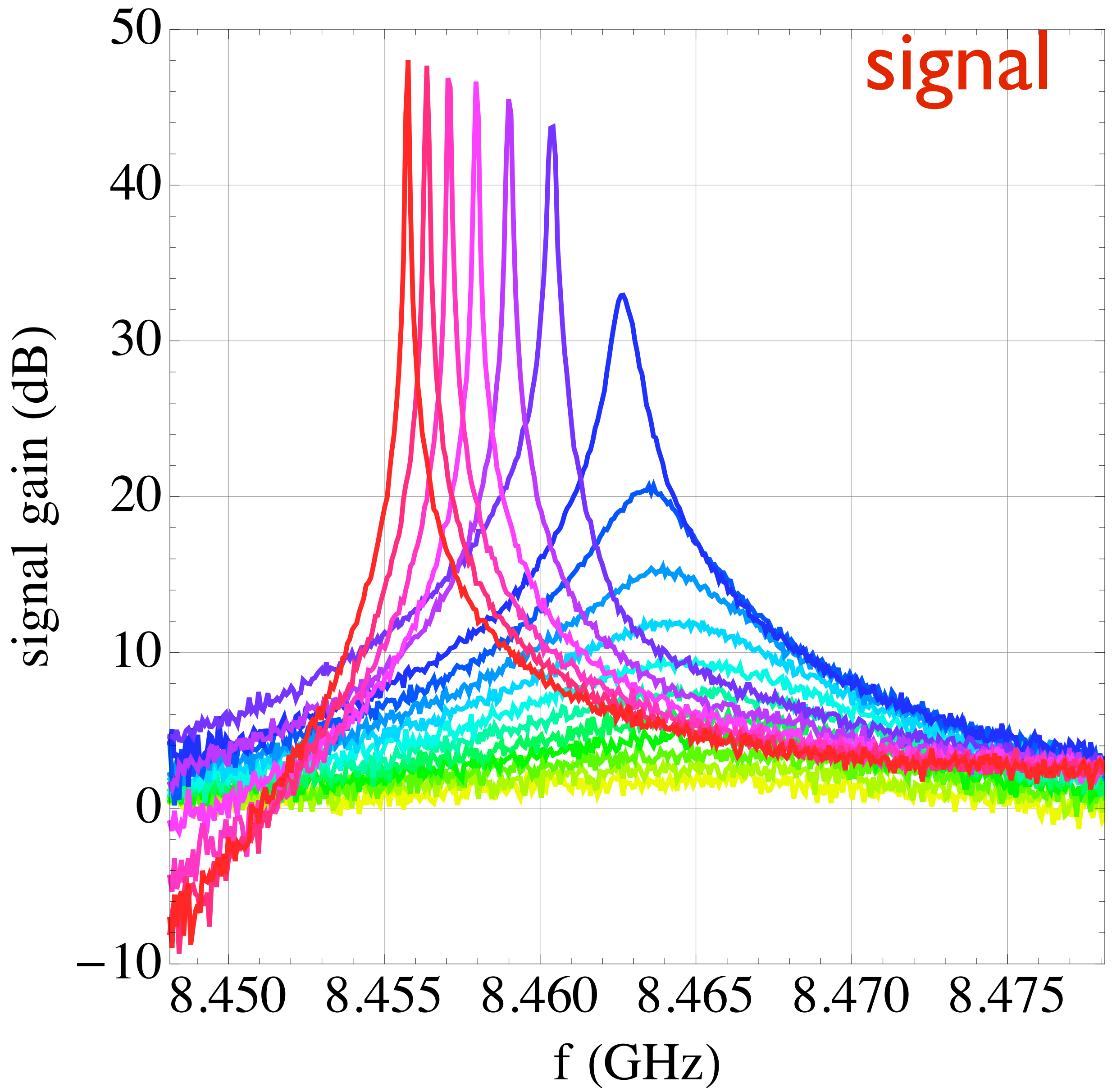
Gain as a function of pump power

35 mK

$$f_{\text{pump}} = 14.071\ \text{GHz}, I_{\text{coil}} = 3\ \mu A$$



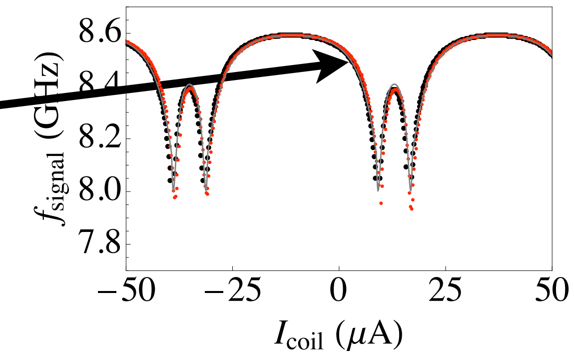


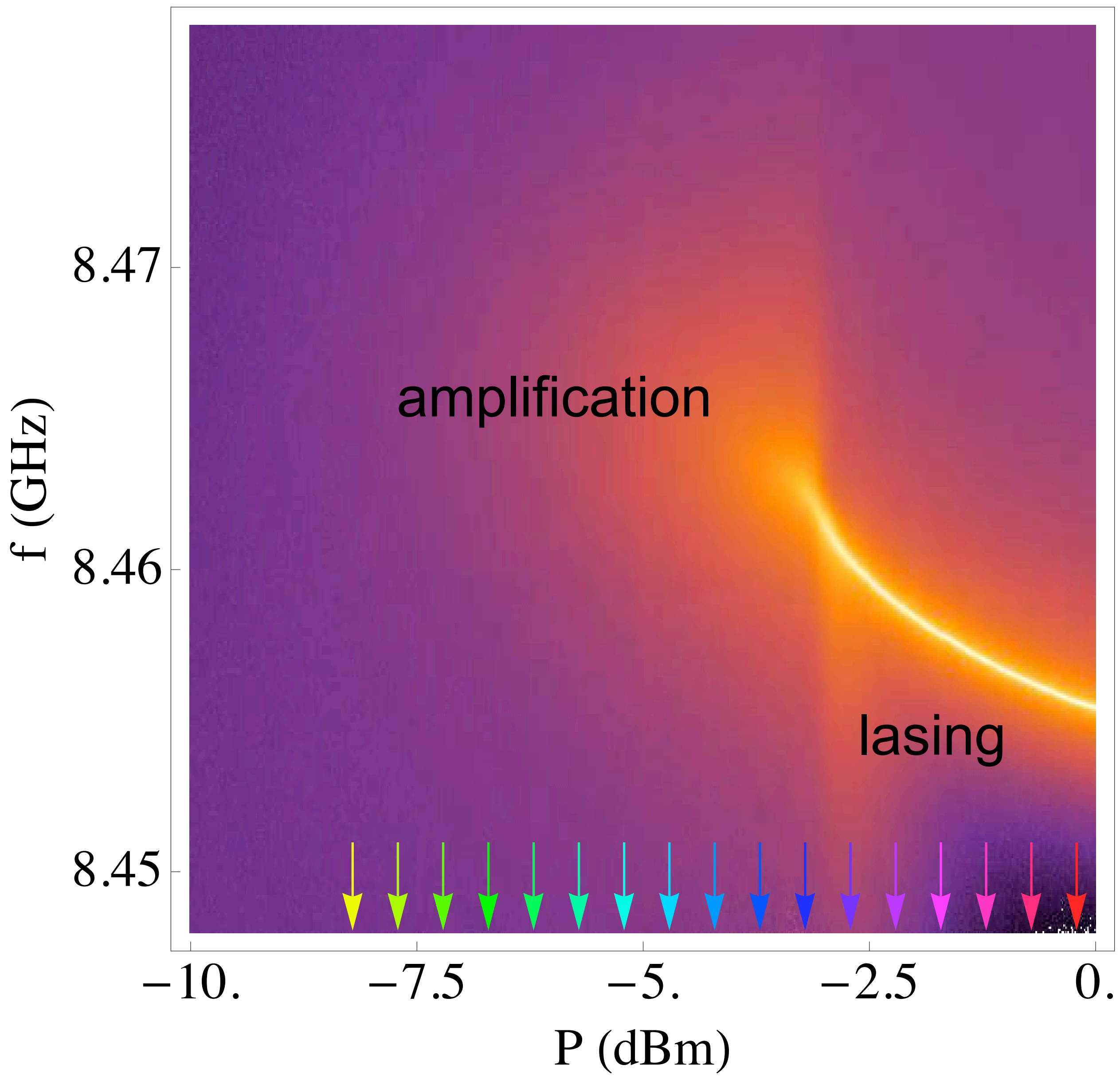


