Quantum error correction and the future of solid state qubits

David DiVincenzo
RWTH Aachen and
Forschungszentrum Juelich

Paris, 6/2011

"In a machine such as this there are very many other problems due to imperfections. . . . At least some of these problems can be remedied in the usual way by techniques such as error correcting codes . . . But until we find a specific implementation for this computer, I do not know how to proceed to analyze these effects."

R.P. Feynman
"Quantum Mechanical Computers"
Optics News, February 1985

Outline

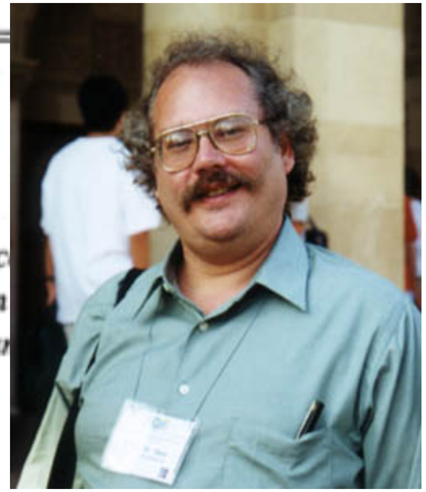
Basics of quantum error correction
- digitizing quantum errors
- a discrete group theory, finding good codes
- circuits for error correction, fault tolerance

Codes for integrated circuits
- surface code
- the basic device experiments
- new tools for fault tolerance:
  making and braiding holes

PHYSICAL REVIEW A

ATOMIC, MOLECULAR, AND OPTICAL PHYSICS

IES, VOLUME 52, NUMBER 4 OCTOBER 1995



RAPID COMMUNICATIONS

Communications section is intended for the accelerated publication of important new results. Sinc
are given priority treatment both in the editorial office and in production, authors should explain
justifies this special handling. A Rapid Communication should be no longer than 4 printed pages a
t. Page proofs are sent to authors.

Scheme for reducing decoherence in quantum computer memory

Peter W. Shor*

AT&T Bell Laboratories, Room 2D-149, 600 Mountain Avenue, Murray Hill, New Jersey 07974

(Received 17 May 1995)

Recently, it was realized that use of the properties of quantum mechanics might speed up certain computations dramatically. Interest has since been growing in the area of quantum computation. One of the main difficulties of quantum computation is that decoherence destroys the information in a superposition of states contained in a quantum computer, thus making long computations impossible. It is shown how to reduce the effects of decoherence for information stored in quantum memory, assuming that the decoherence process acts independently on each of the bits stored in memory. This involves the use of a quantum analog of error-correcting codes.

Two great things about this paper:

1) Made evident the fact (clarified by others) that quantum errors are discrete.

For a one-qubit system:

General continuous-time Bath-System quantum evolution

$$\boldsymbol{T}\int_0^t dt'\exp\left(\sum_{i=1} B_i(t')\otimes S_i(t')\right) =$$

$$B_I\otimes I + B_X\otimes X + B_Y\otimes Y + B_Z\otimes Z$$

X, Y, Z are Pauli matrices

If error correction procedure corrects for “bit flip” (X), “π-phase error” (Z), then it also corrects Y=iZX, and, by linearity, corrects the most general system-bath coupling.

Two great things about this paper:

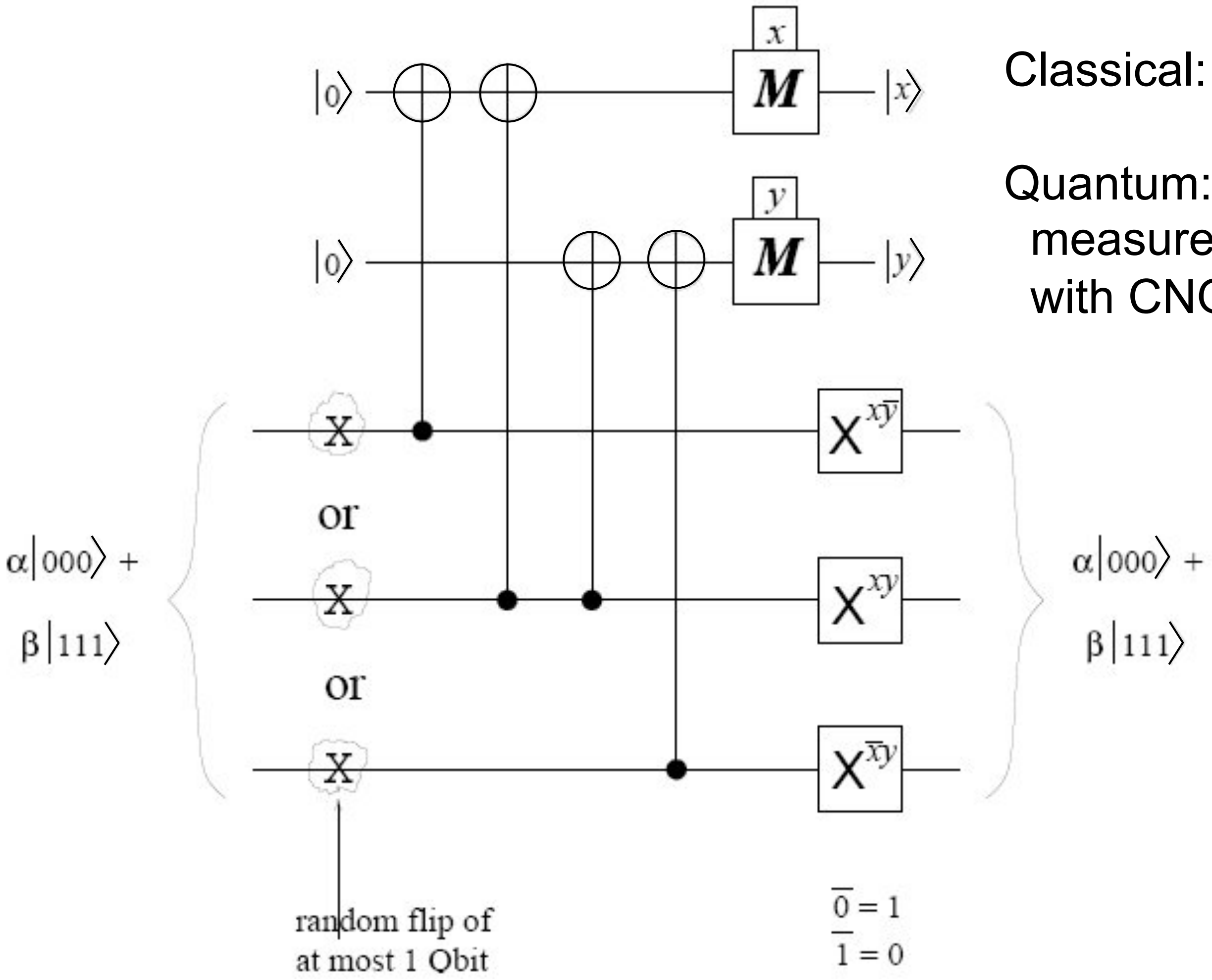
2) Found a code that corrects against single-qubit error.

$$|0\rangle \rightarrow \frac{1}{2\sqrt{2}}(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle),$$

$$|1\rangle \rightarrow \frac{1}{2\sqrt{2}}(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle). \tag{3.1}$$

Triple-repetition code inside itself.

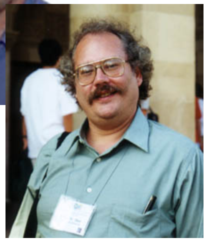
Quantum error correction implemented by quantum circuits



Classical: two parity checks

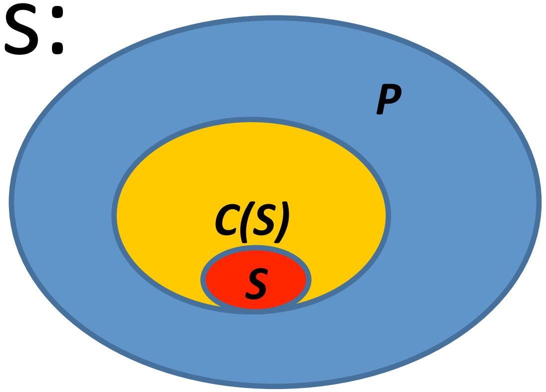
Quantum: two non-demolition measurements, accomplished with CNOT circuit

The three-bit repetition code

Calderbank-Shor [also Gottesman], 1997

A group theory for codes: stabilizers



Groups:

1) Pauli group P: all products of Pauli operators on a set of qubits, e.g., IXZXXYZ…
2) Stabilizer group S: abelian subgroup of P.
   States protected by code are +1 eigenstates of S
3) Centralizer C(S): operators in P that commute with all operators in S.
   (a non-abelian group).

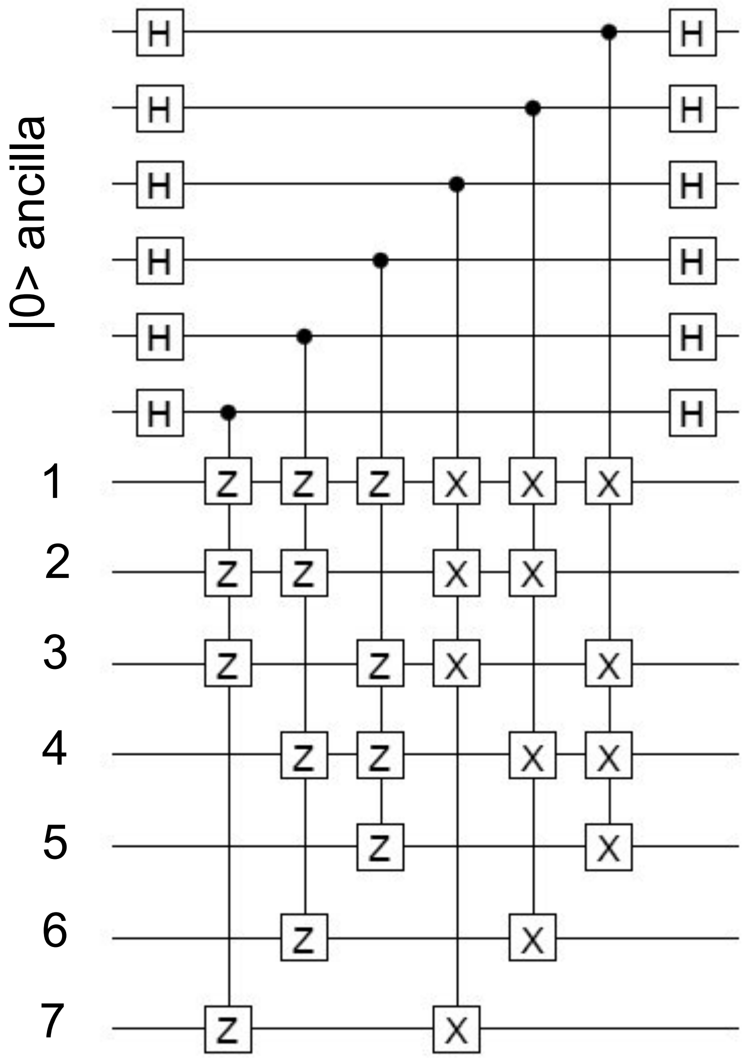
The theorem: errors can be detected as long as they are not in C(S)\S.

This assured that there would be a huge number of codes.

The early favorite: Steane 7-qubit code

Most efficient CSS code that corrects one general quantum error (X, Y, Z)

All gates are essentially CNOTs



Error correction: circuit does non-demolition measurement of operators

$$M(1)=Z_1 Z_2 Z_3 Z_7$$
$$M(2)=Z_1 Z_2 Z_4 Z_6$$
$$M(3)=Z_1 Z_3 Z_4 Z_5$$
$$M(4)=X_1 X_2 X_3 X_7$$
$$M(5)=X_1 X_2 X_4 X_6$$
$$M(6)=X_1 X_3 X_4 X_5$$

Distressingly difficult experiment!

Lots of qubits, lots of long-distance coupling (regularity is not geometric)

Analysis of fault tolerance:

Consider algorithm requiring N qubits and T time steps. Without error correction, the probability of failure for a run of the algorithm is estimated as

$$TNp$$

Not small unless $p<10^{-15}$ for runs of interest. Consider a code which will correct one error, so that $p_{eff}= Cp^2$. *C* is a couting factor near 10,000. Now the probability of failure is

$$TNCp^2$$

Slightly improved for small *p,* but not good enough. But there are many codes, Including ones that correct *x* errors. We can choose *x* with a knowledge of *N*. So the failure probability becomes

$$T(N)NC[x(N)]\, p^{\,x(N)+1} = poly(N)C[x(N)]\, p^{\,x(N)+1}$$

So long as *C[x]* doesn't grow too fast with *x*, then *x* can be chosen such that for some finite *p,* this expression can always be made <<1.

