Ultrastrong coupling circuit QED: vacuum degeneracy and quantum phase transitions

Cristiano Ciuti

Université Paris Diderot-Paris 7 and CNRS,
Laboratoire Matériaux et Phénomènes Quantiques (MPQ), UMR7162 ,
75013 Paris France

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Acknowledgments for the work presented today

Pierre Nataf (Ph.D. student) [*Circuit QED*]

S. De Liberato (PhD 2009) [*Cavity QED in semiconductors*]

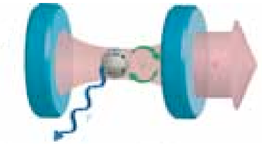
D. Hagenmüller (PhD student) [*Cavity QED in semiconductors*]

I. Carusotto (University of Trento, Italy)

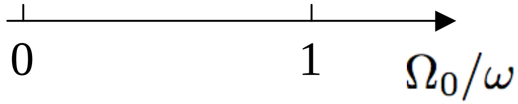
…. (mentioned during the talk)

Outline

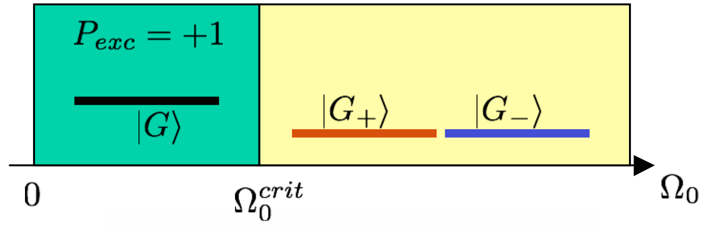
- Introduction on cavity and circuit QED

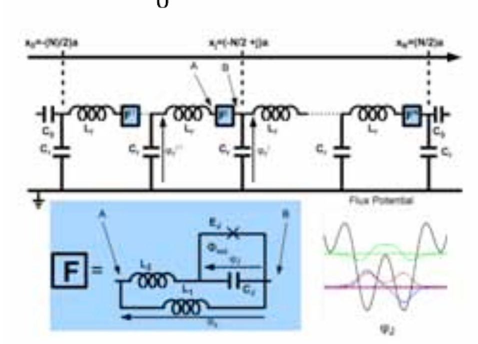


- Ultrastrong coupling regime



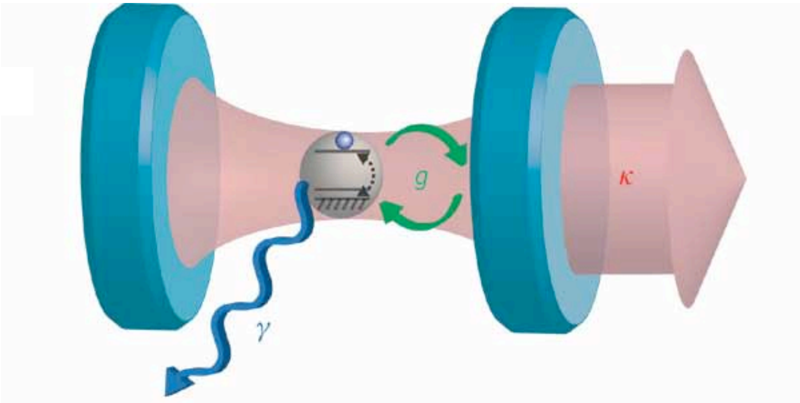
- Quantum phase transitions and vacuum degeneracy in circuit QED





- Conclusions and perspectives

Cavity quantum electrodynamics

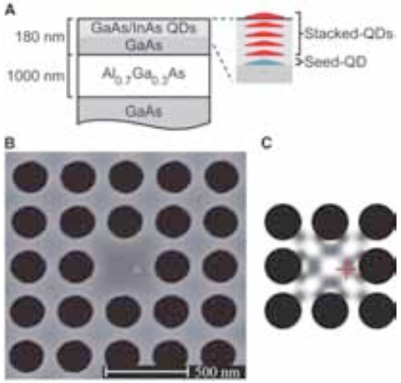


Cavity quantum electrodynamics (cavity QED) is the study of the interaction between light confined in a reflective cavity and atoms or other particles, under conditions where the quantum nature of light photons is significant.