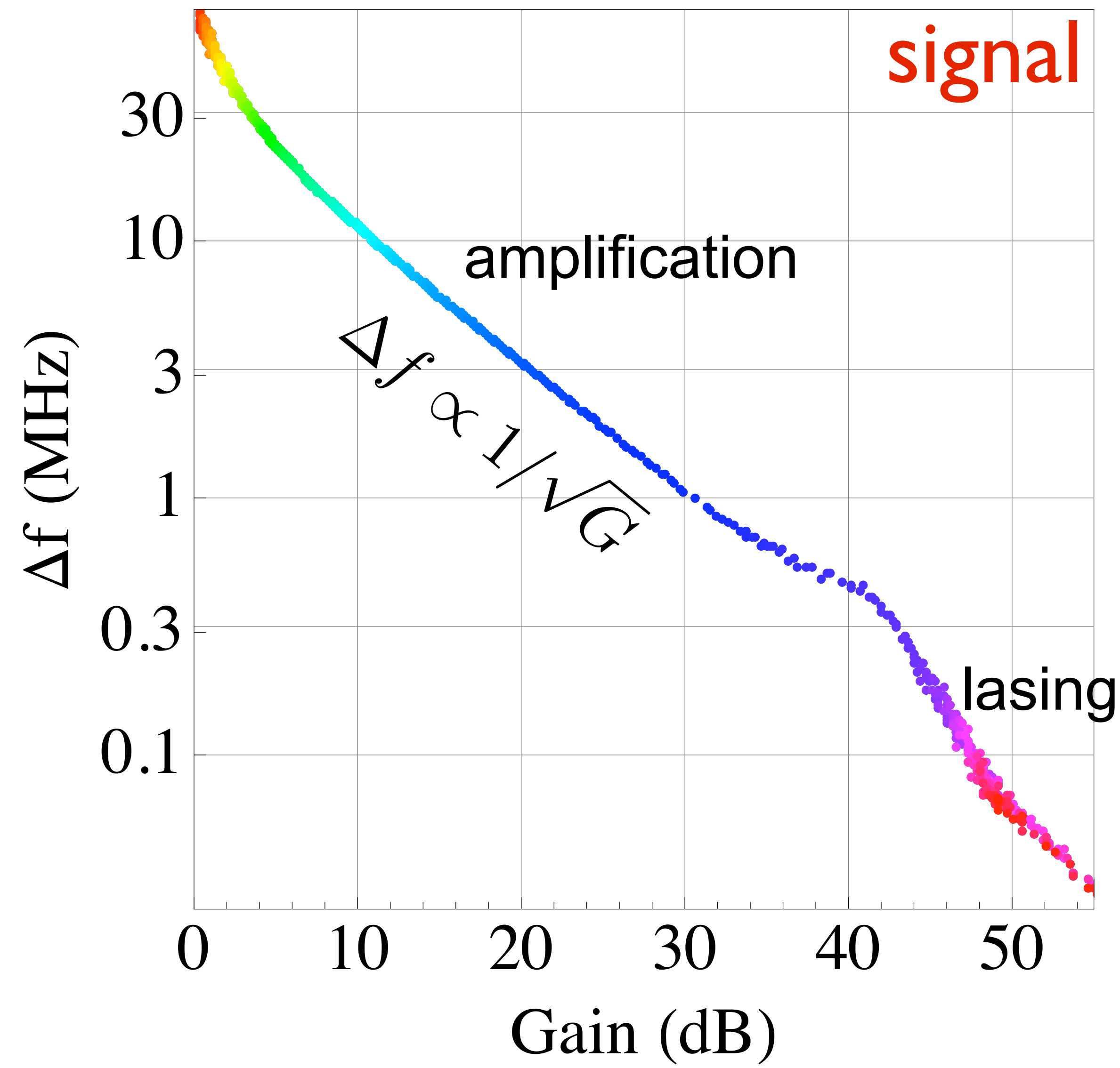
Gain as a function of pump power

35 mK

$$f_{\text{pump}} = 14.071 \text{ GHz}, I_{\text{coil}} = 3\ \mu A$$



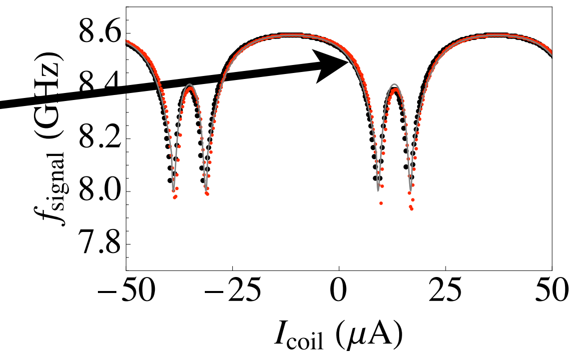