However:

However:

For many families of codes the counting factor grows incredibly fast with x:

$$C[x] \approx x^{cx}$$

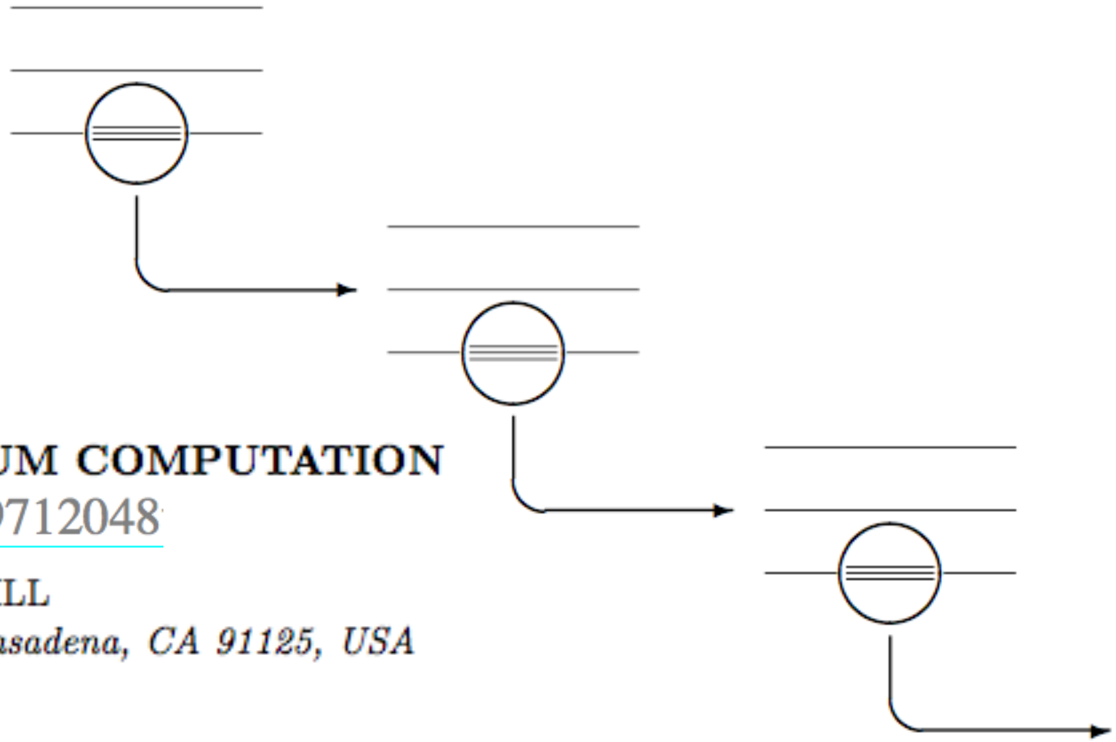
One solution: for special sequences of codes, those produced by concatenation,
The scaling is better:

$$C[x] \approx c^{x};$$

$$p_{th} \approx 1/c.$$

For various codes, this gave $p_{th} \approx 10^{-4}$ *or* 10^{-5}.

Concatenation:
regular, but produces
very non-local circuits



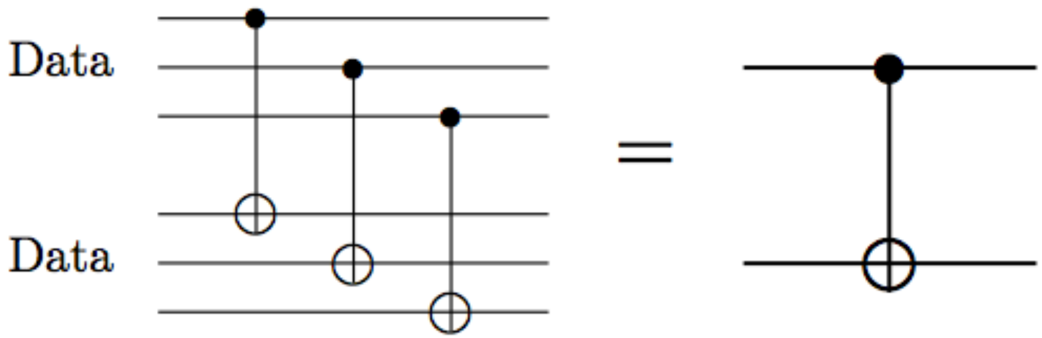
FAULT-TOLERANT QUANTUM COMPUTATION
arXiv:quant-ph/9712048

JOHN PRESKILL
California Institute of Technology, Pasadena, CA 91125, USA

Figure 14: Concatenated coding. Each qubit in the block, when inspected at higher resolution, is itself an encoded subblock.

-gates of coded qubits: transversal construction
(also geometrically highly nonlocal)

Even more to do to get
universal computation
-magic states
-teleported gates



Other development of 1996-7:

Quantum error correction with imperfect gates

A. Yu. Kitaev

L.D.Landau Institute for Theoretical Physics,
117940, Kosygina St. 2
e-mail: kitaev@itp.ac.ru

September 25, 1996

Abstract

Quantum error correction can be performed fault-tolerantly. This allow a quantum state intact (with arbitrary small error probability) for arbit time at a constant decoherence rate.

In *Quantum Communication, Computing, and Measurement*, O. Hirota *et al.*, Eds. (Plenum, New York, 1997).

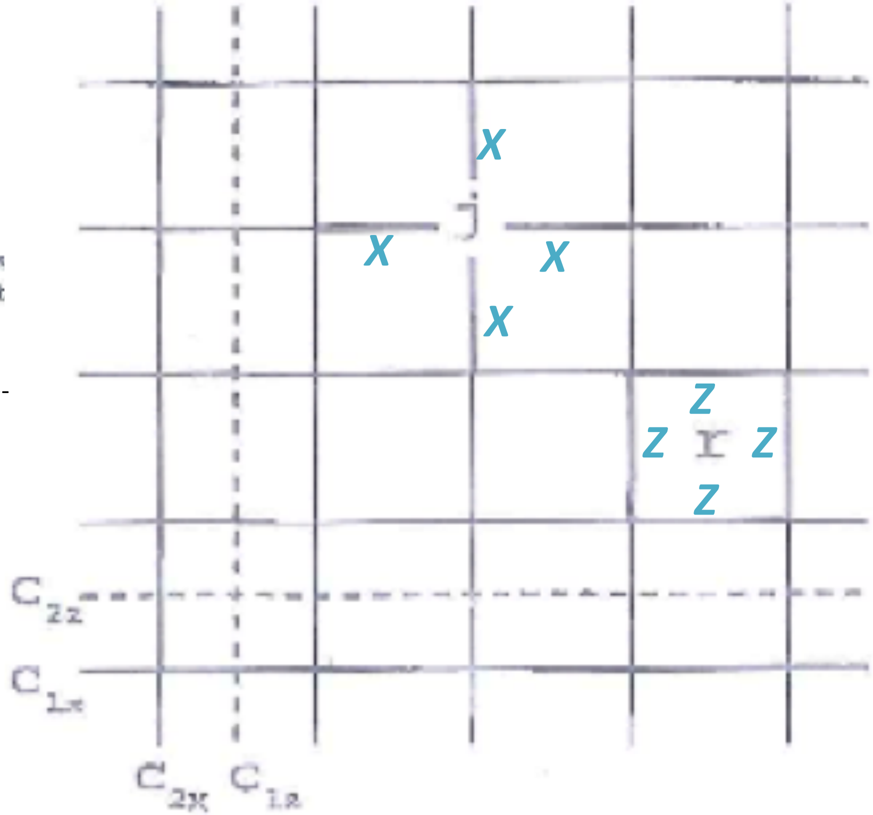


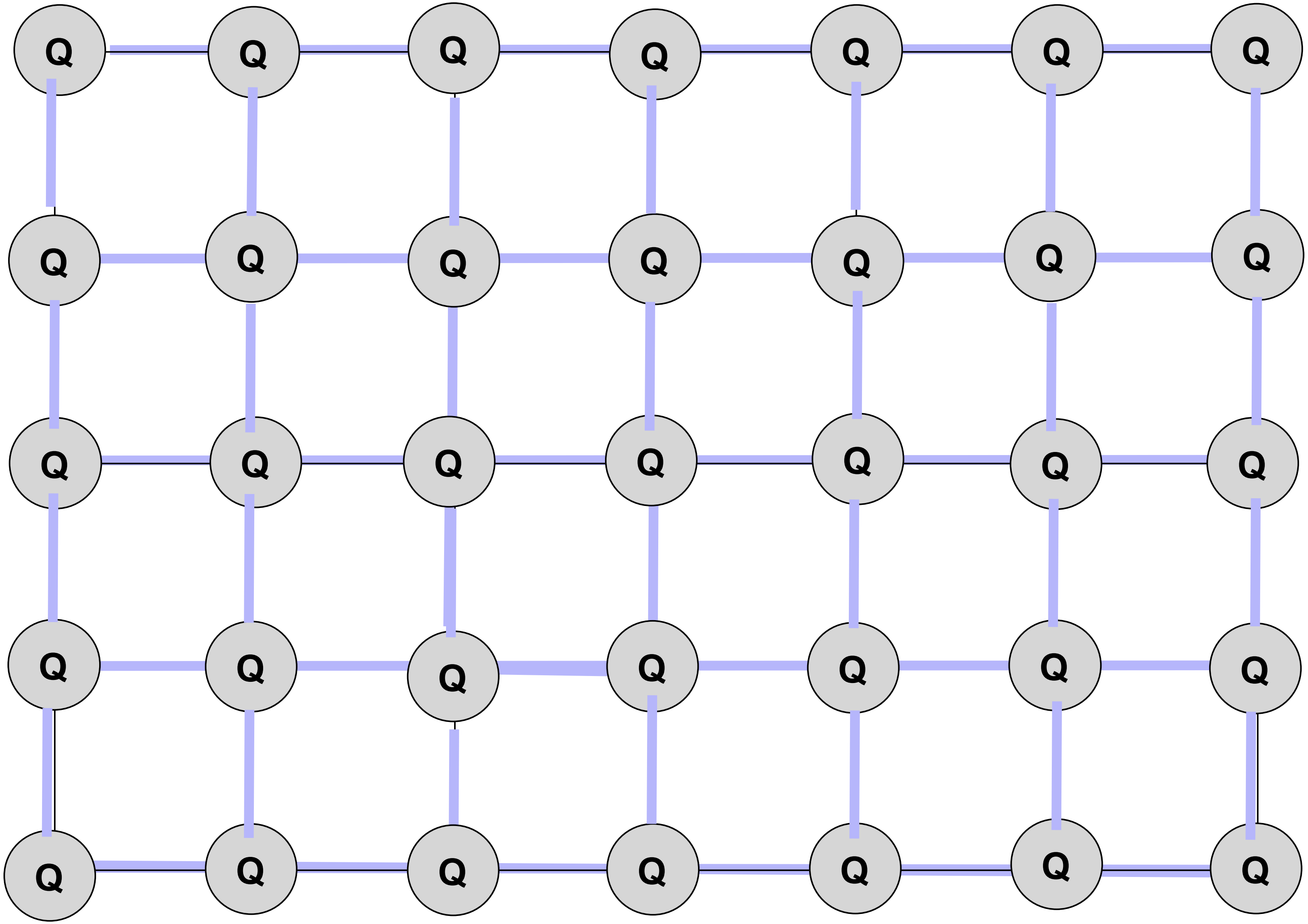
Figure 1: The toric code TOR(5).

Stabilizer generators XXXX, ZZZZ;

Stars and plaquettes of interesting 2D lattice Hamiltonian model

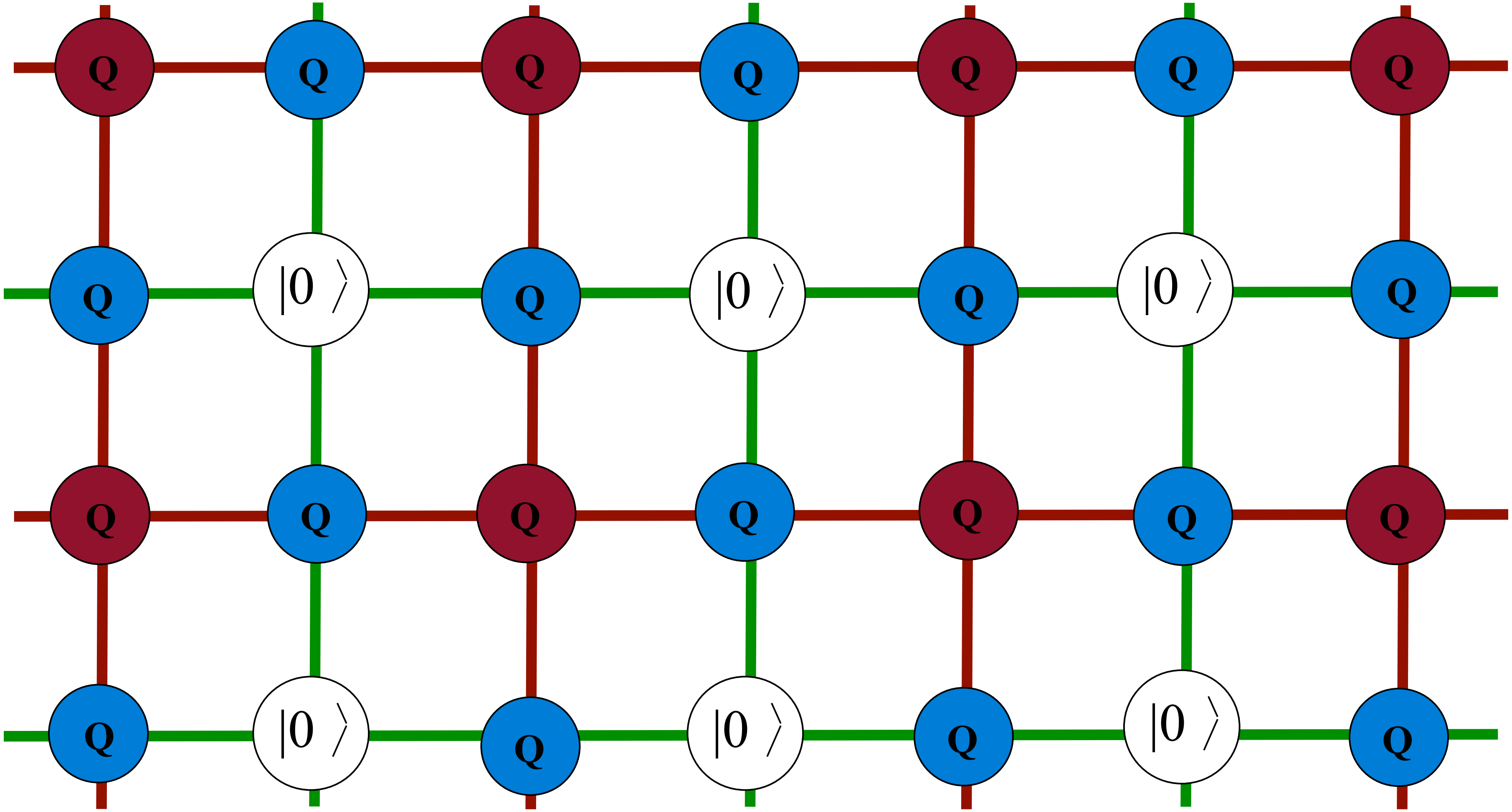
Toric Code/Surface Code

Surface code error correction: qubits (abstract) in fixed 2D square arrangement ("sea of qubits"), only nearest-neighbor coupling are possible