See for example:
S. Haroche, J.-M. Raimond,
Exploring the quantum: atoms, cavities, photons, (Oxford Press, 2006).

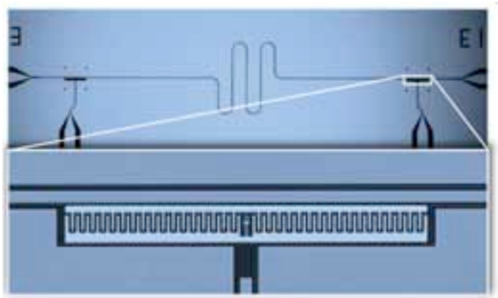
H.J. Kimble, Nature 453, 1023-1030 (19 June 2008).

Examples of solid-state cavity QED systems



• Quantum dots or quantum wells in dielectric cavities (semiconductor cavity QED)

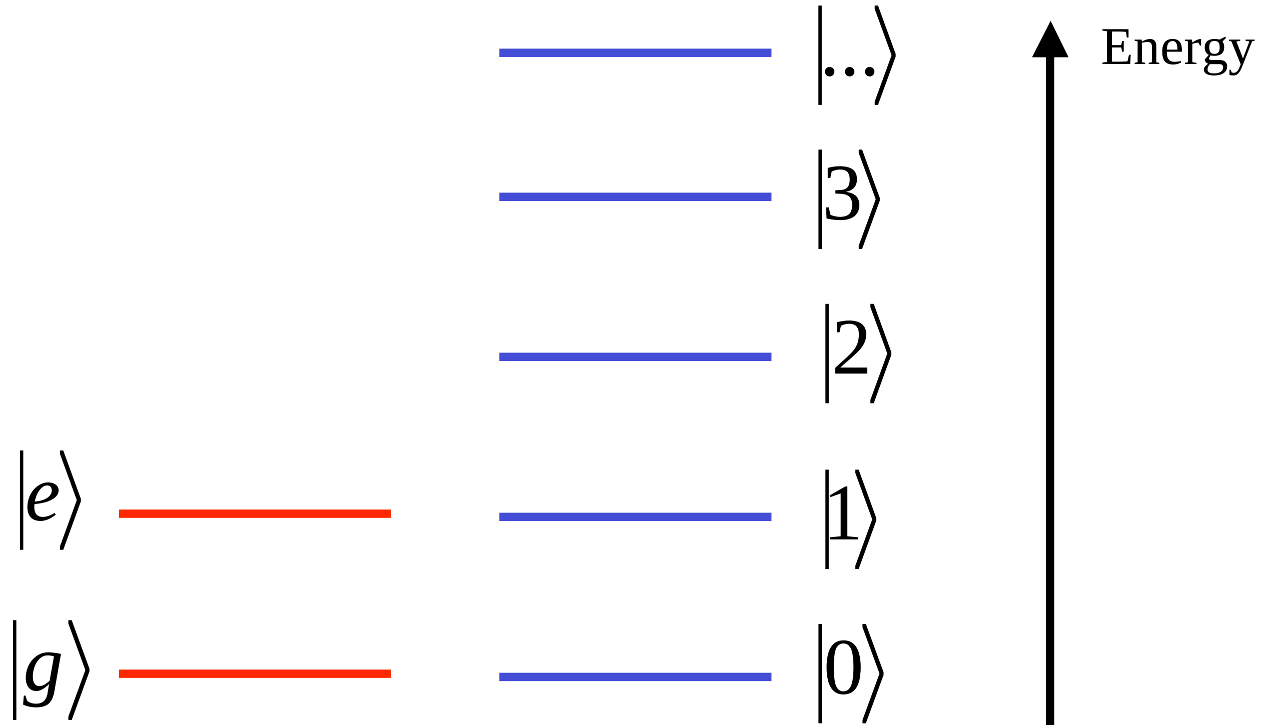
See for example:
- A. Badolato et al., Science 308, 5725 (2005).
- J. P. Reithmaier et al., Nature (London) 432, 197 (2004).
- T. Yoshie et al., Nature (London) 432, 200 (2004).
- E. Peter et al., Phys. Rev. Lett. 95, 067401 (2005).
- …



• Josephson junction artificial atoms in transmission line resonators (superconducting circuit QED)

See for example:
- M.H. Devoret, Lectures at Collège de France (years 2008, 2009)
- R. J. Schoelkopf, S. M. Girvin, Nature 451, 664 (2008).