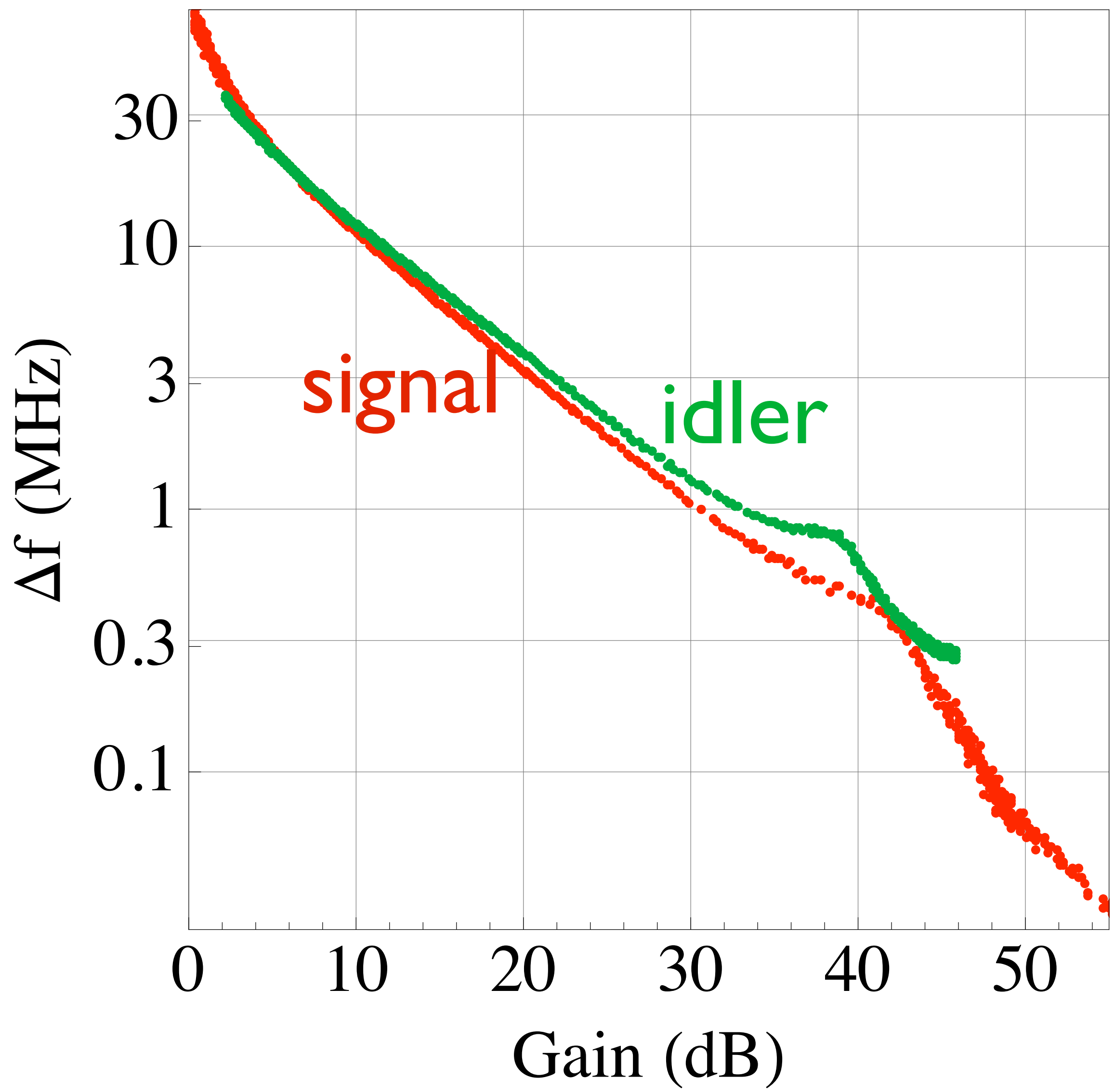


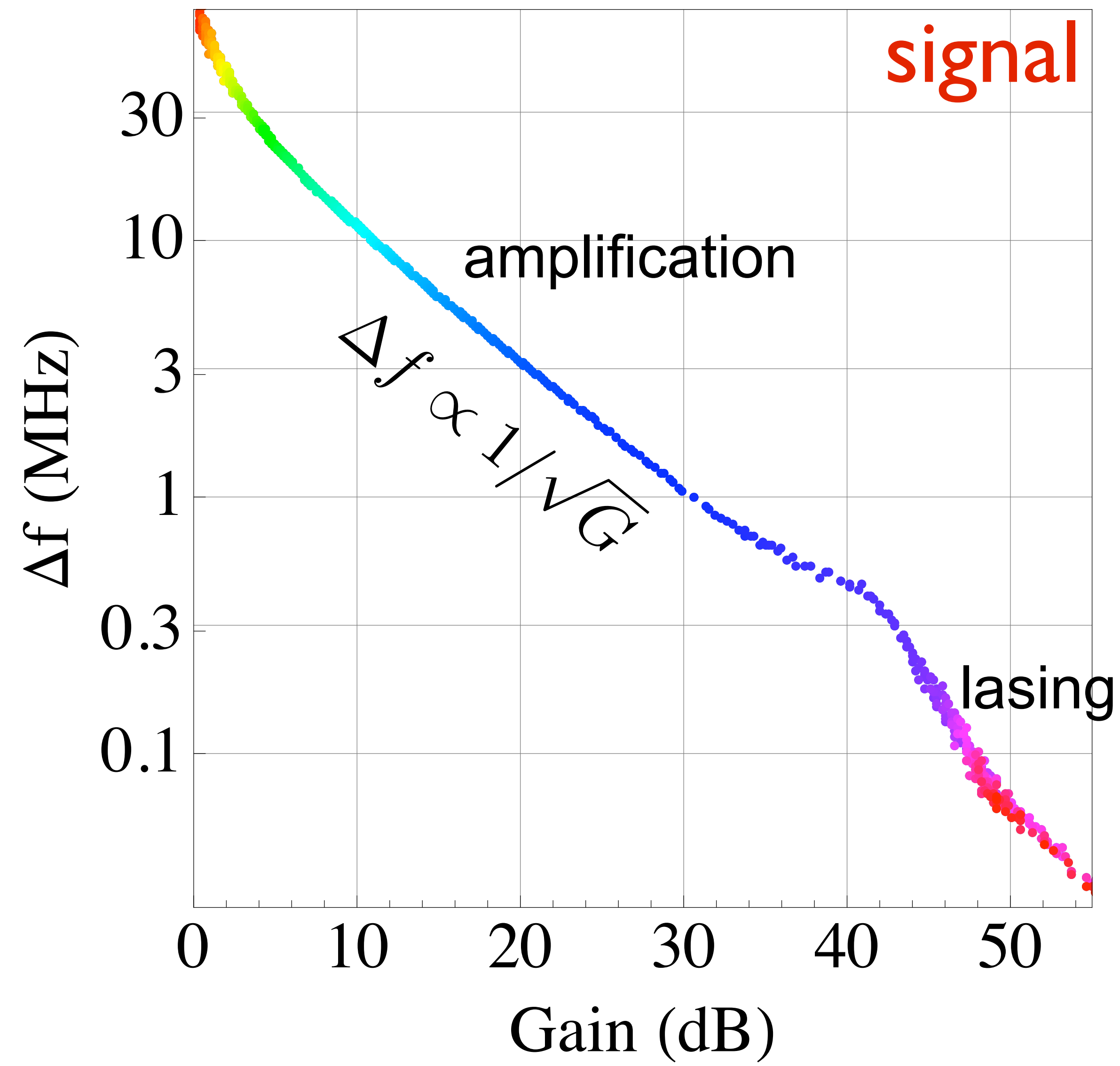
Gain as a function of pump power

35 mK

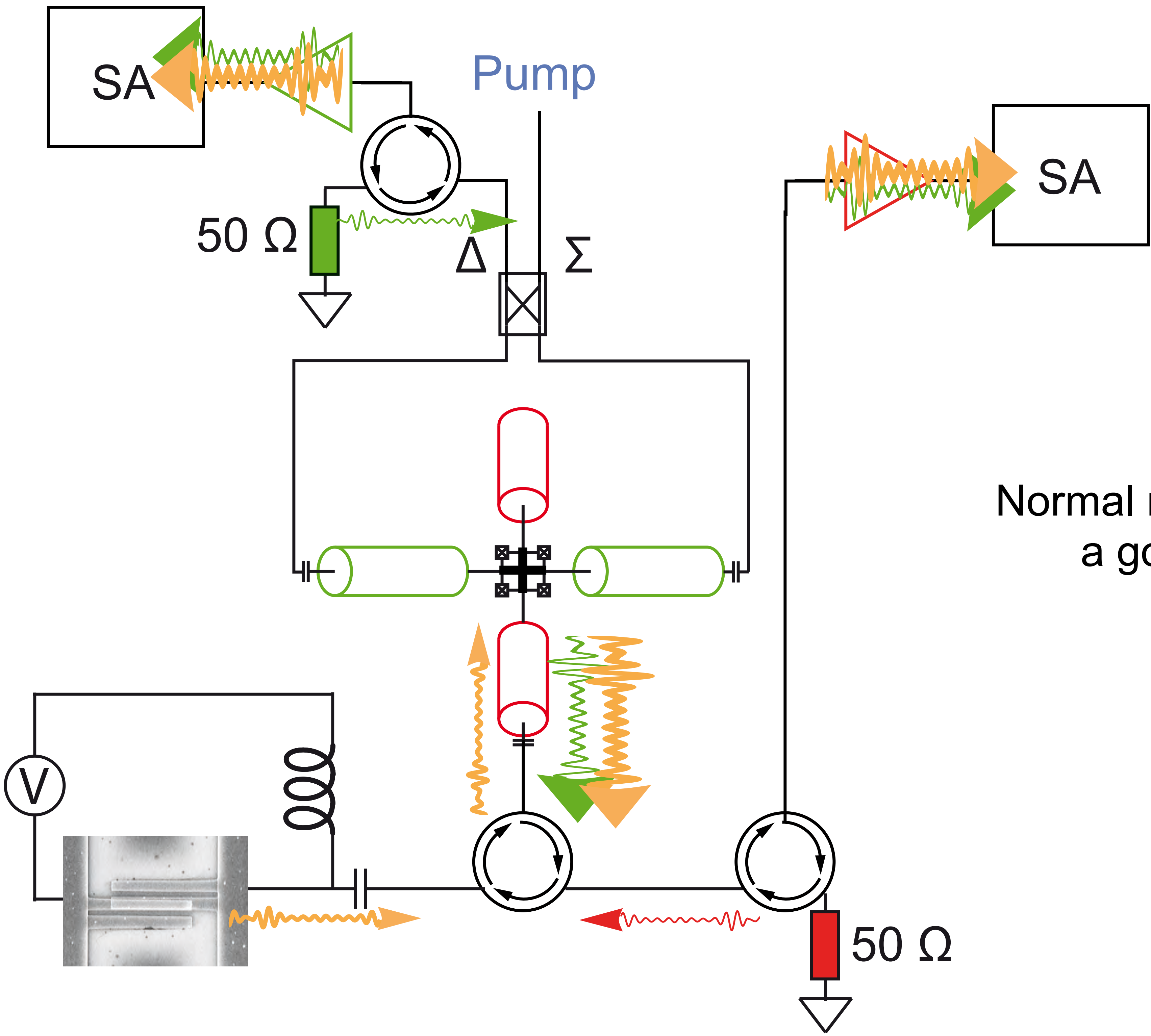
$f_{\mathrm{pump}} = 14.071\ \mathrm{GHz}, I_{\mathrm{coil}} = 3\ \mu A$