Colorized thanks to
Jay Gambetta and John Smolin

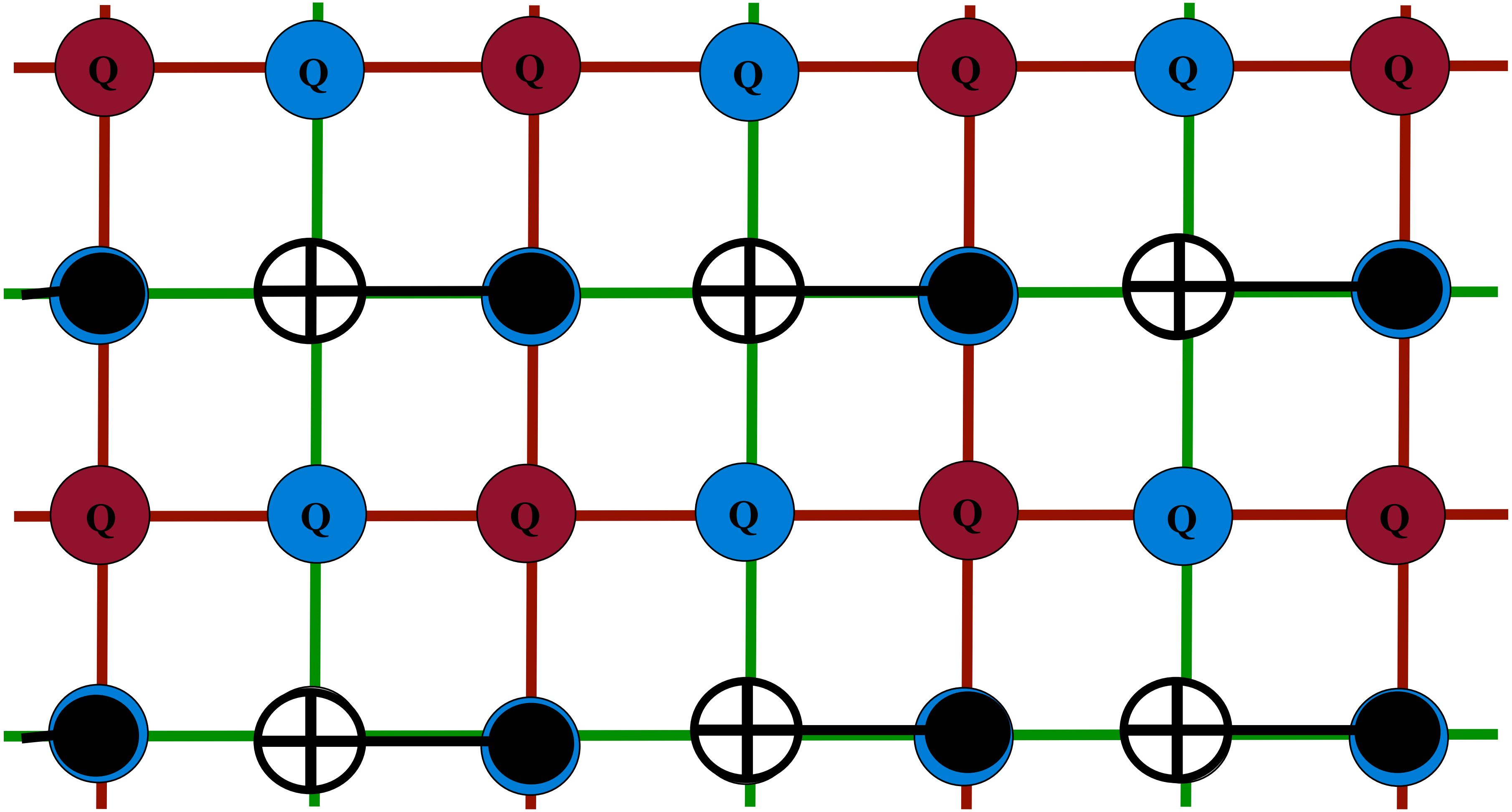
Surface code



Initialize Z syndrome qubits to $|0\rangle$

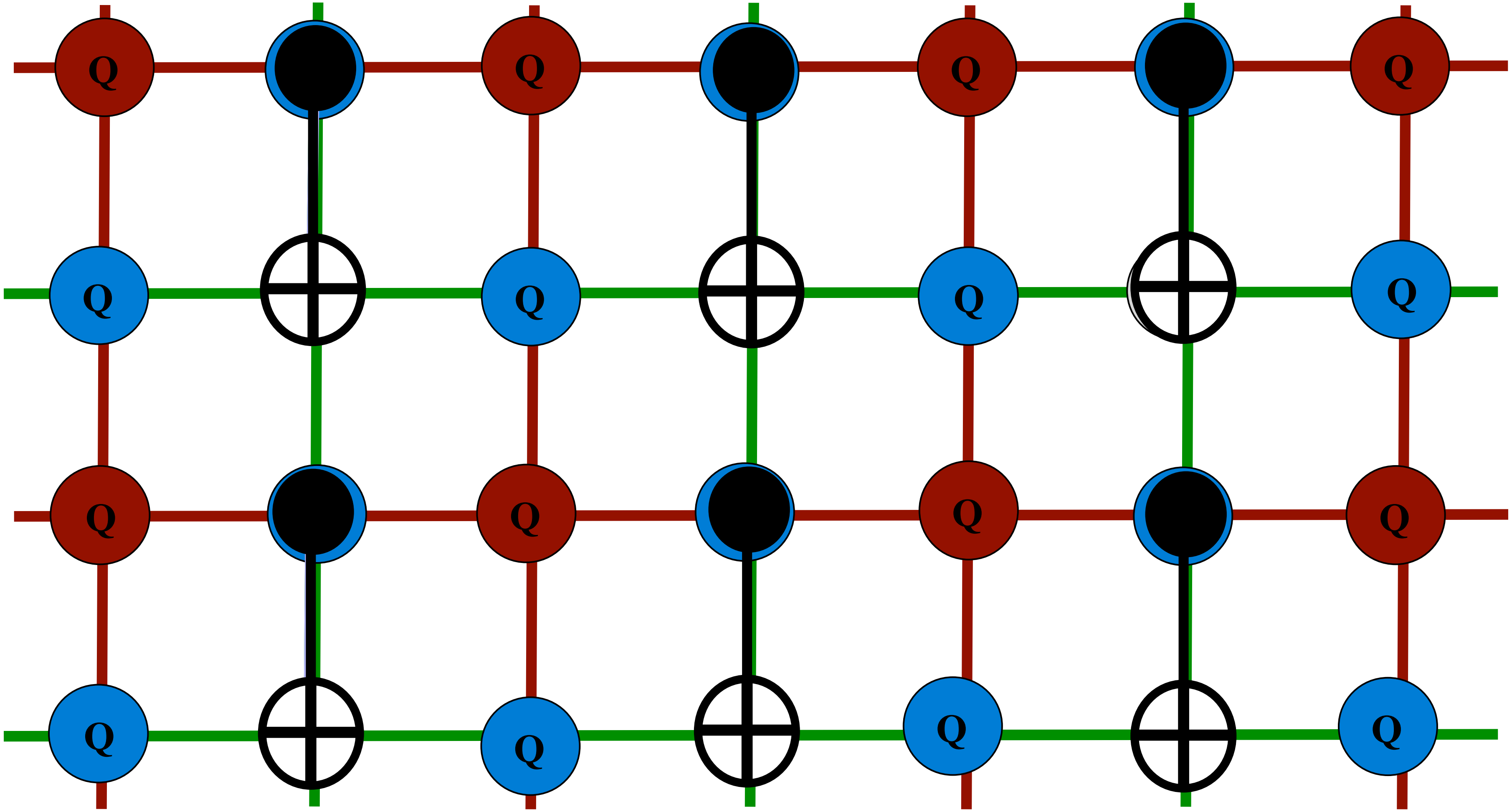
Implementing the "surface code":
-- in any given patch, independent of the quantum algorithm to be done:

Surface code



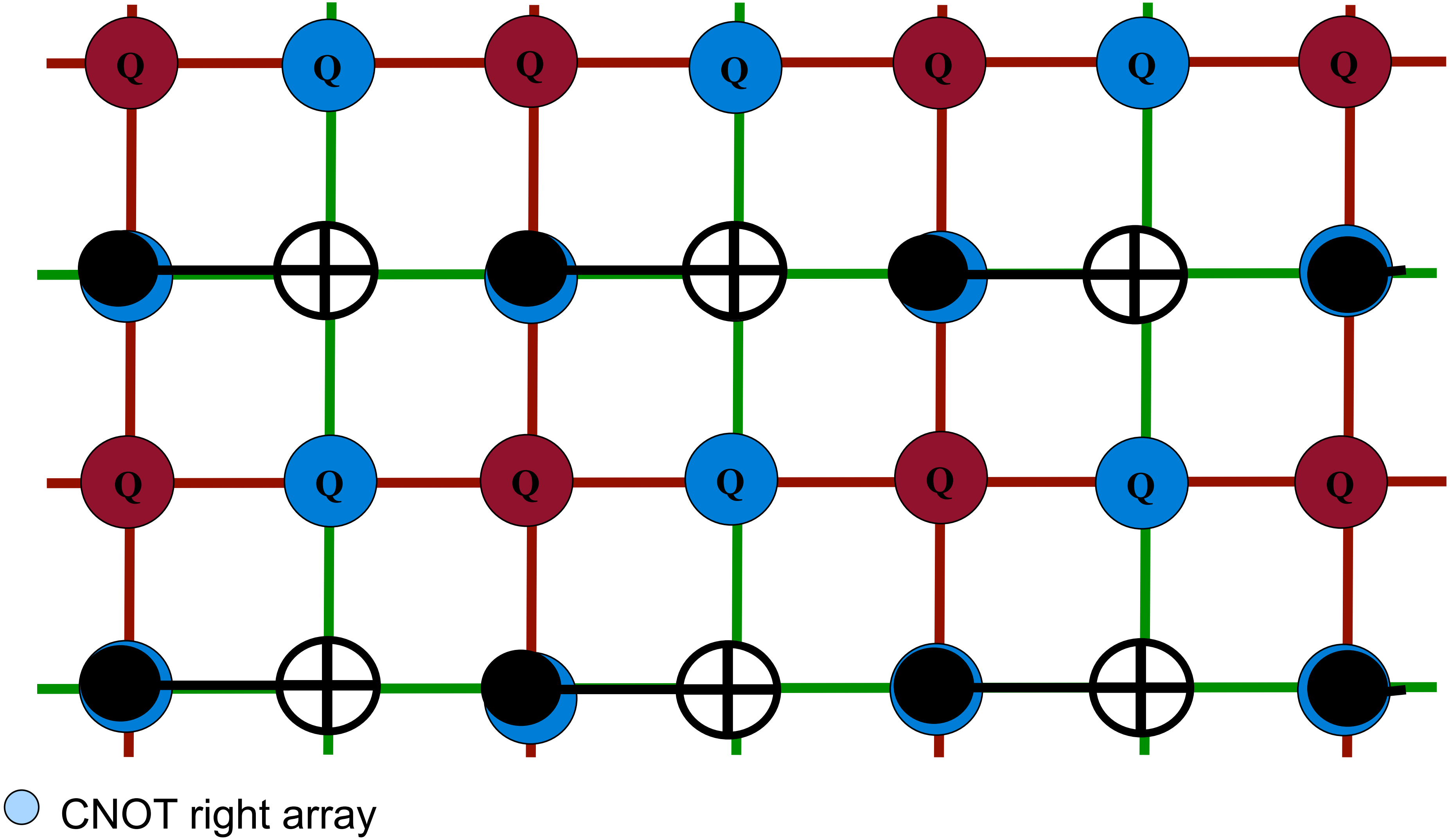
CNOT left array

Surface code

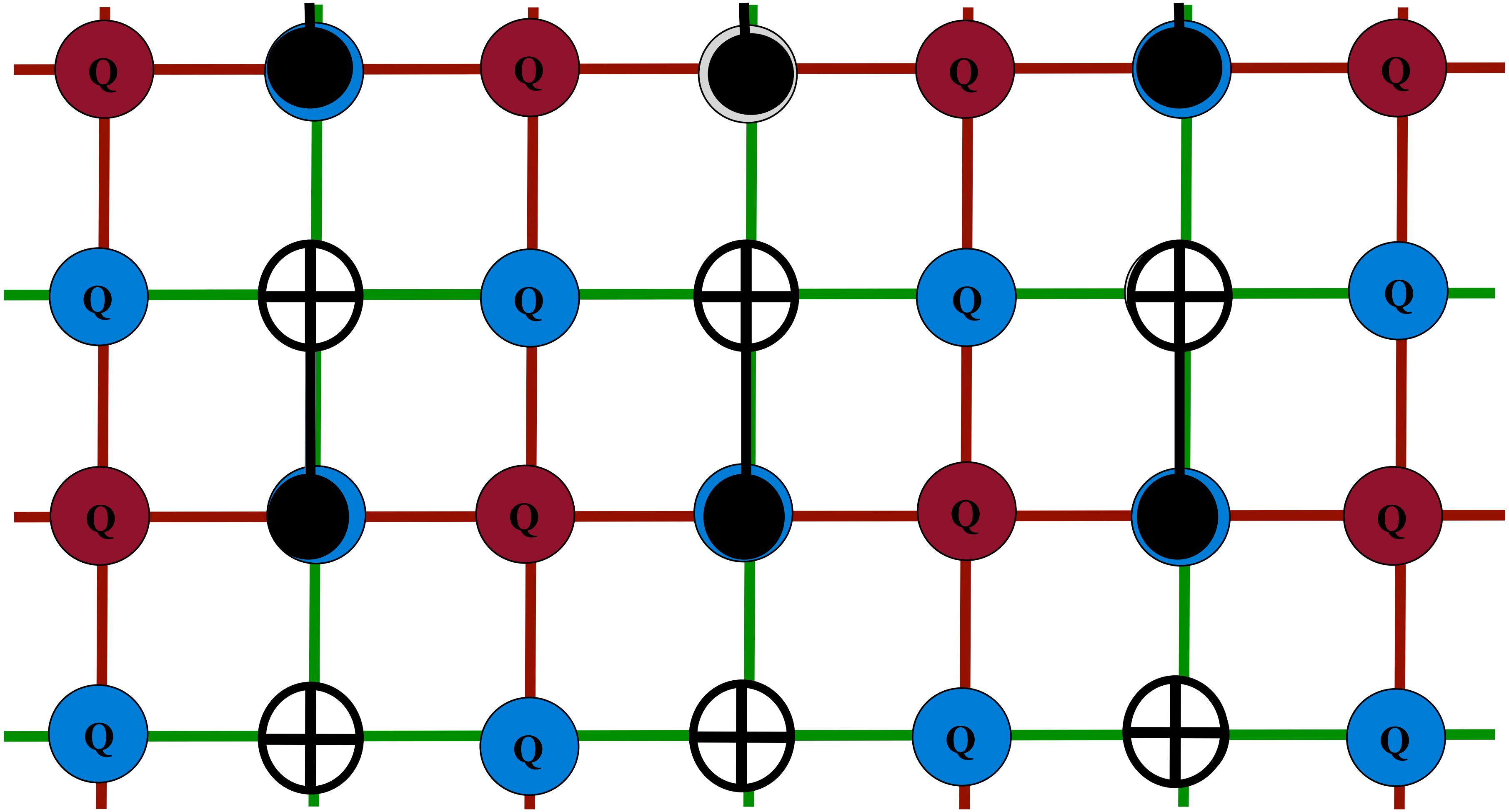


CNOT down array

Surface code

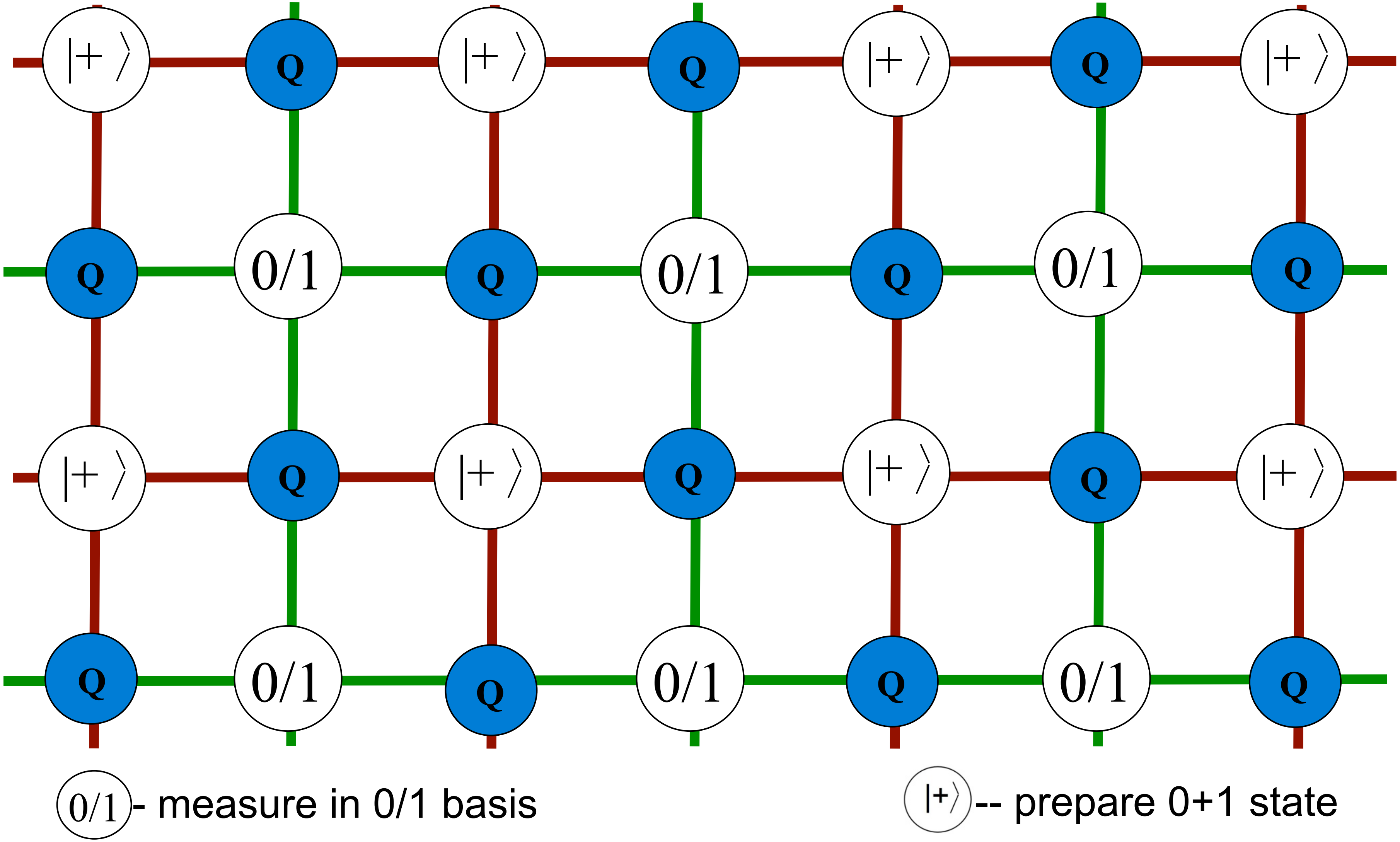


Surface code fabric

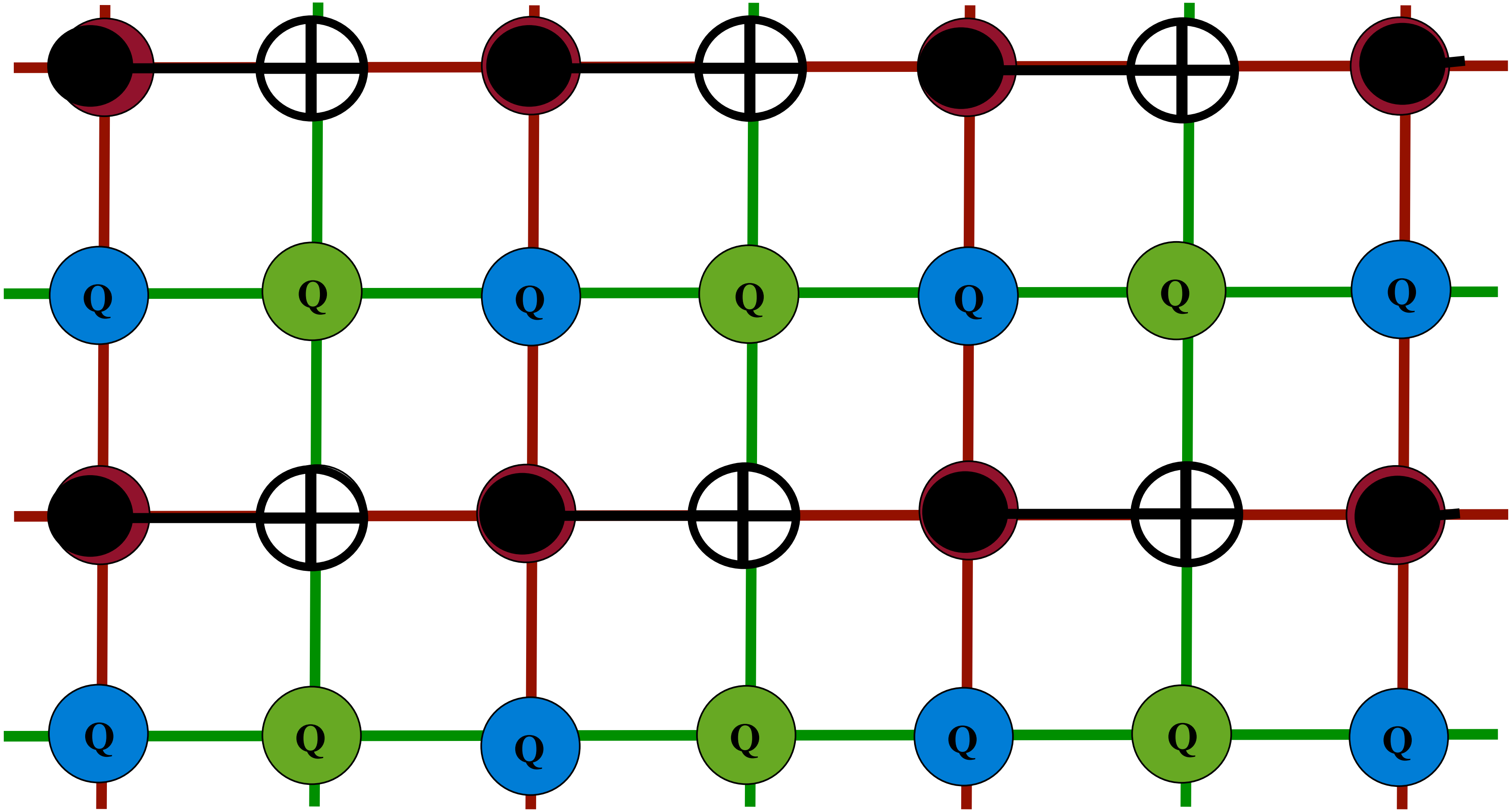


CNOT down array

Surface code fabric

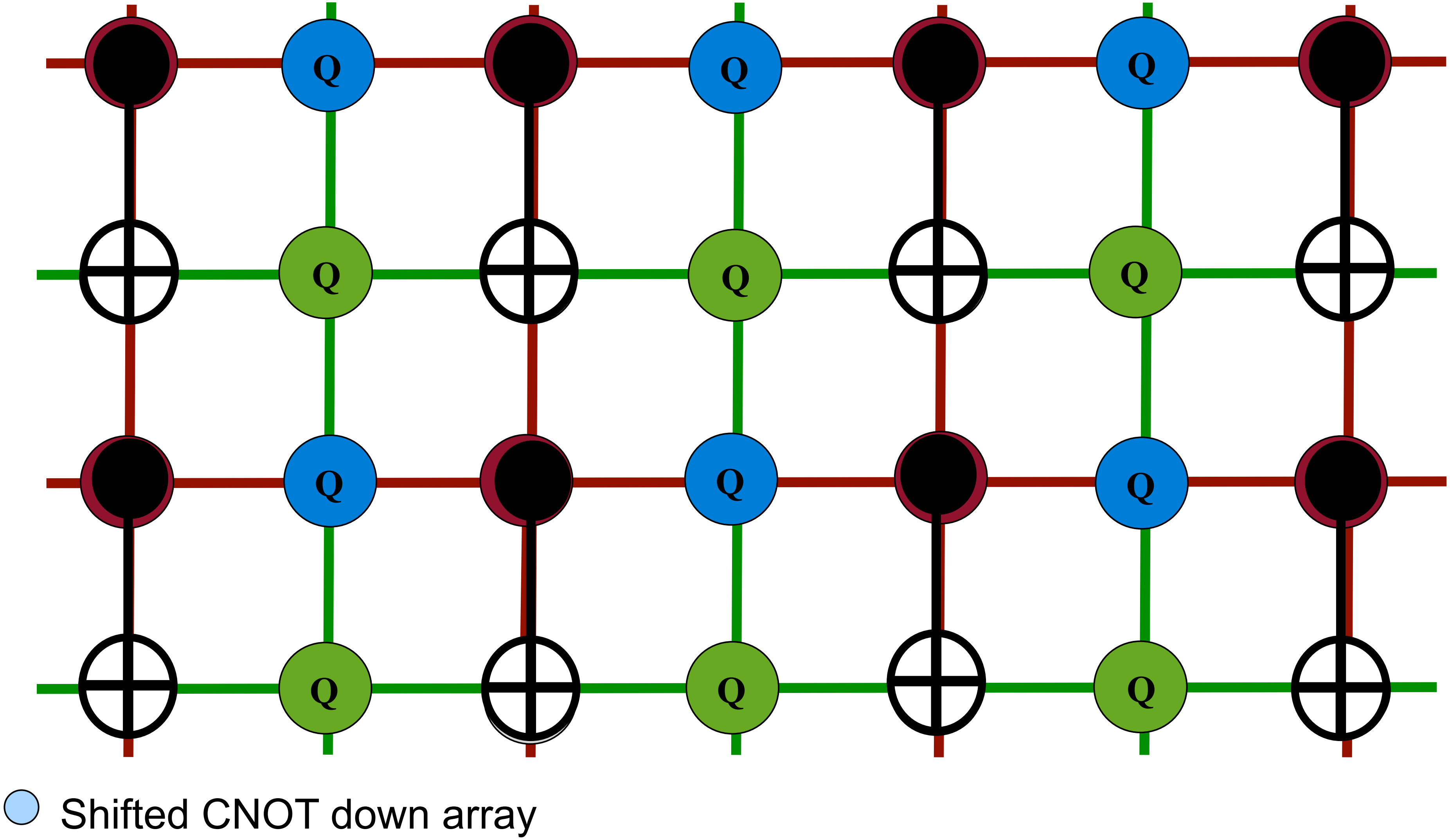


Surface code

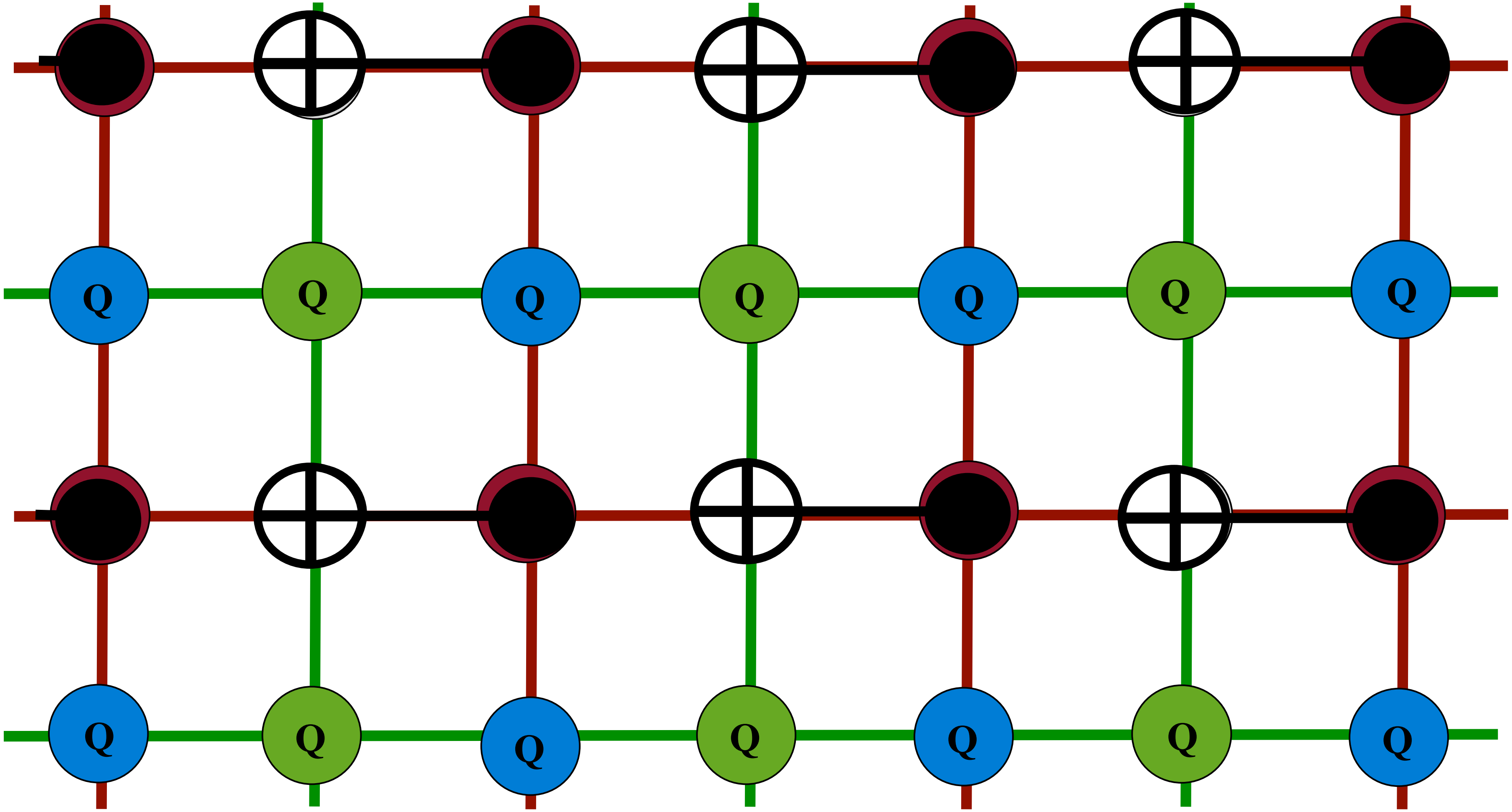


Shifted CNOT right array

Surface code

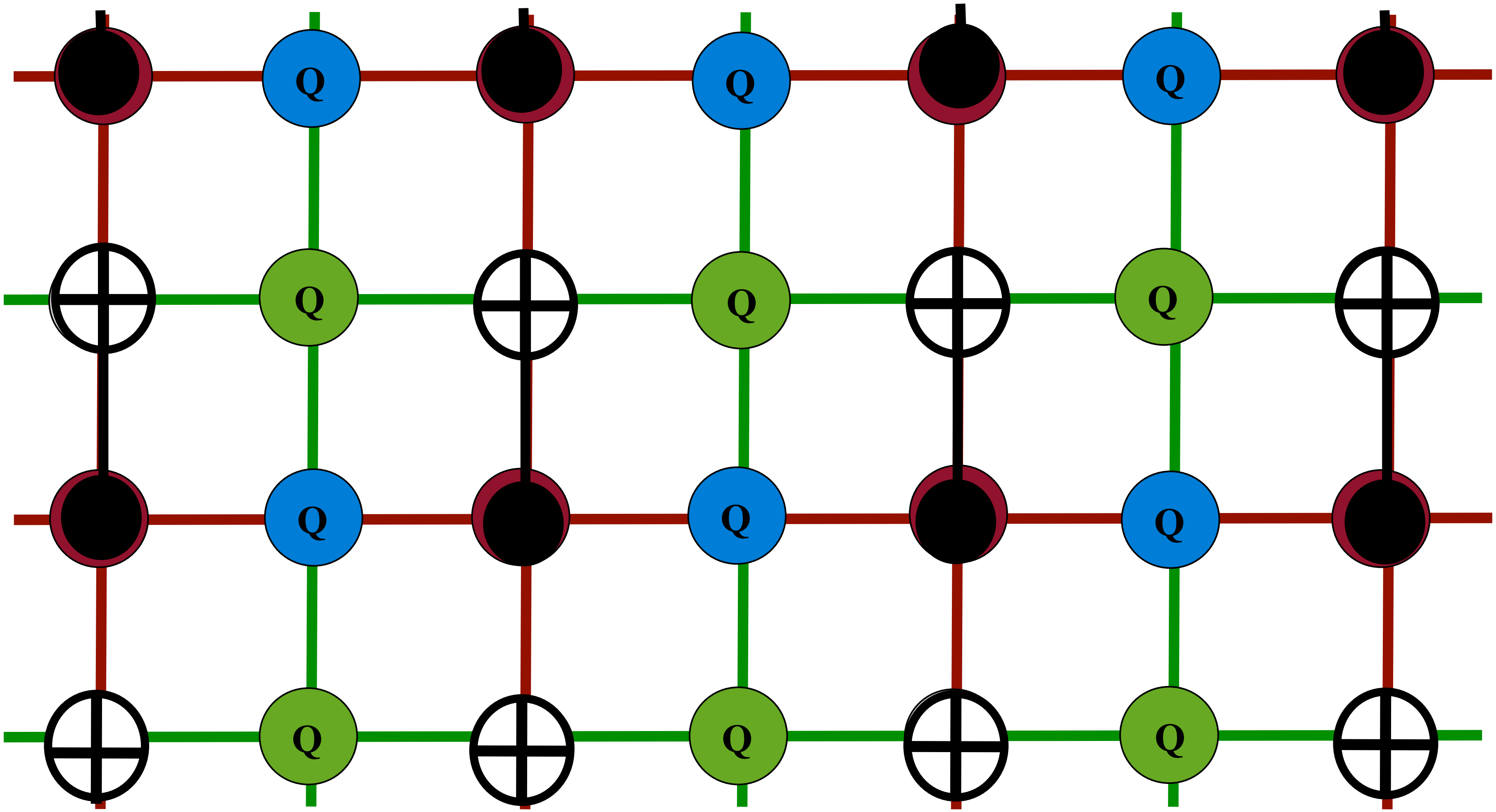


Surface code



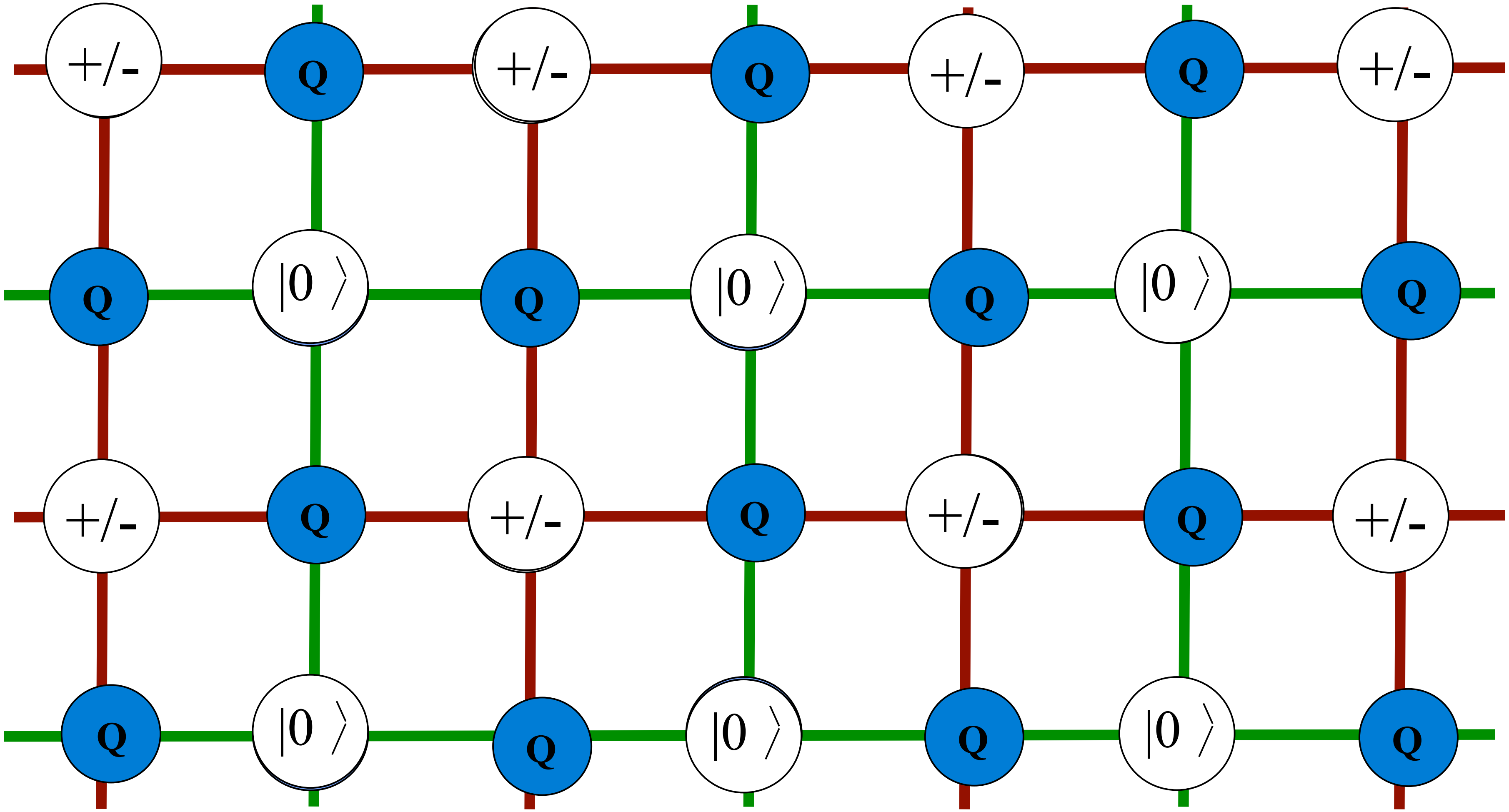
Shifted CNOT left array

Surface code



Shifted CNOT up array

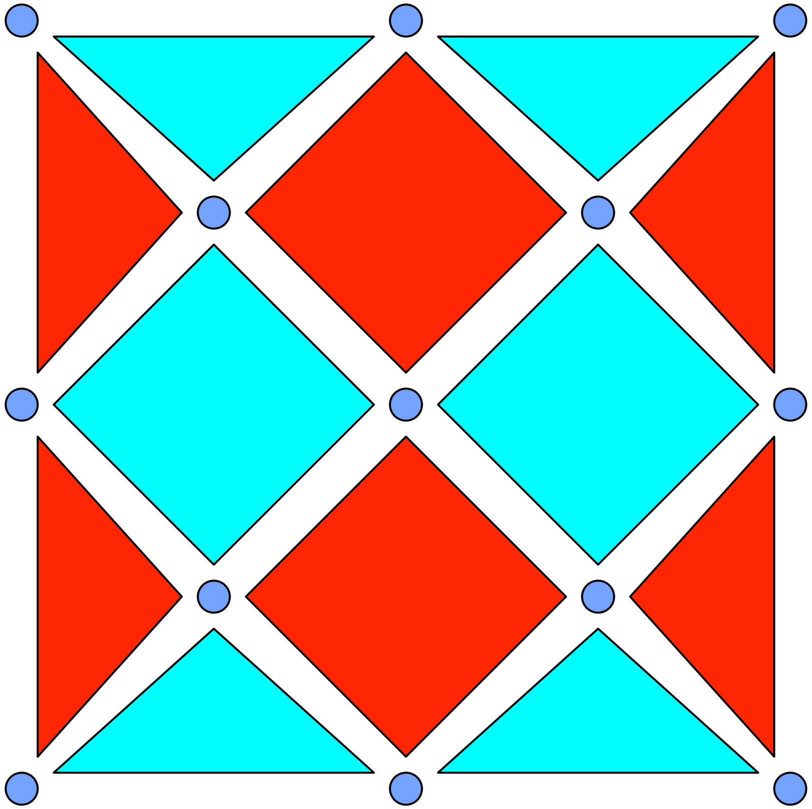
Surface code fabric



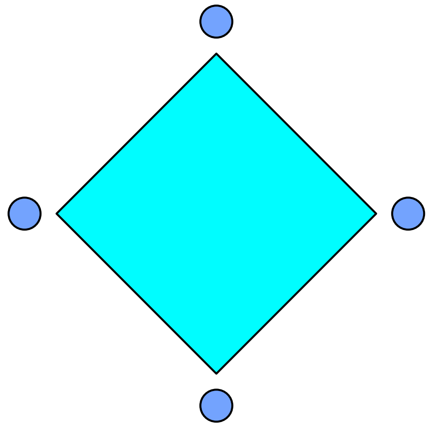
Repeat over and over....

Another view of the 2D Surface Code