The simplest cavity QED system: a 2-level atom + a photon mode



Two-level system

(fermionic system
Anharmonic spectrum)

Cavity photon Fock space

(Bosonic field, harmonic oscillator)

Jaynes-Cummings Hamiltonian and conserved number

$$H_0 = const. + \hbar\omega_e |e\rangle\langle e| + \hbar\omega_g |g\rangle\langle g| + \hbar\omega_{cav} a^\dagger a$$

$$H_{int} = \hbar\Omega_0 \left(|e\rangle\langle g| a + a^\dagger |g\rangle\langle e| \right)$$

Photon Absorption

Photon Emission

$$\hat{N}_{exc} = a^\dagger a + |e\rangle\langle e|$$

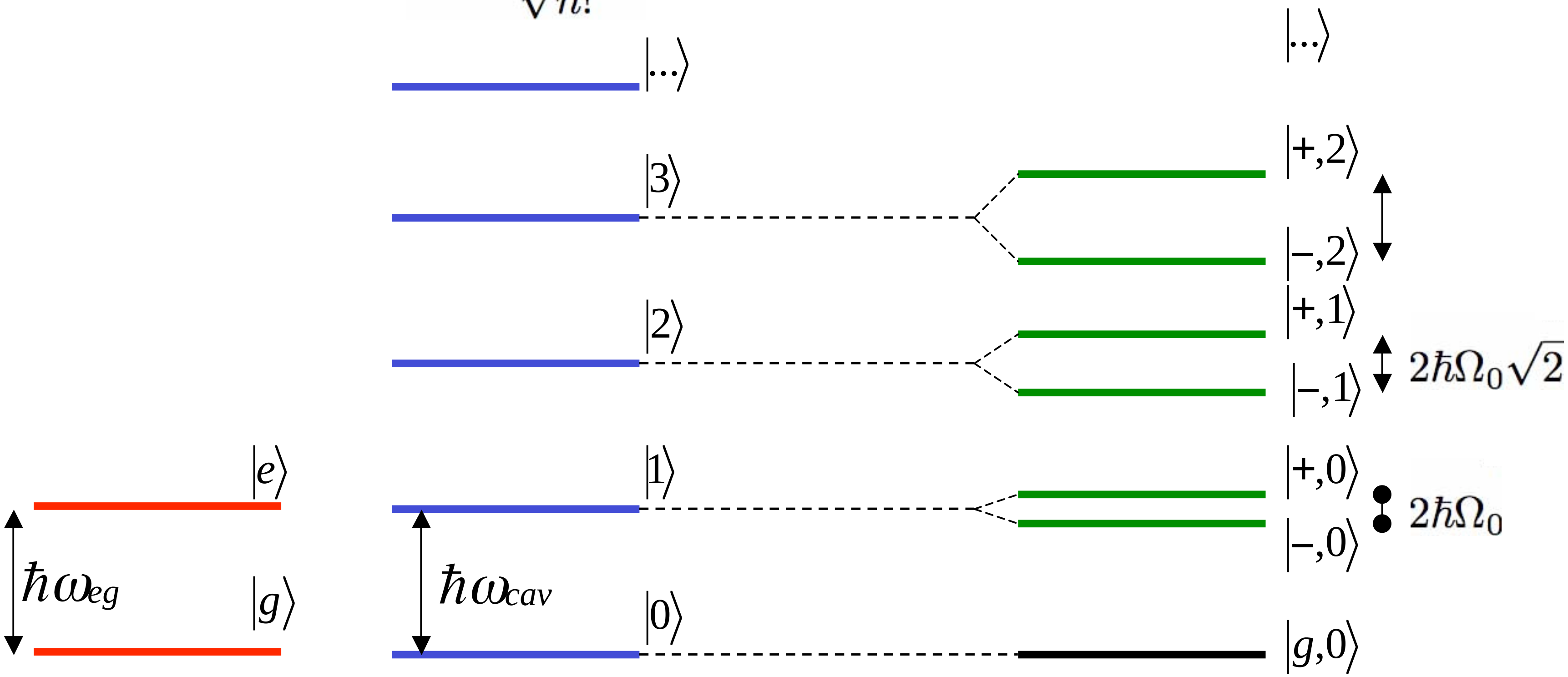
Total excitation number (cavity + atom)

$$[\hat{N}_{exc}, H_0 + H_{int}] = 0$$

Conserved by JC Hamiltonian !