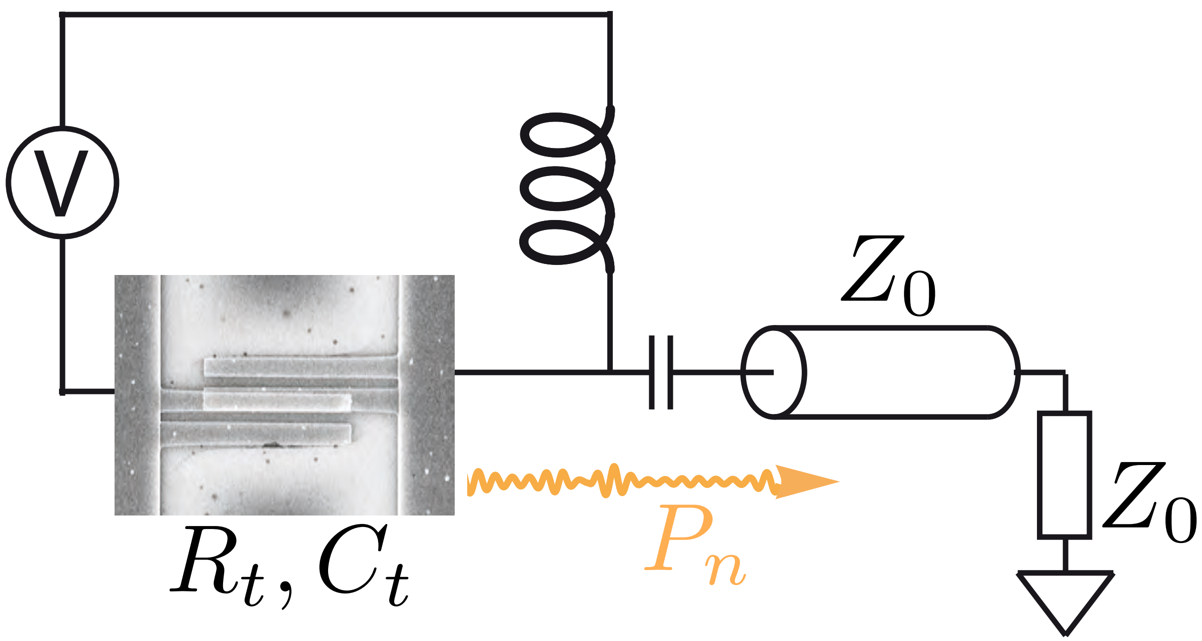


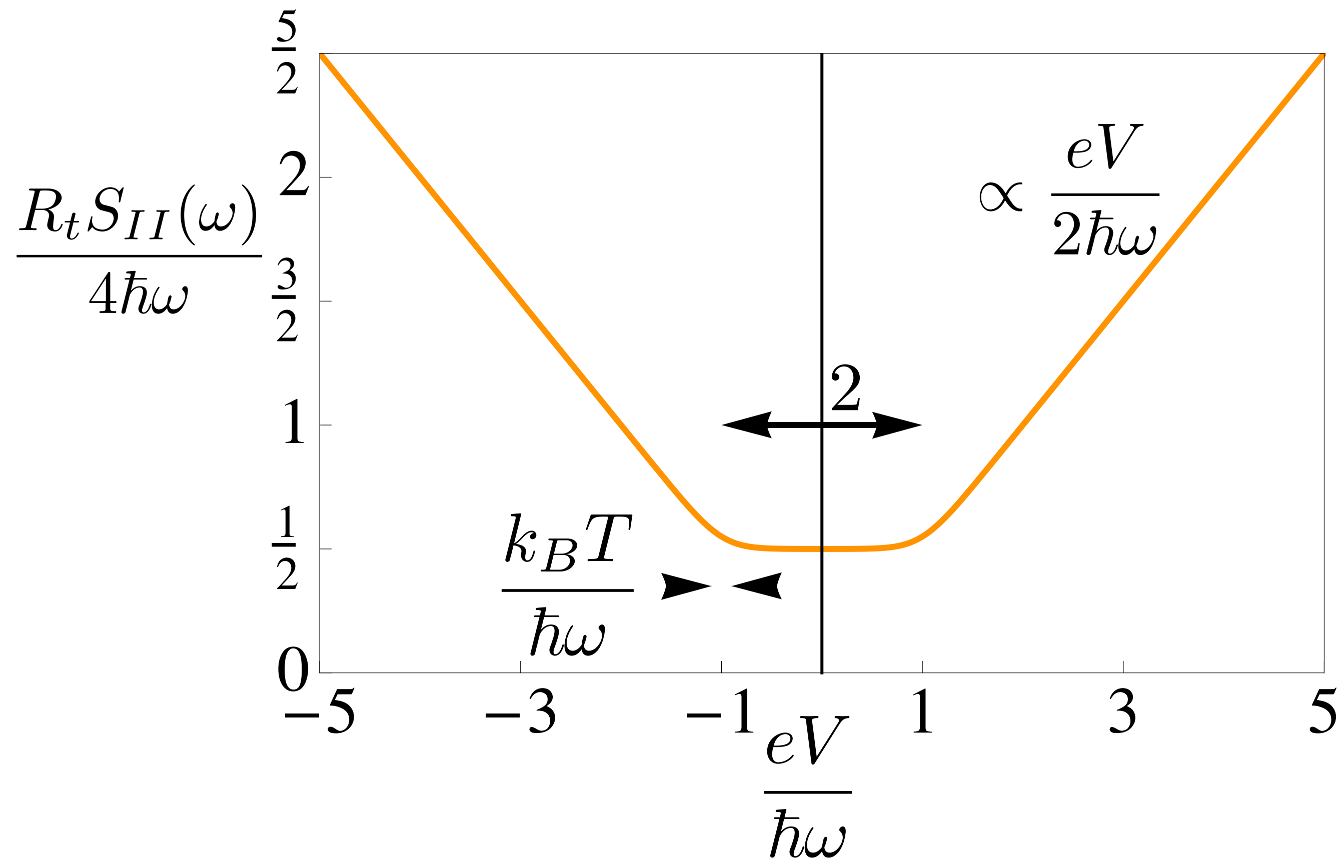
Noise calibration



Normal metal tunnel junction:
a good noise source

Noise measurement





$$\hbar\omega \gg k_B T \Rightarrow R_t S_{II}(V, \omega_S) = \mathrm{Max}(2|eV|, 2\hbar\omega_S)$$

$$\neq \sqrt{(eV)^2 + (\hbar\omega_S)^2}$$

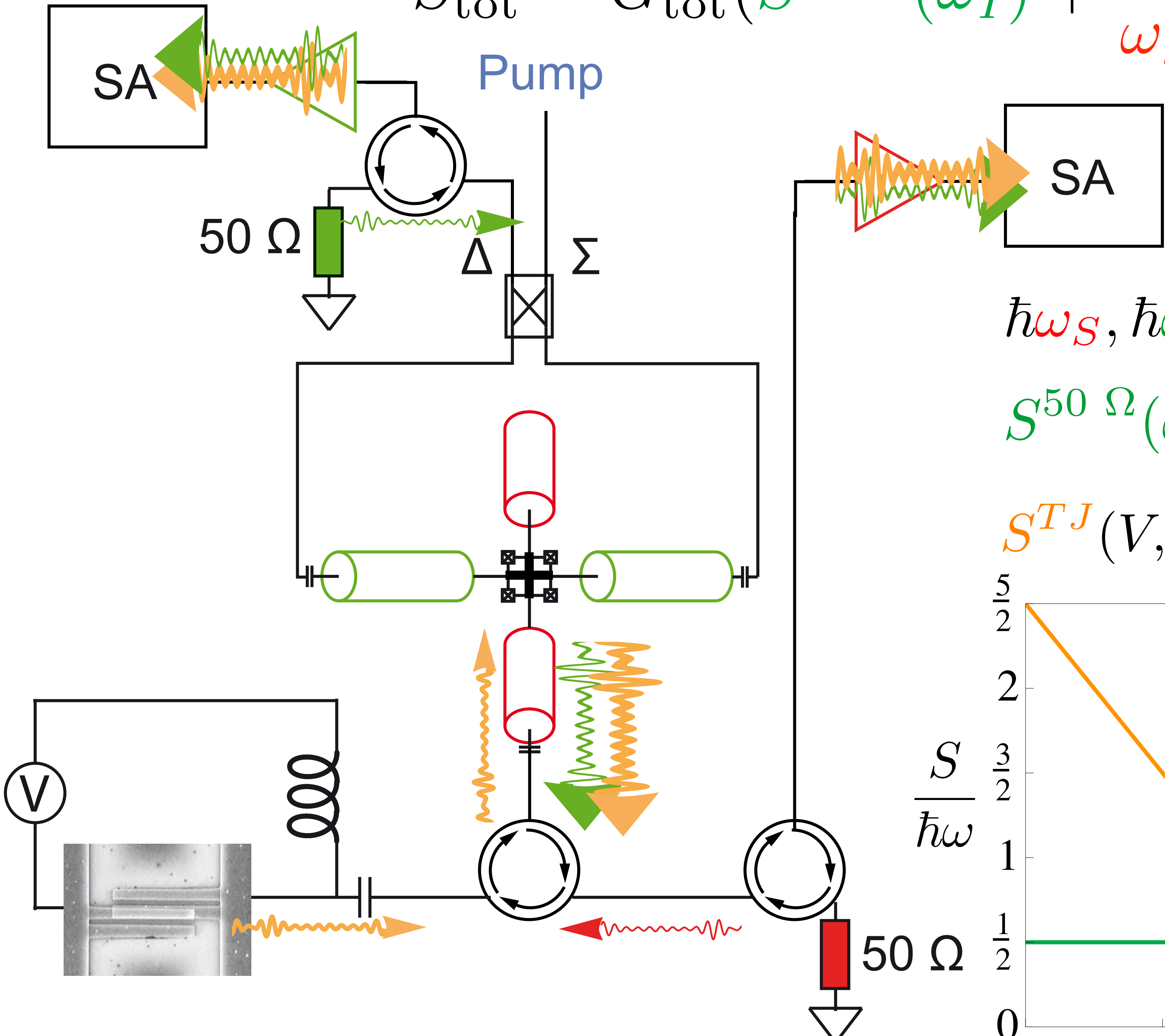
$$P_n(\omega_s) = \frac{Z_0 S_{II}(\omega_s)}{4}\Delta\omega$$

if $R_t = Z_0$ and $R_t C_t \omega_S \ll 1$

perfect matching

Noise measurement

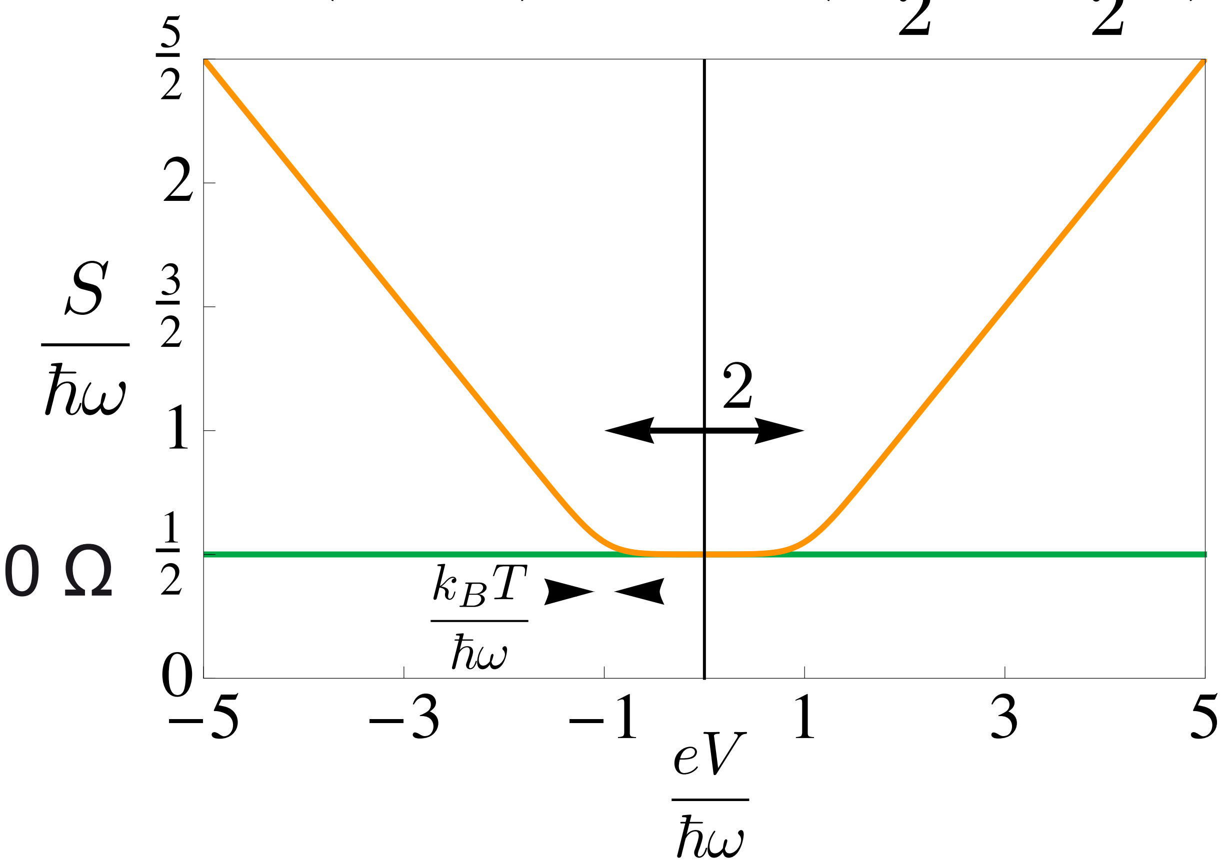
$$S_{\text{tot}} = G_{\text{tot}}\left(S^{50\ \Omega}(\omega_I) + \frac{\omega_I}{\omega_S} S^{TJ}(\omega_S)\right)$$