With these 13 qubits, one gets a standard code that will correct for one error:



4-qubit QND parity measurement:



Red diamond: the same in the conjugate basis

S. Bravyi and A. Yu. Kitaev, "Quantum codes on a lattice with boundary," Quantum Computers and Computing **2**, 43-48 (2001).
M. H. Freedman and D. A. Meyer, "Projective plane and planar quantum codes," Found. Comp. Math. **1**, 325 (2001)

Observations:

Calculated fault tolerant threshold:

$p \approx 0.7\%$

Now $p > 1$ %, according to Wang, Fowler, Hollenberg, Phys. Rev. A 83, 020302(R) (2011)

Crosstalk assumed "very small", not analyzed

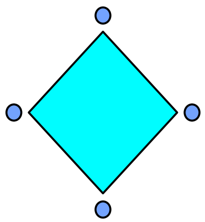
Residual errors decrease exponentially with lattice size

Gates: CNOT only (can be CPHASE), no one qubit gates

If measurements slow: more ancilla qubits needed, no threshold penalty

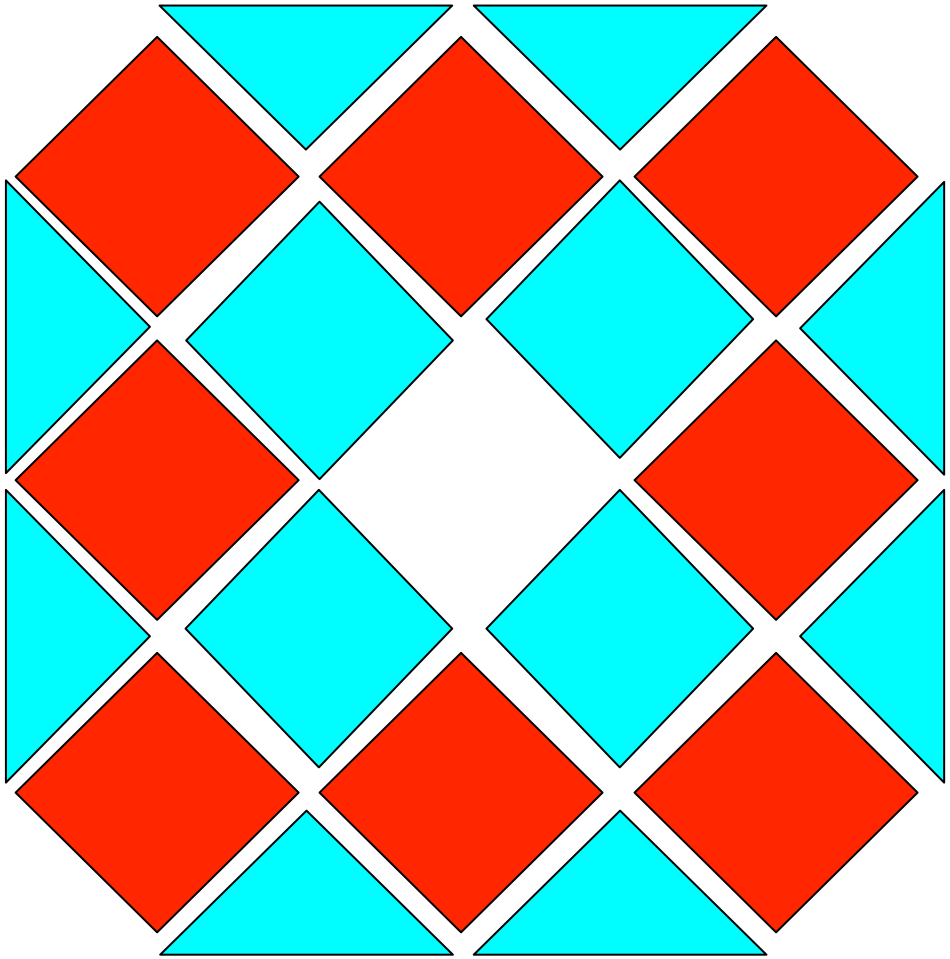
NB: Error threshold for 4-qubit Parity QND measurement is around

$2\% < p < 12\%$



How to compute with the surface code:

24-qubit structure



8 ZZZZ checks
12 XXXX checks

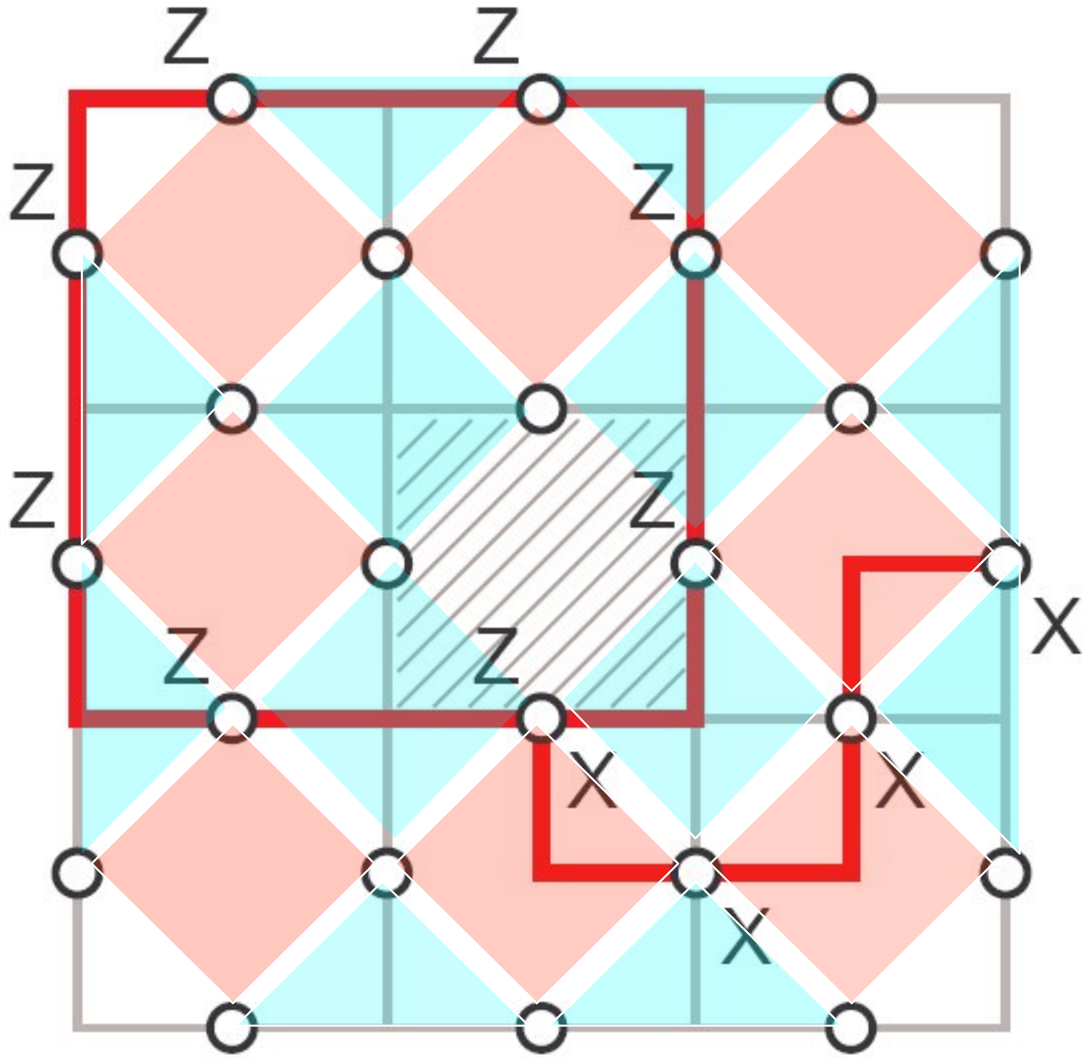
To get a qubit:

Stop measuring one Plaquette

(Freedman and Meyers, 1998)

Austin G. Fowler, Ashley M. Stephens, and Peter Groszkowski, "High threshold universal quantum computation on the surface code," Phys. Rev. A 80, 052312 (2009).

Logical gates (X and Z) on the "hole" qubit



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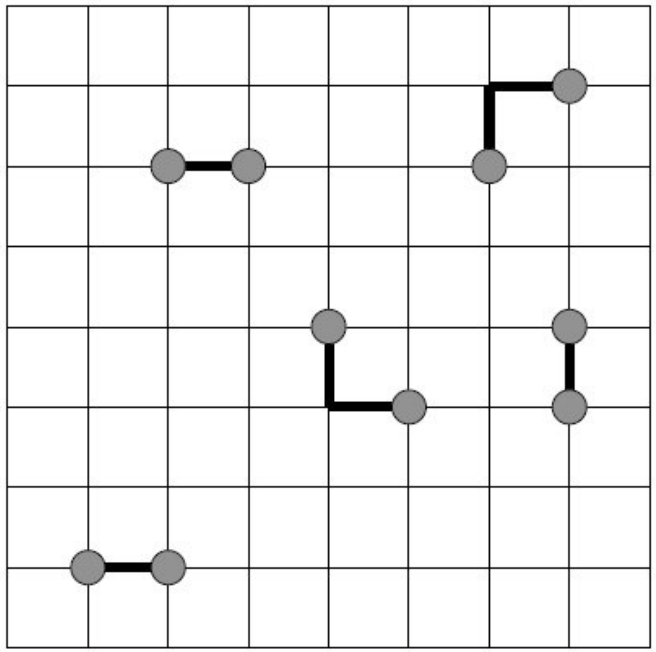
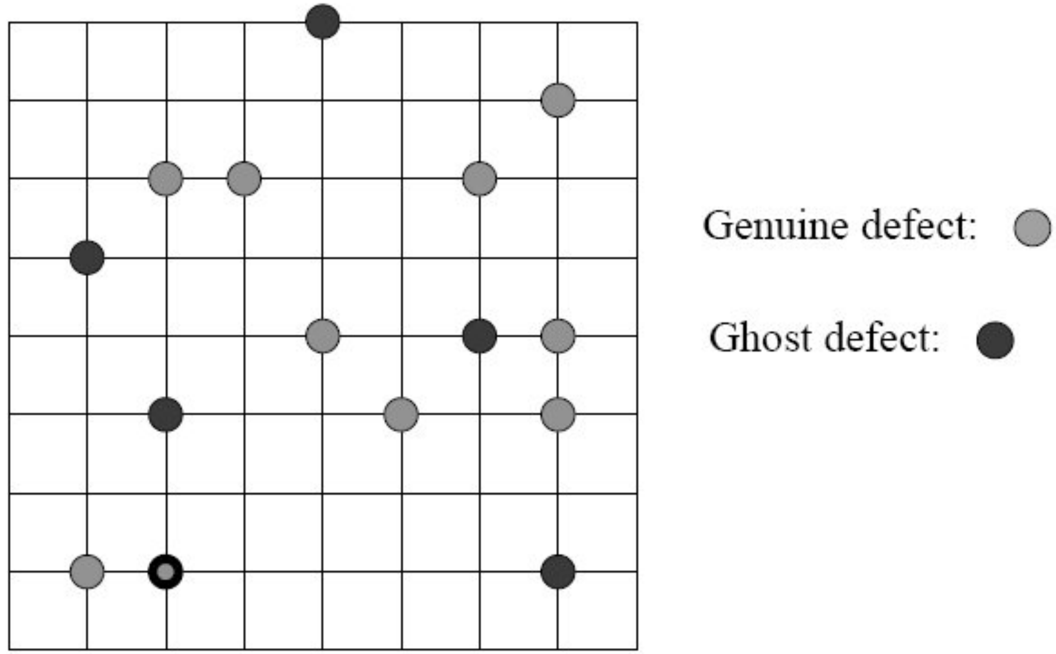
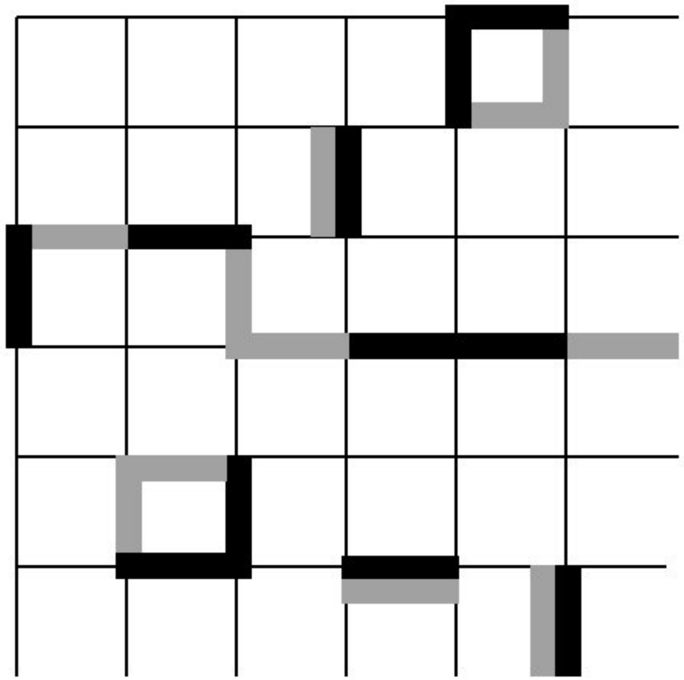