Vacuum Rabi splitting in the frequency domain

Number (Fock) states $|n\rangle = \frac{a^{\dagger n}}{\sqrt{n!}}|0\rangle$

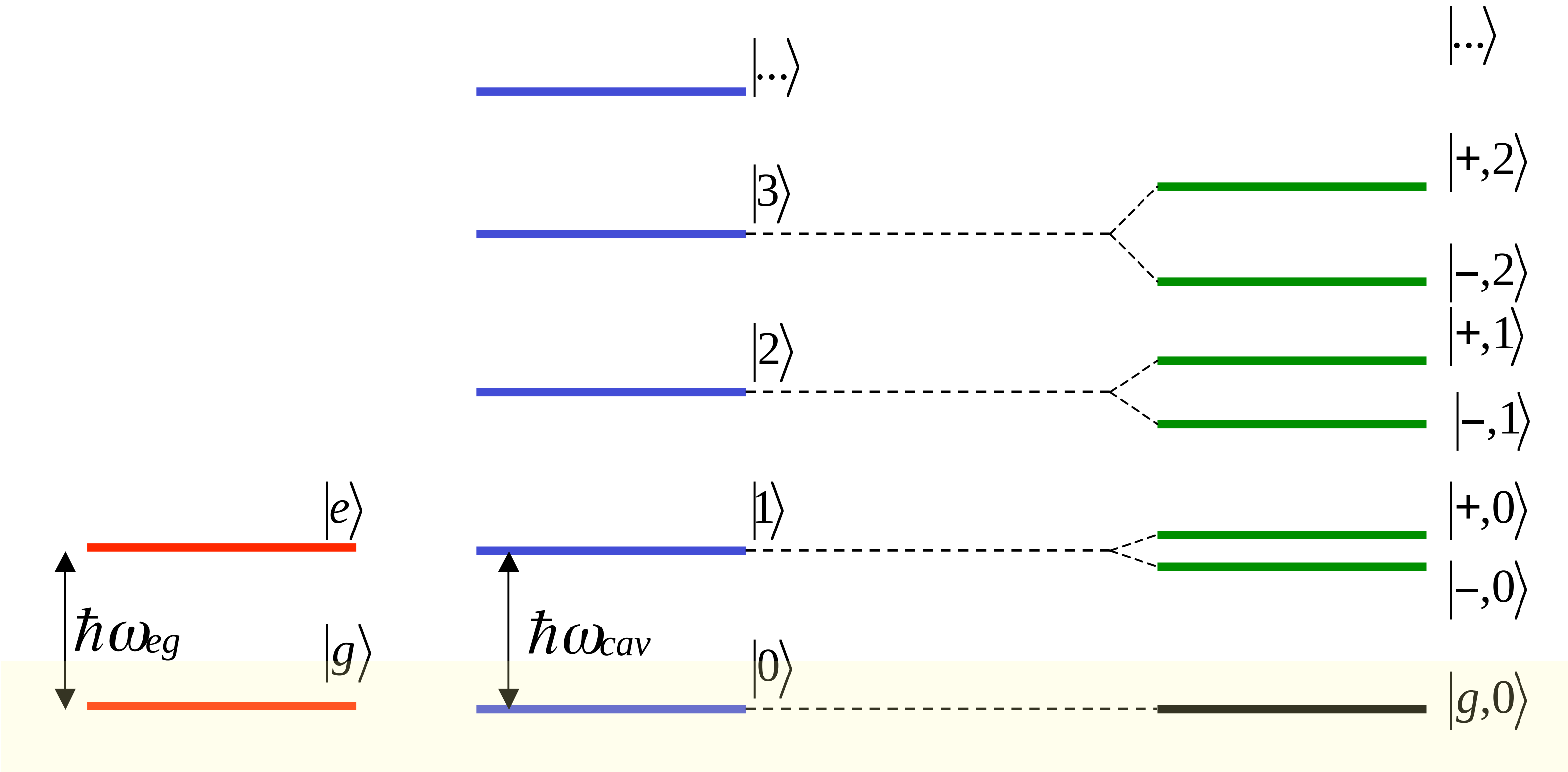


Resonant case

Energies: $H|\pm,n\rangle = (n\hbar\omega \pm \hbar\Omega_0\sqrt{n+1})|\pm,n\rangle$

Eigenstates $|\pm,n\rangle = \frac{1}{\sqrt{2}}(|e,n\rangle \pm |g,n+1\rangle)$