$$\hbar\omega_S, \hbar\omega_I \gg k_B T$$

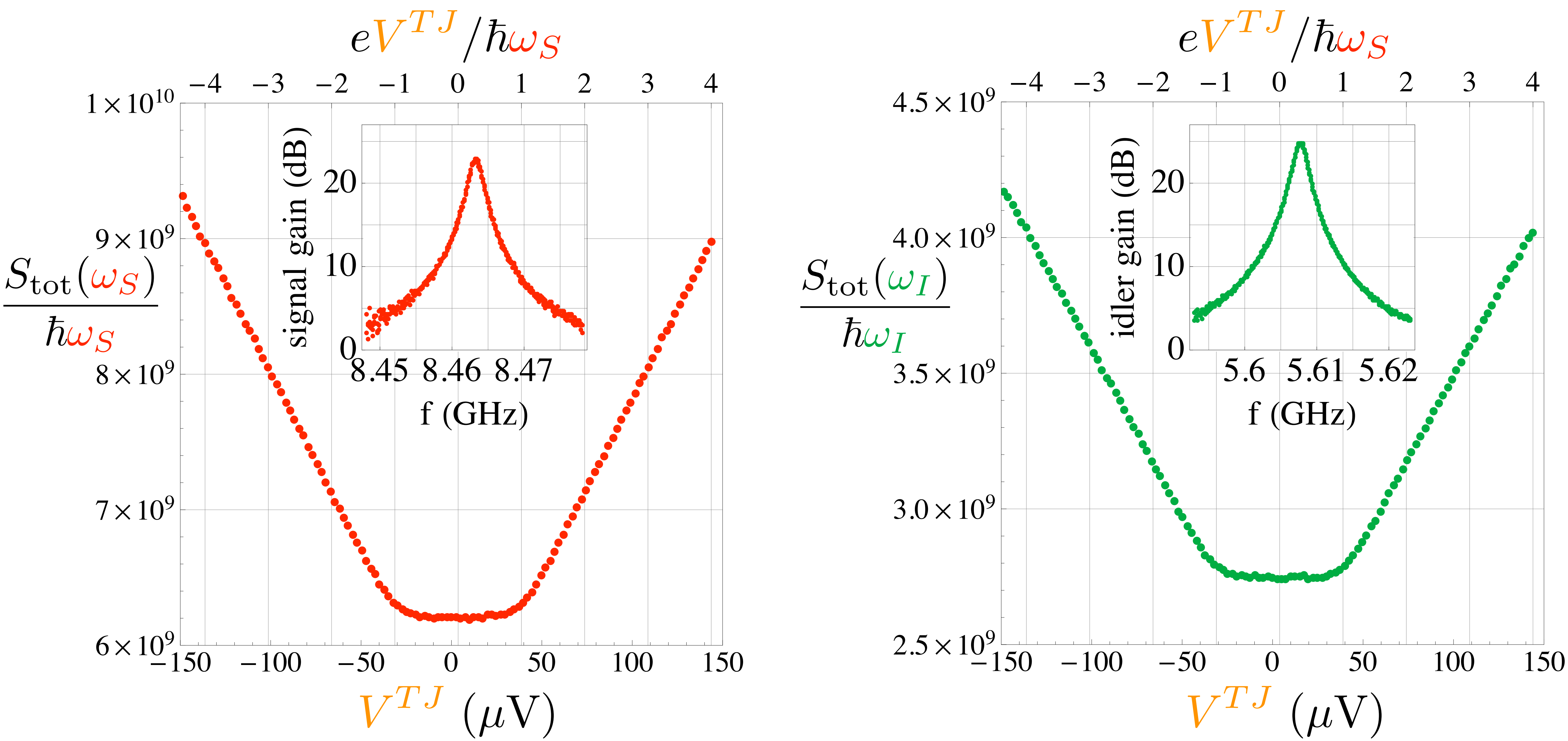
$$S^{50\ \Omega}(\omega_I) = \frac{\hbar\omega_I}{2}$$

$$S^{TJ}(V, \omega_S) = \text{Max}\left(\frac{|eV|}{2}, \frac{\hbar\omega_S}{2}\right)$$



Noise measurement

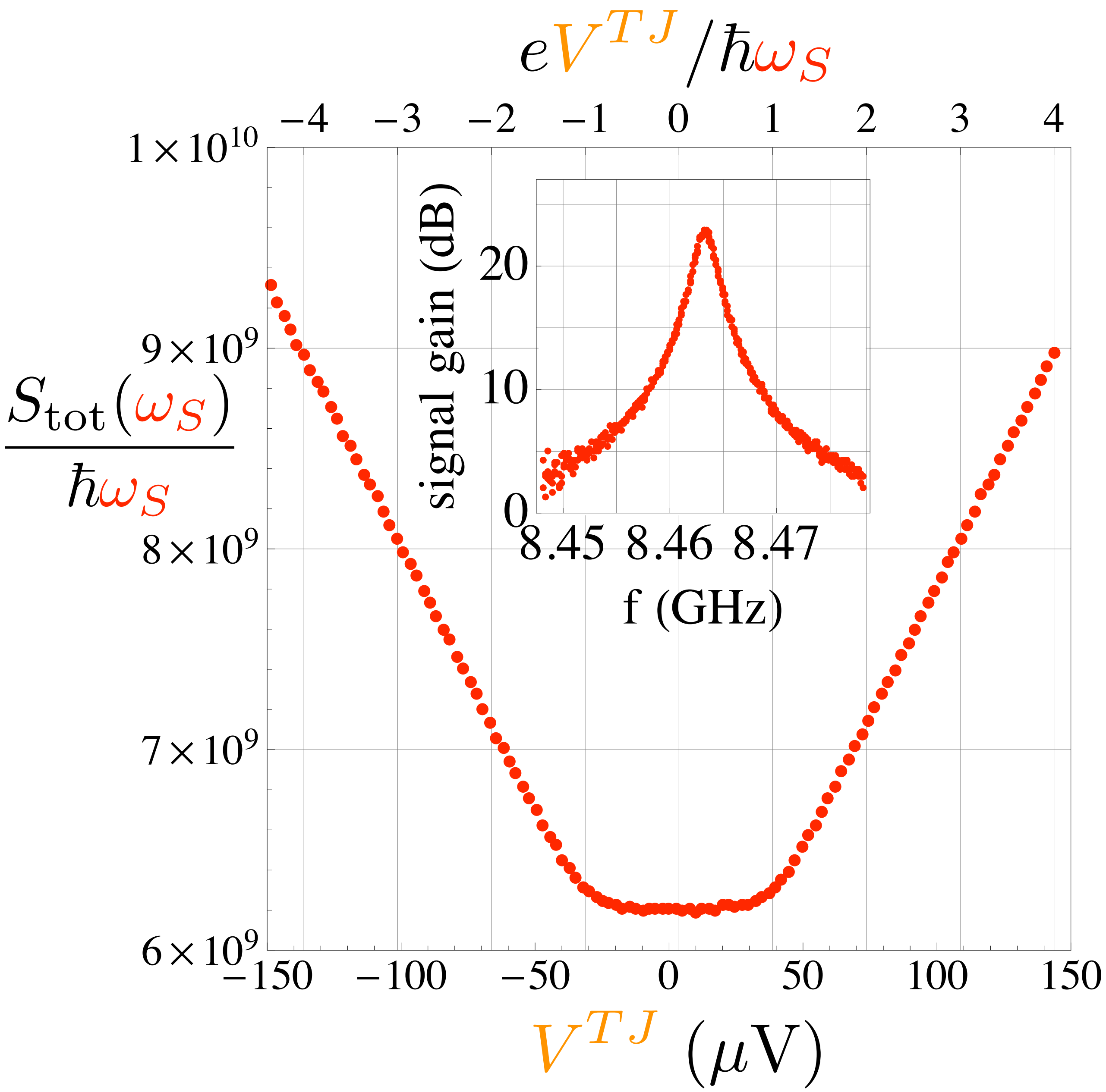
$f_{\text{pump}} = 14.071$ GHz, $P_{\text{pump}} = -3.56$ dBm, $I_{\text{coil}} = 3\ \mu A$