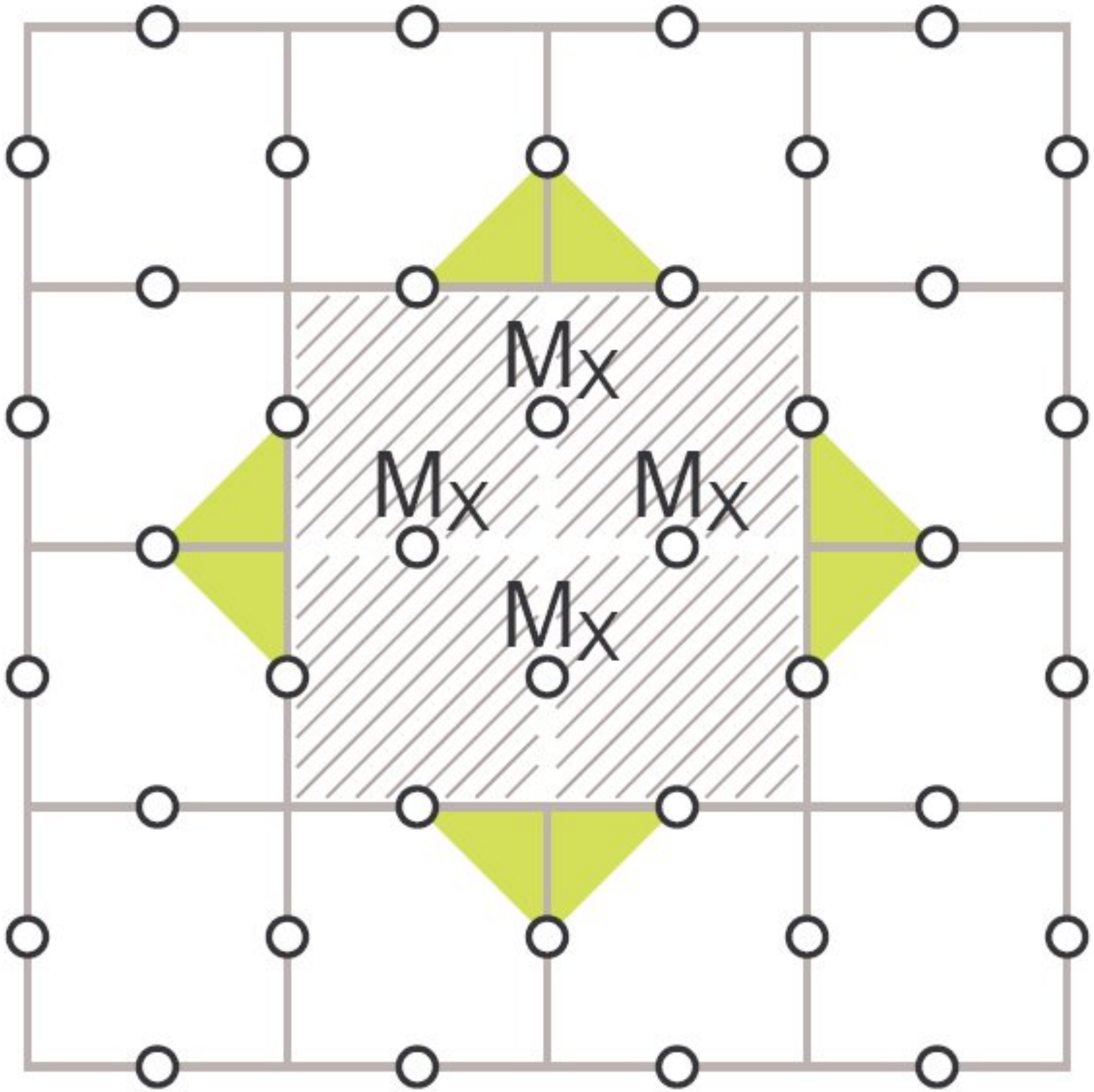


E. Dennis, A. Kitaev, A. Landahl, and J. Preskill,
"Topological quantum memory,"
J. Math. Phys. **43**, 4452–4505 (2002).

No ghost defects: 2D random bond Ising,
FM/PM transition at 10% error rate

With ghosts: 3D random plaquette model,
Transition at 1% error rate

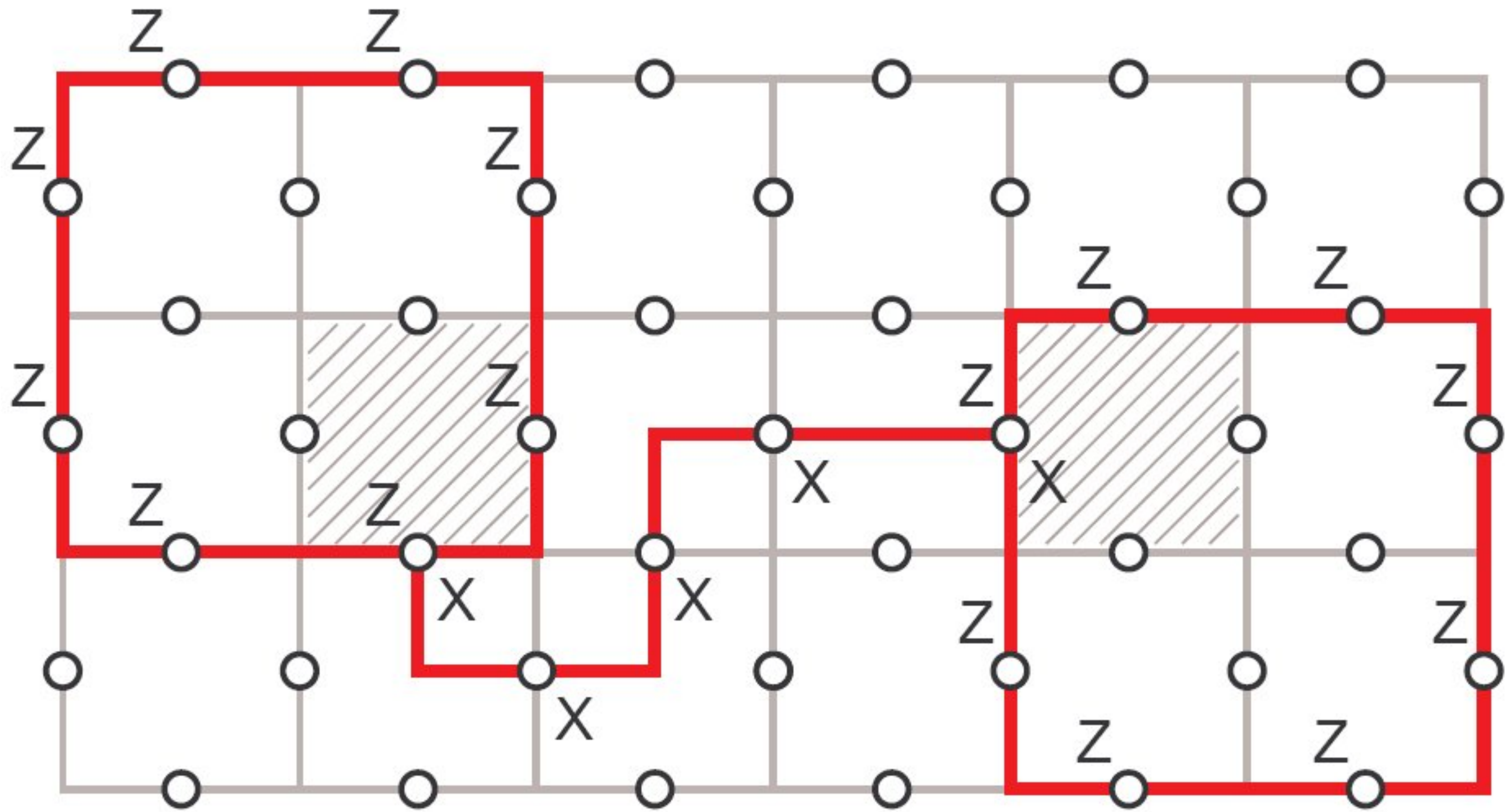
Experimentalists can skip the next 6 slides



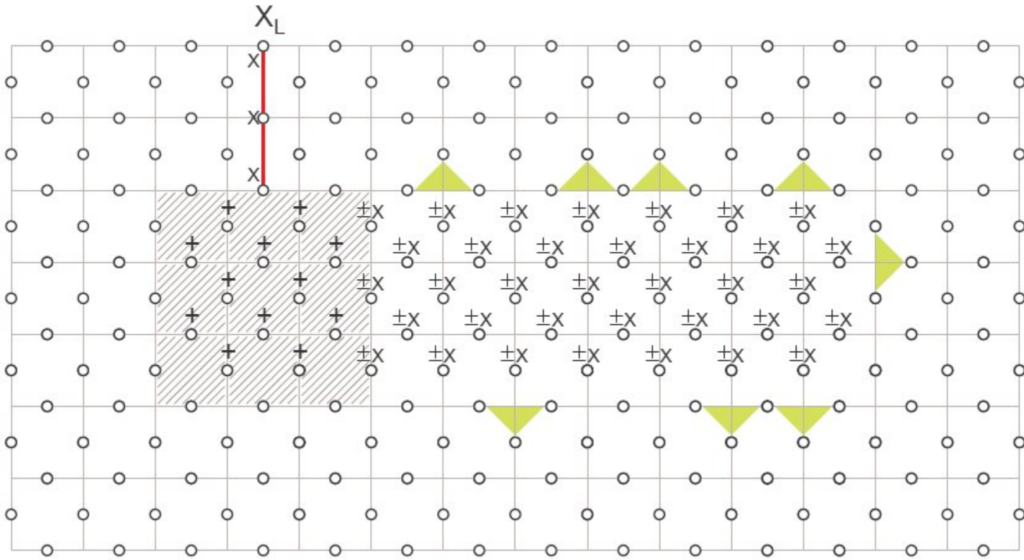
Larger hole is also OK.

Has greater error-correcting capability

Convenient to associate a qubit with a *pair* of holes.



a.)