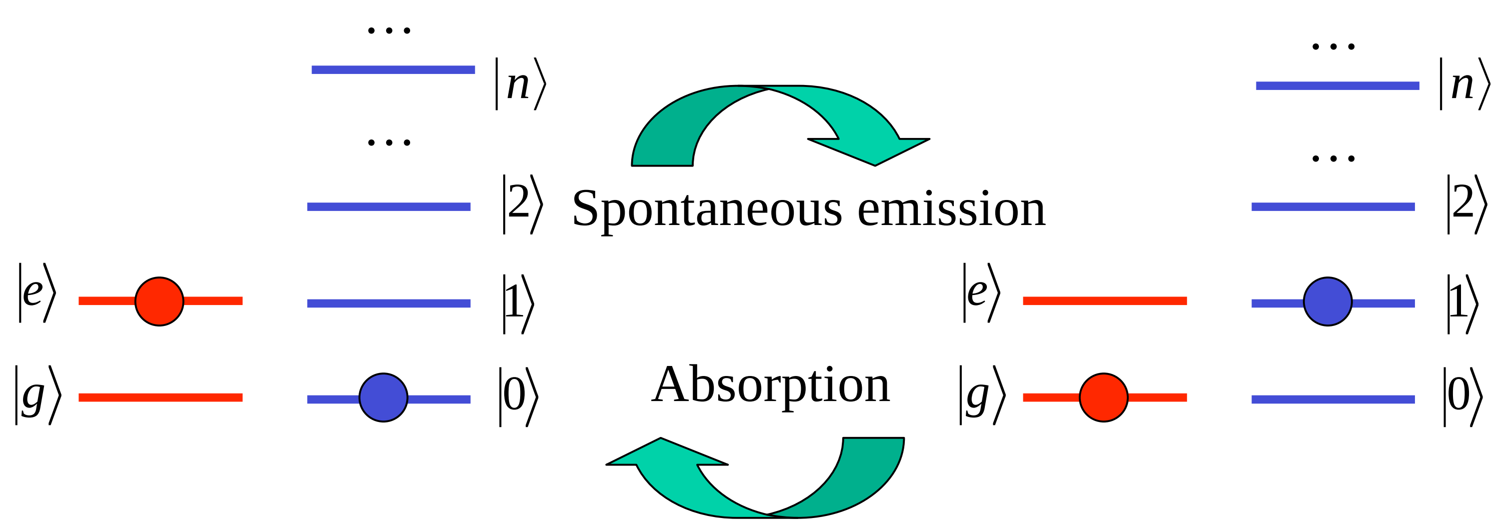
What about the vacuum (ground state) ? Unchanged



The vacuum is NOT changed by light-matter interaction in the Jaynes-Cummings Model !

$$|g,0\rangle = |g\rangle \otimes |0\rangle$$

Vacuum Rabi coupling (electric dipole case)



- Reversible exchange of energy between the atom and the photon field.