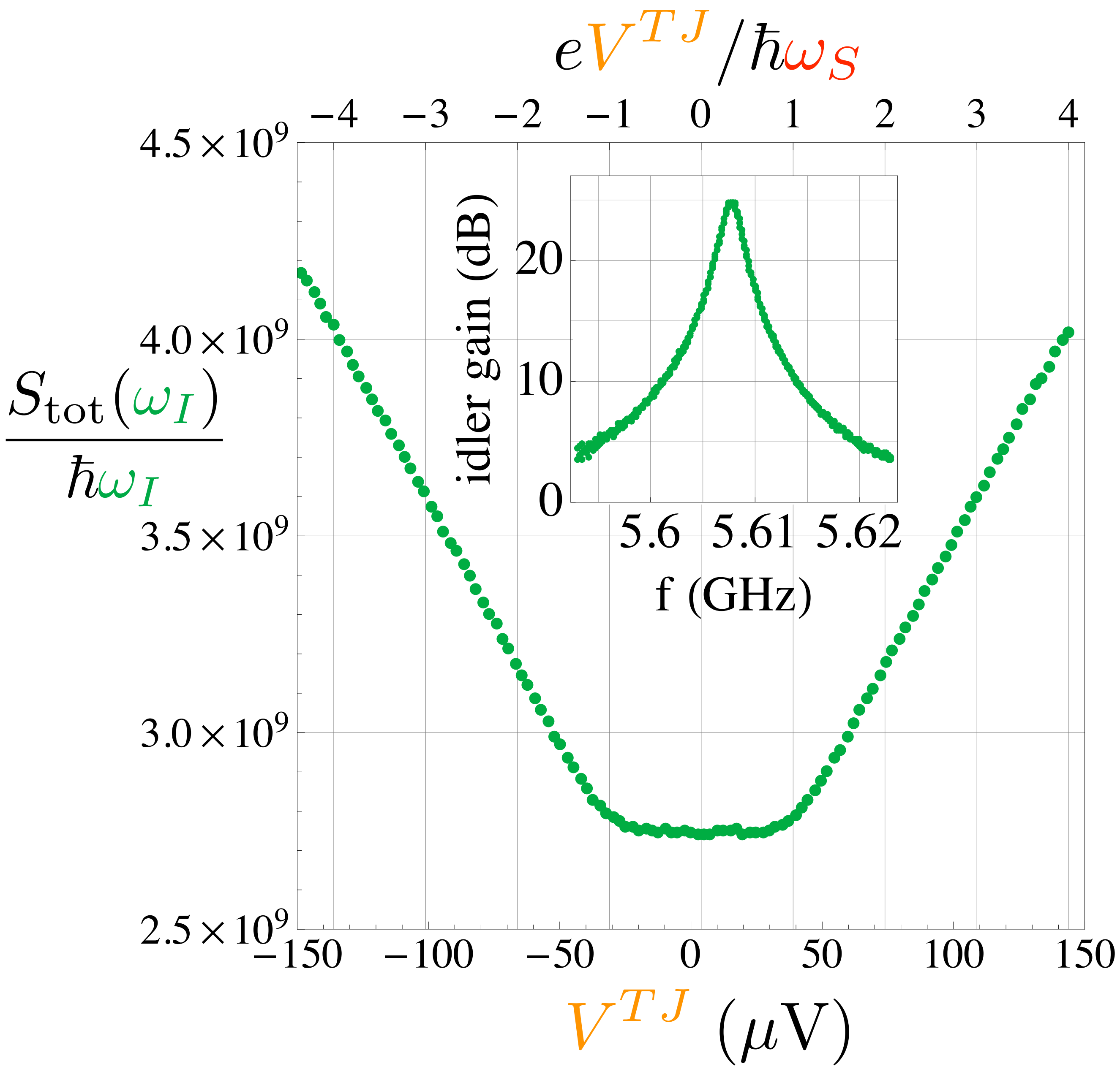


slope change at $eV^{TJ} = \hbar\omega_S$ even for $S_{\text{tot}}(\omega_I)$

Noise measurement

$f_{\text{pump}} = 14.071$ GHz, $P_{\text{pump}} = -3.56$ dBm, $I_{\text{coil}} = 3\ \mu A$



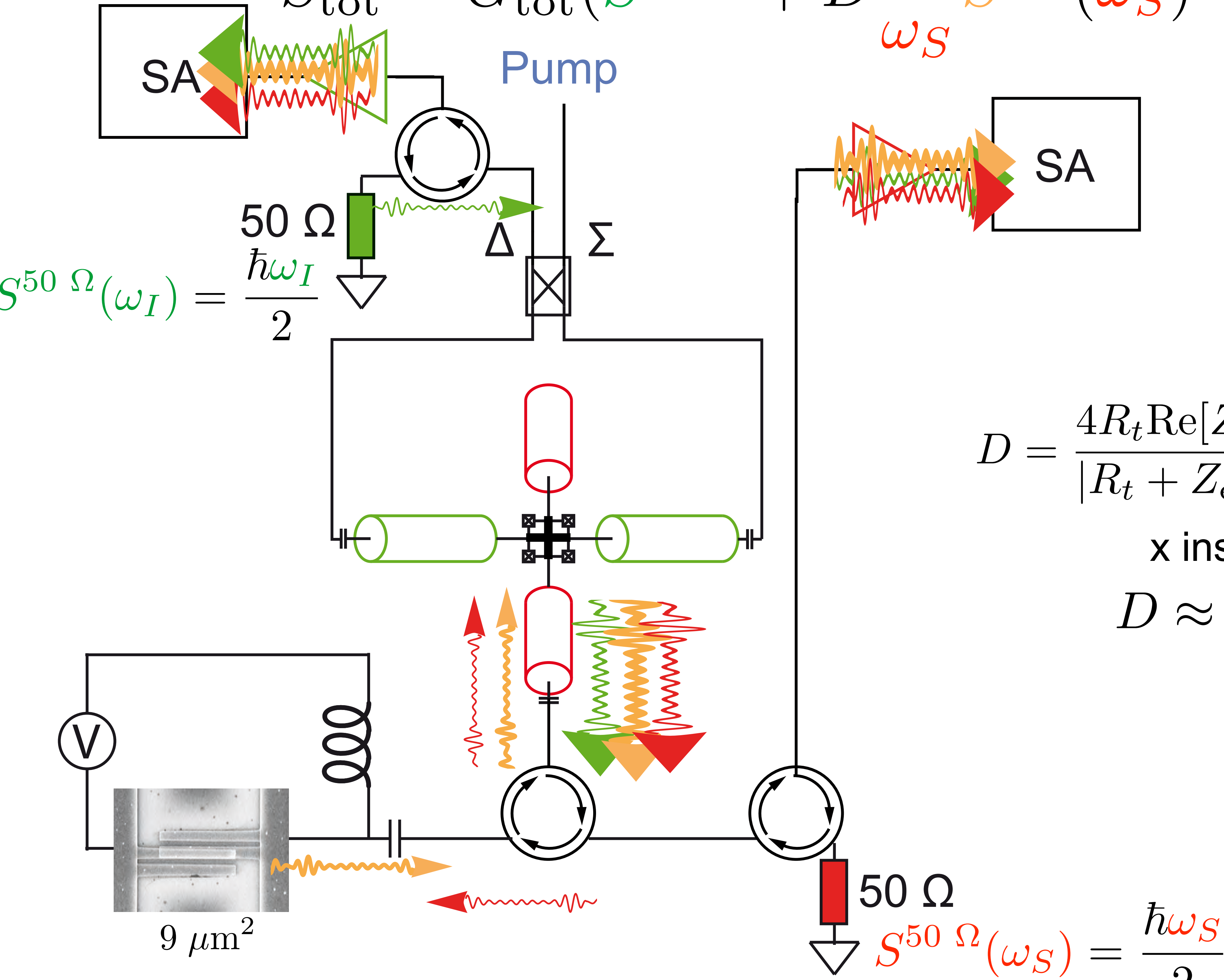


Can we use it to calibrate the added noise ?

YES, but need to determine the impedance matching of the junction

Noise measurement

$$S_{\text{tot}} = G_{\text{tot}}\left(S^{50\ \Omega} + D\frac{\omega_I}{\omega_S}S^{TJ}(\omega_S) + (1-D)\frac{\omega_I}{\omega_S}S^{50\ \Omega}\right)$$



$$D = \frac{4R_t\text{Re}[Z_{\text{env}}(\omega_S)]}{|R_t + Z_{\text{env}}(\omega_S)|^2} \approx 0.38 - 0.71$$