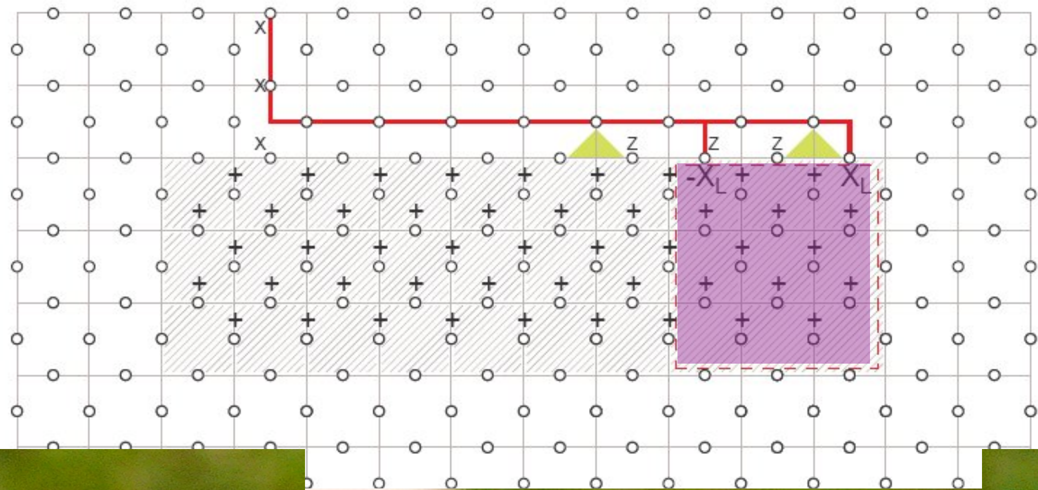


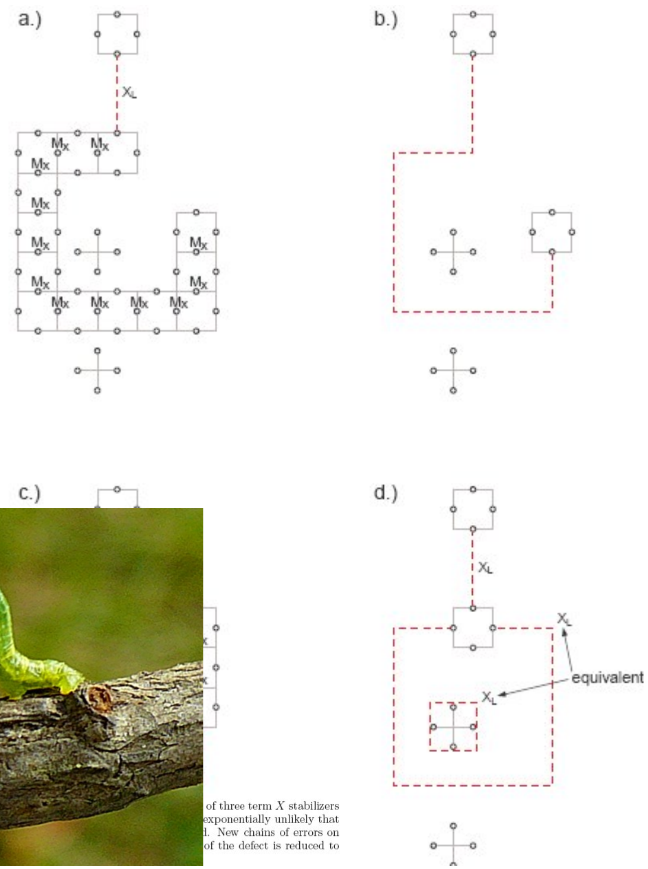
Holes movable by enlargement and reshrinkage

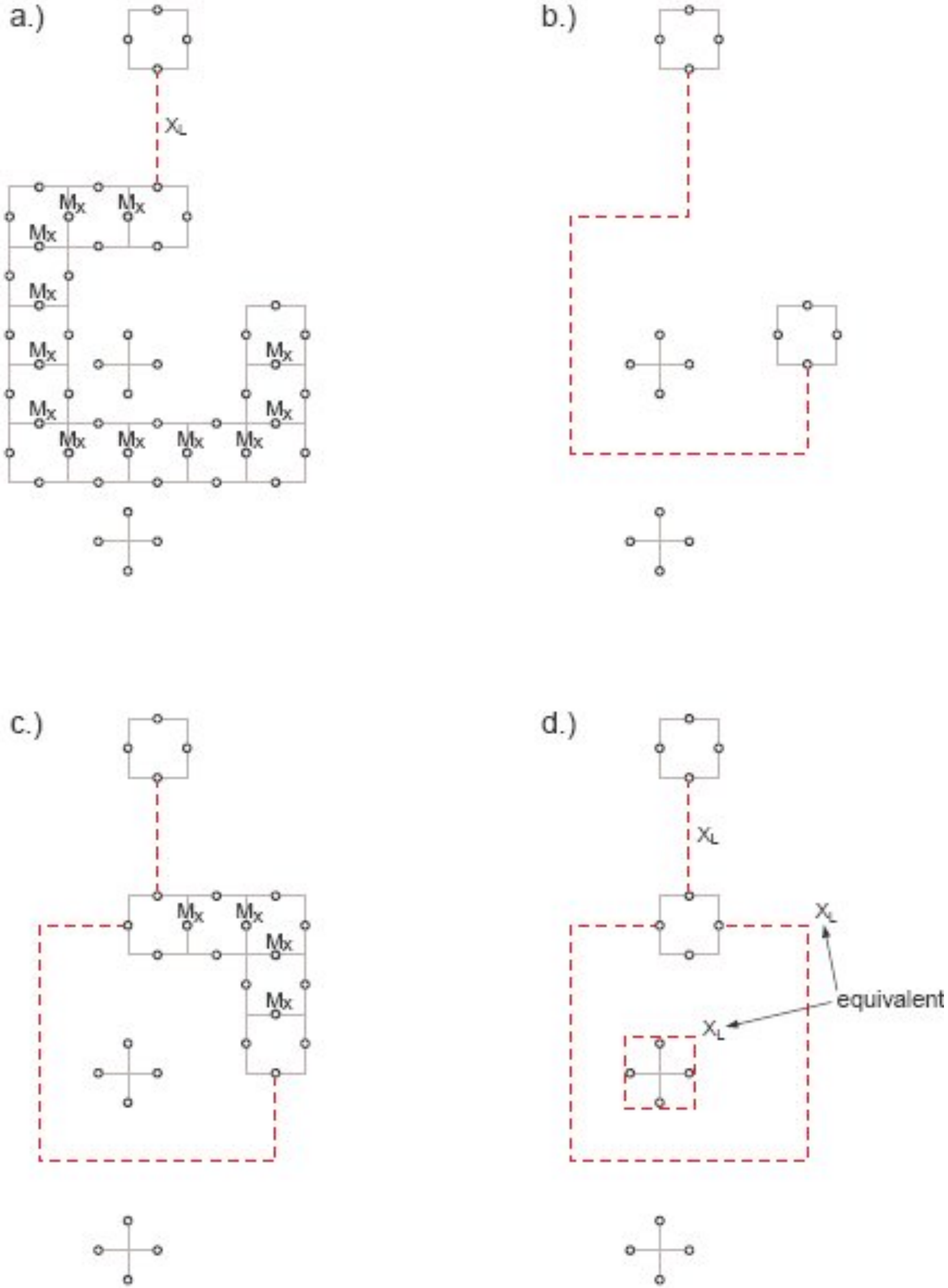
Accomplished by changing pattern of operations/measurements

Doesn’t interfere with collection of Error-correction information

b.)







CNOT: braid a hole from one qubit in between the holes embodying the other qubit.

Error correction continues throughout gate operation

Superior to “tranversal” technique

Subsystem concept leads to many new codes…

Five-Squares Code

PHYSICAL REVIEW A **81**, 032301 (2010)

Topological subsystem codes

H. Bombin

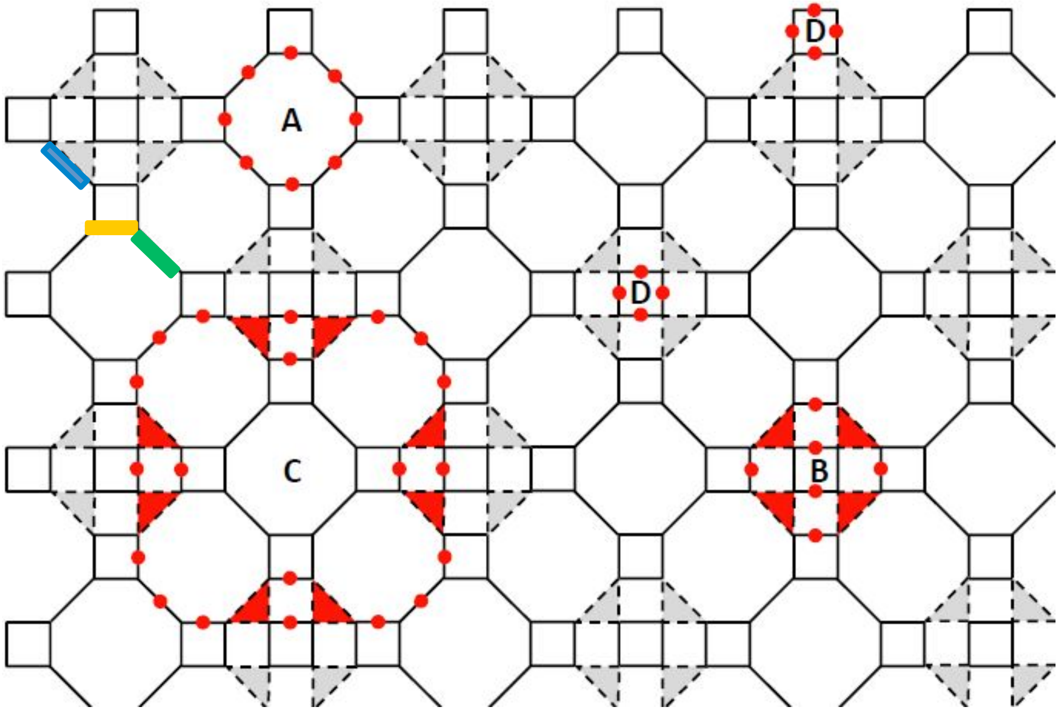
Suchara, Bravyi, Terhal, arxiv:1012:0425

Qubits are at vertices. Periodic boundary conditions (torus).

G=< K_e > where K_e are 2-qubit Pauli link operators.

- Dashed links are ZZ.
- Solid links around squares are XY (clockwise say).
- Solid links between squares are ZZ.

Note that solid links anticommute when they overlap on one qubit.



Subsystem stabilizer codes

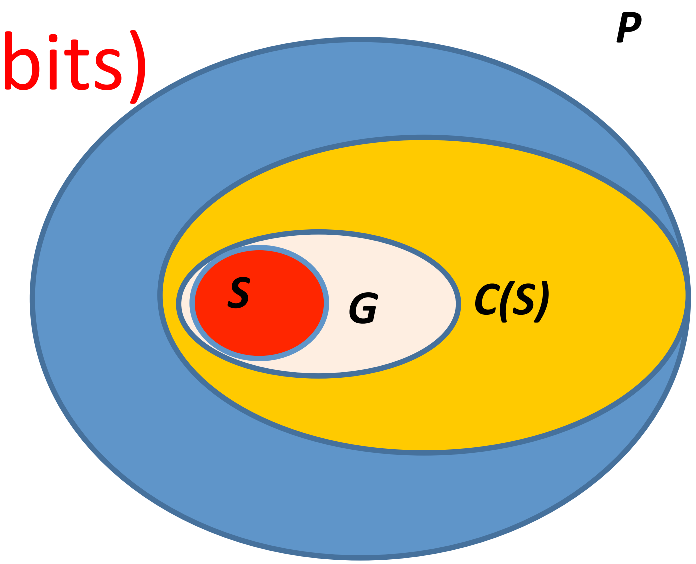
Non-Abelian group *G* with elements <G_i> where G_i have, say, local support on a lattice.

Abelian center of group *S*.

Operators in *C(S)* (in but not in *G*) are logical operators for protected qubits.

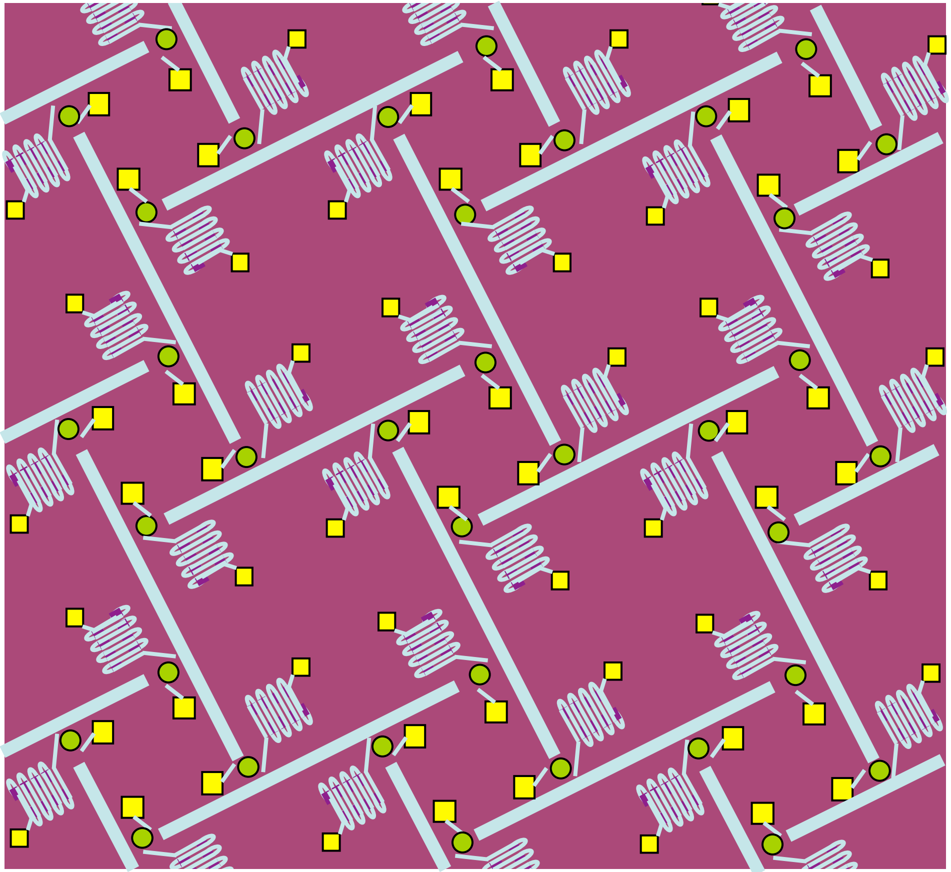
Operators in *G* are logical operators

for gauge qubits (extra unused qubits)



Conclusion: quantum error correction in your future

- Original insights still being played out
- Maybe a good evolutionary path to quantum computer hardware



Concept (IBM) of surface code fabric with Superconducting qubits and coupling resonators

"In a machine such as this there are very many other problems due to imperfections. . . . At least some of these problems can be remedied in the usual way by techniques such as error correcting codes . . . But until we find a specific implementation for this computer, I do not know how to proceed to analyze these effects."

R.P. Feynman
"Quantum Mechanical Computers"
Optics News, February 1985