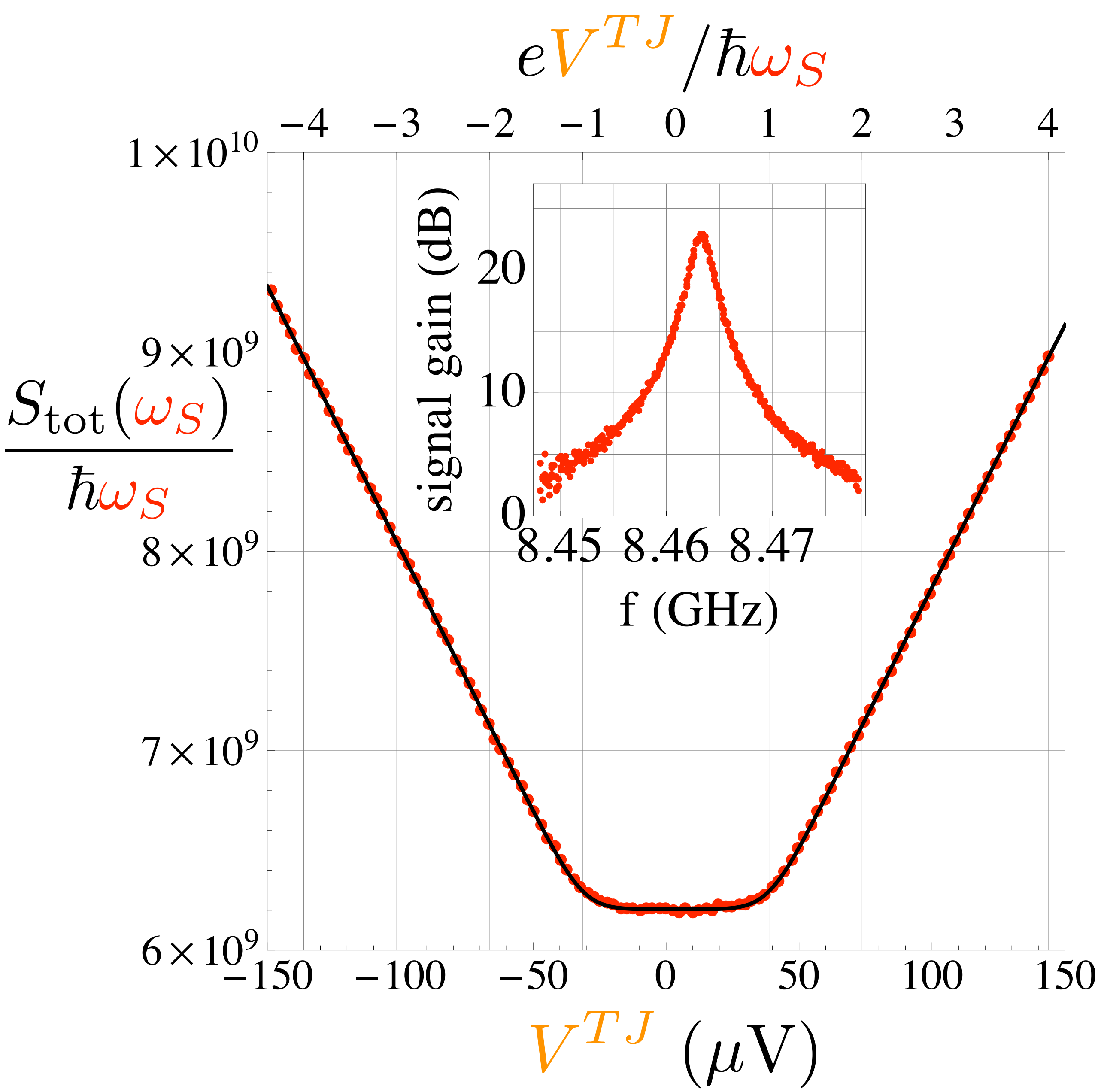
x insertion loss

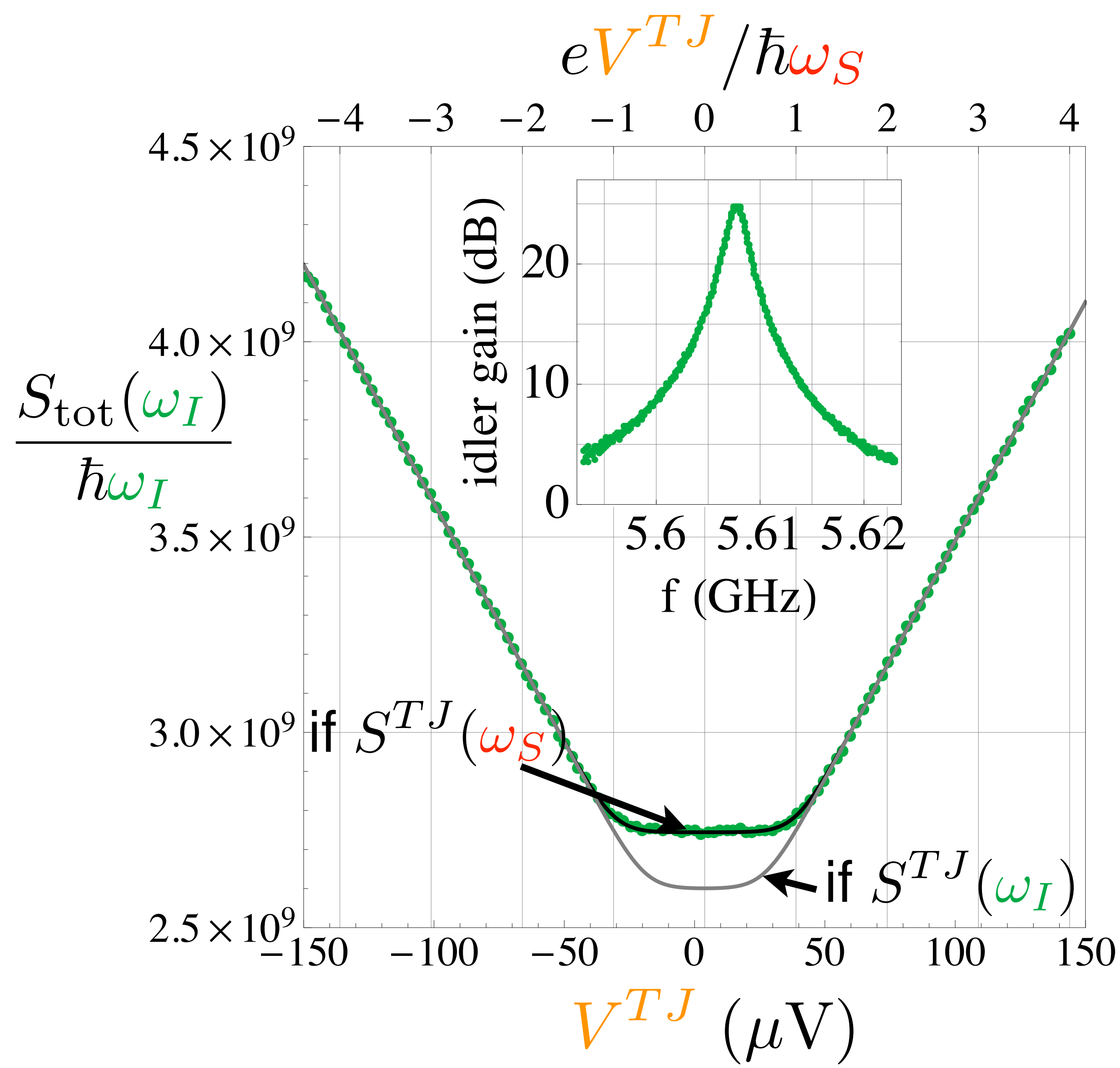
$$D \approx 0.2 - 0.71$$

$C_t \approx 0.5 - 1$ pF, $R_t = 44\ \Omega$, $Z_0 = 50\ \Omega$, $\omega/2\pi = 8.6$ GHz

Noise measurement

$$f_{\text{pump}} = 14.071\ \text{GHz}, P_{\text{pump}} = -3.56\ \text{dBm}, I_{\text{coil}} = 3\ \mu A$$

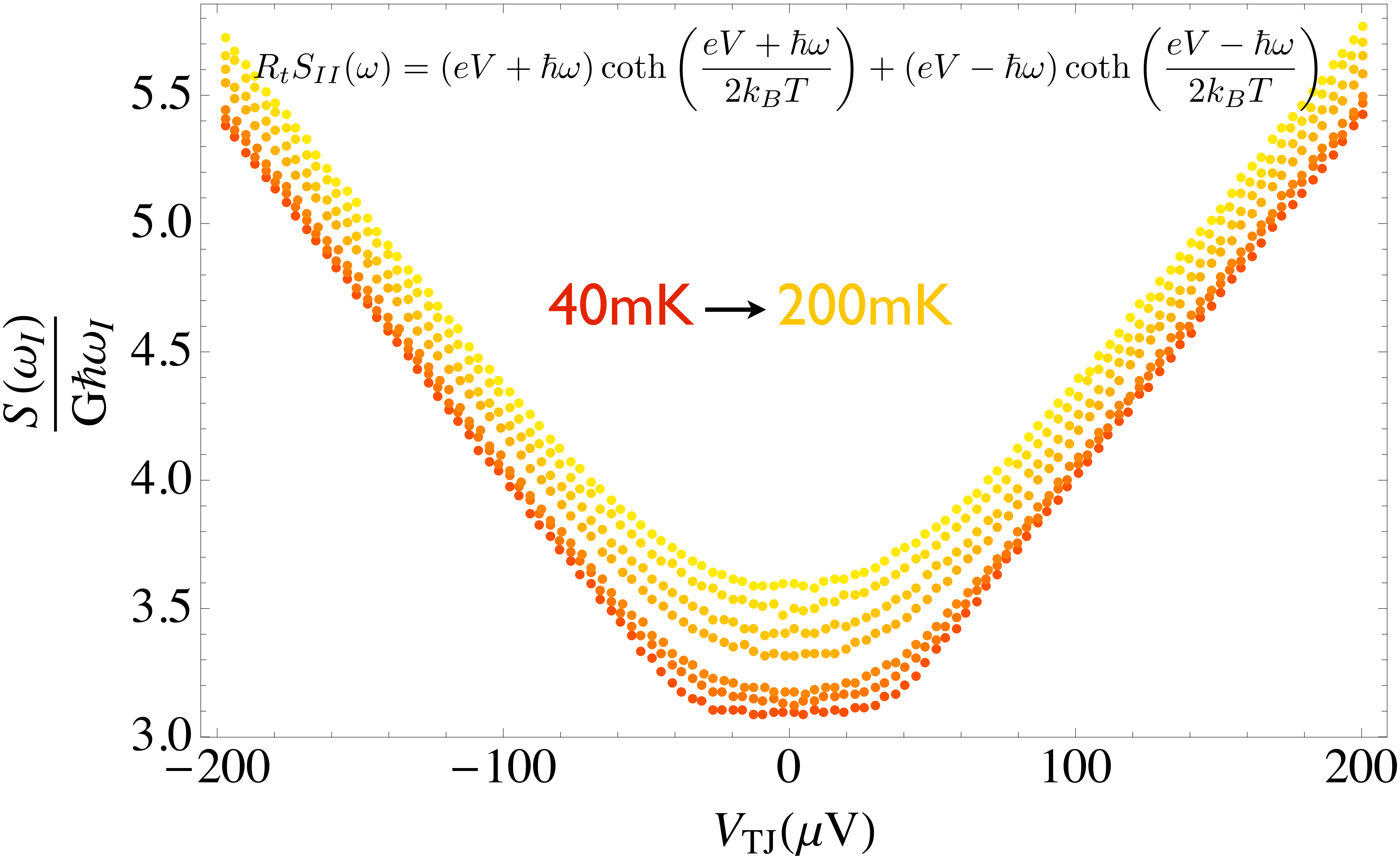




fit $D = 0.30, T^{TJ} = 40\ \text{mK}$

fit $D = 0.31, T^{TJ} = 40\ \text{mK}$

Noise as a function of temperature

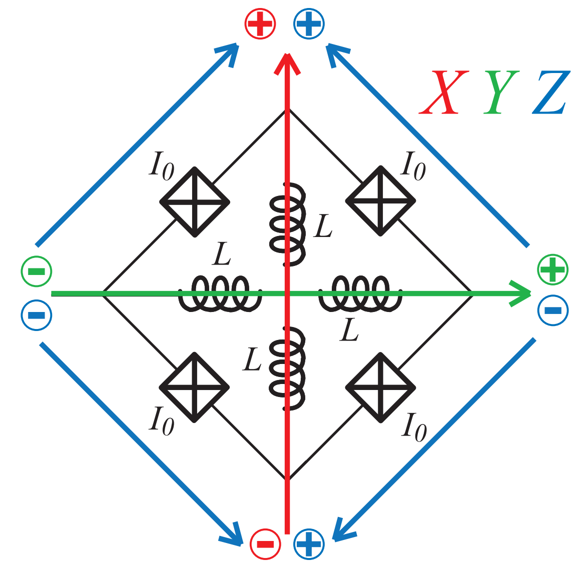


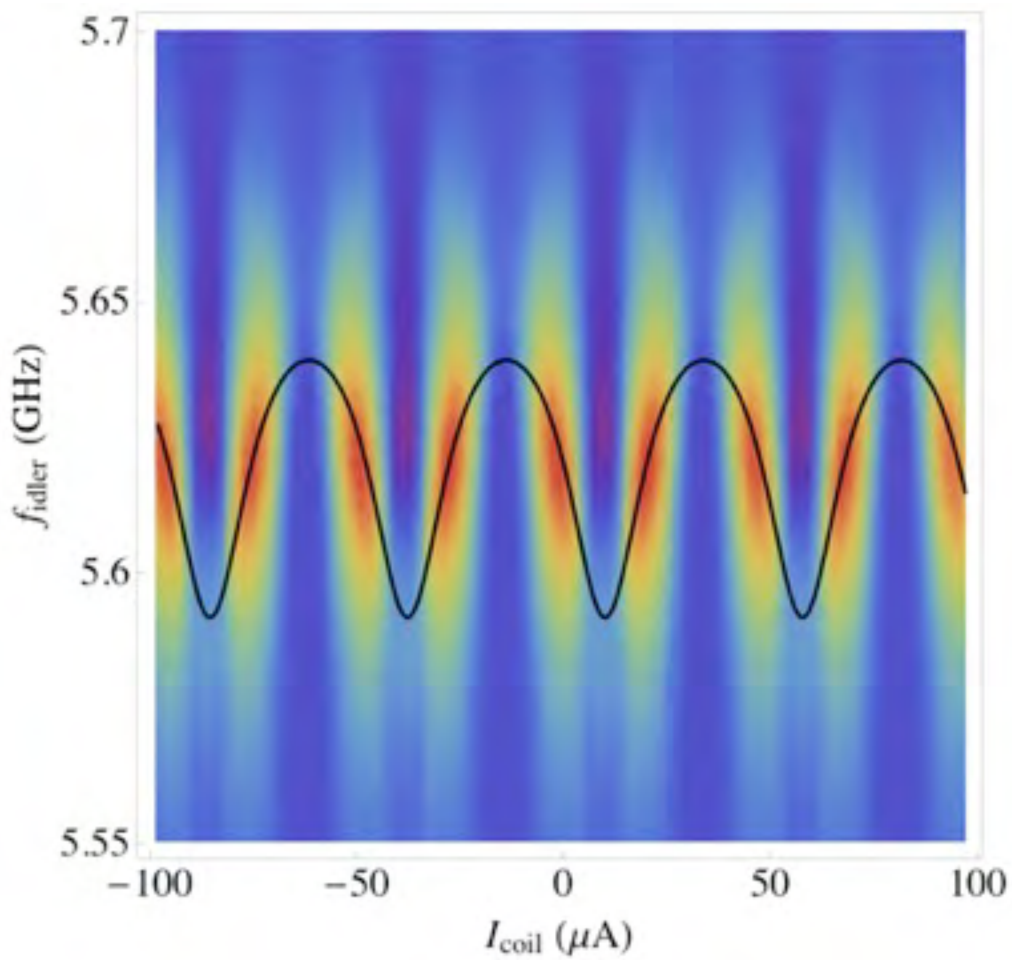
note: other junction and amplifier

Conclusions

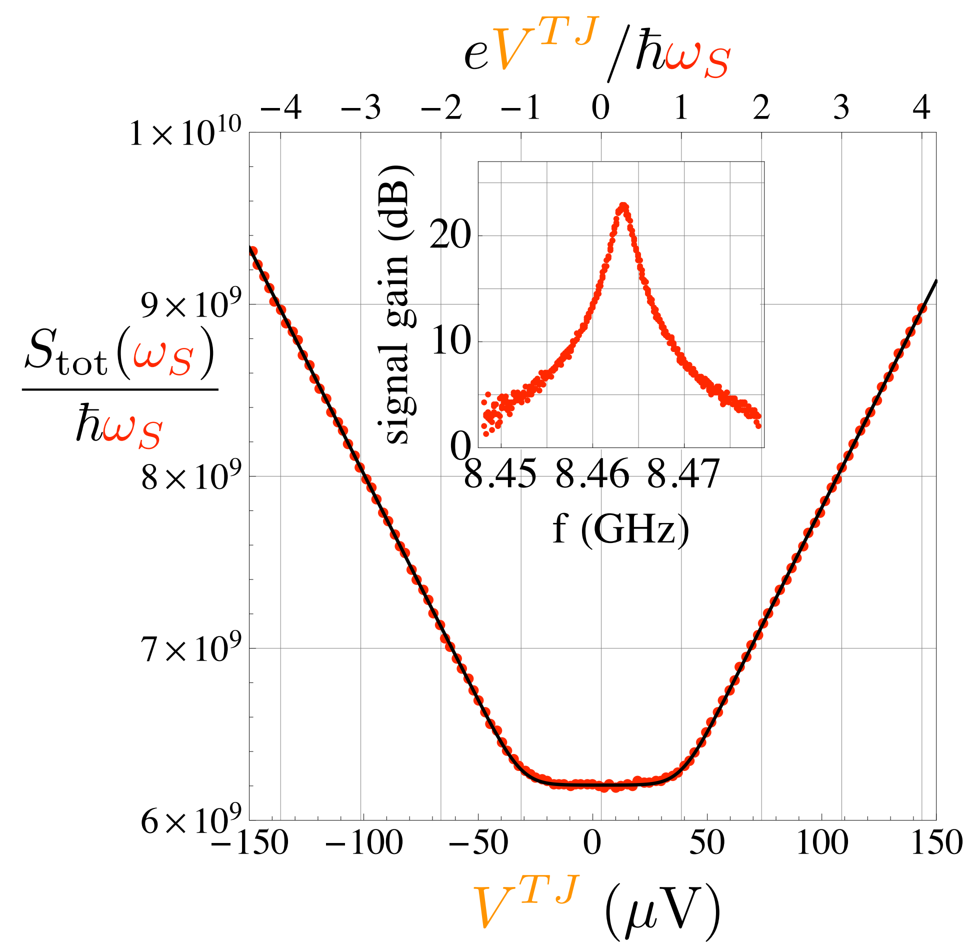
Ring of 4 Josephson junctions in cavity realizes a non-degenerate parametric amplifier for microwave photons

[Bergeal et al., Nat. Phys. (2010)]
[Bergeal et al., Nature (2010)]





Bandwidth tunability and stability achieved using additional inductances



Proper calibration of attenuation between noise source and amp needed to prove **quantum limit** is reached

Thanks !

Thanks !

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Michel Devoret

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David Darson

Former members

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Lola El Sahmarany (2010)
François Nguyen (2009)

Discussions

Devoret's group (Yale University, USA)
Hybrid Quantum Circuits group (LPA-ENS)
Mesoscopic physics group (LPA - ENS)
Quantronics Group (CEA Saclay, France)
Mazyar Mirrahimi (INRIA, Paris, France)
Cristiano Ciuti (Paris 7, France)
Theory group (LPA-ENS)
...

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Roland Lefevre (Observatoire de Paris)
Roger Gohier (INSP)