- Coupling quantified by vacuum Rabi frequency

$$\hbar\Omega_0 = dE_0$$

Electric dipole

Electric field
vacuum fluctuations

Vacuum Rabi frequency: back-of-the-envelope calculation

$$\hbar\Omega_0 = dE_0$$

$$\mathbf{d} = e\int dV\,\phi_e^\star(\mathbf{r})\,\mathbf{r}\,\phi_g(\mathbf{r})$$

Electric dipole of two-level transitions

$$d \sim eL_{at}$$

Atomic size

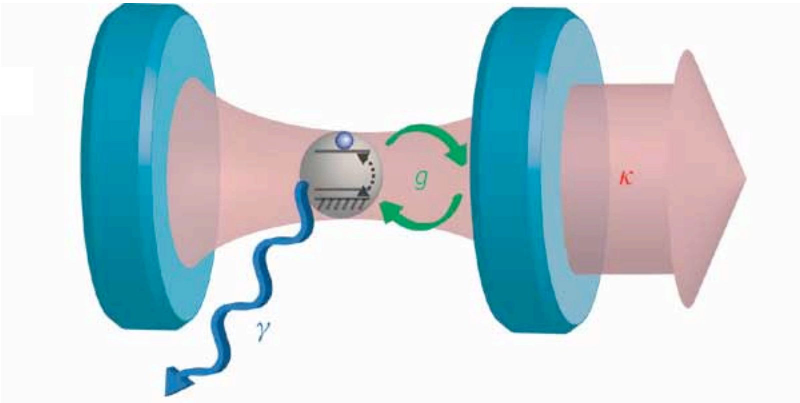
$$\hbar\omega \sim \epsilon_0 \int E^2 dV \sim \epsilon_0 E_0^2 V$$

Energy of one photon

$$E_0 \sim \sqrt{\frac{\hbar\omega}{\epsilon_0 V}}$$

Vacuum electric field

Strong coupling regime



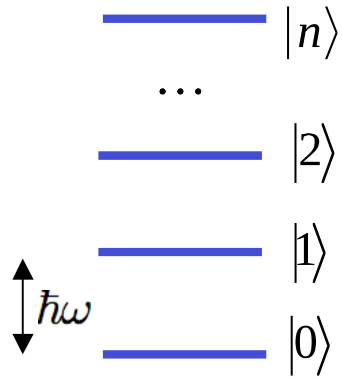
Vacuum Rabi coupling larger than photon and atom losses

$$\Omega_0 > \gamma, \kappa$$

Recipes for strong coupling :
- very small losses OR/AND
- very large coupling

Electrical dipole coupling: limit imposed by fine structure constant

$$\frac{\Omega_0}{\omega} \sim \sqrt{\frac{d^2}{\epsilon_0 V \hbar\omega}}$$



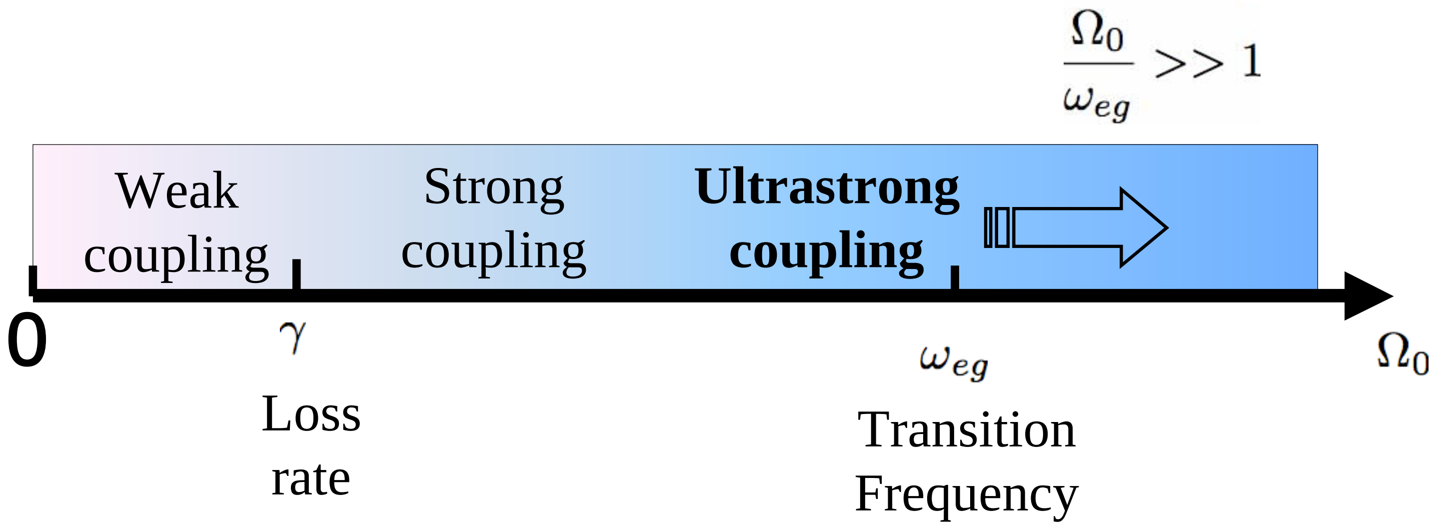
For atoms in a λ^3 cavity:

$$\frac{\Omega_0}{\omega} \sim \sqrt{\frac{e^2 L_{at}^2}{\epsilon_0 h c \lambda^2}} = \frac{L_{at}}{\lambda}\sqrt{4\pi}\sqrt{\alpha} \ll \sqrt{\alpha} \quad \Rightarrow \quad \frac{\Omega_0}{\omega} \ll 1$$

In the case of Rydberg atoms in a microwave cavity: $\frac{\Omega_0}{\omega} \sim 10^{-6}$ (for a single atom)

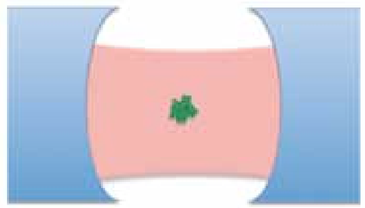
See e.g.: M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Ann. Phys. (Leipzig) 16, 767 (2007))
R. J. Schoelkopf, S. M. Girvin, Nature 451, 664-669 (6 February 2008)

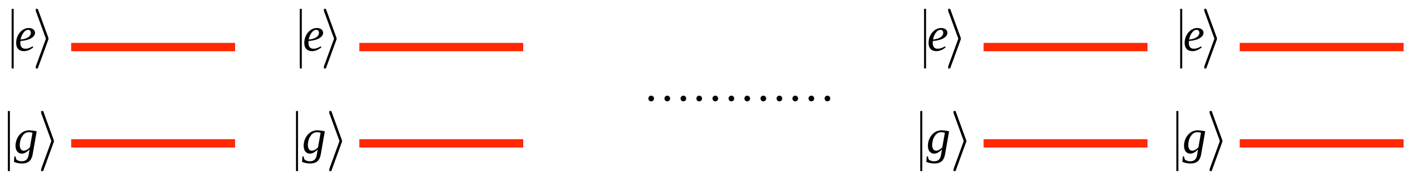
Ultrastrong coupling regime



Collective vacuum Rabi coupling

N identical atoms coupled to the same photon mode





$$\Omega_0 \propto \sqrt{N}$$

-Vacuum Rabi frequency is enhanced by collective excitation

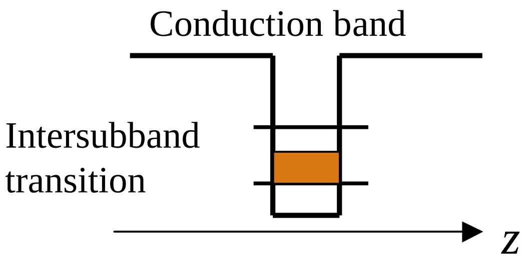
- Collective excitations are bosonic for $N >> 1$

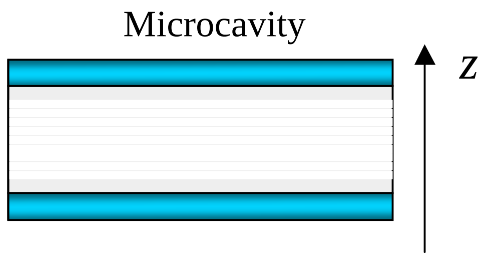
- In principle, anharmonicity is lost …

How to reach ultrastrong coupling in semiconductors

Ultrastrong coupling due to collective vacuum Rabi coupling in **semiconductors**:

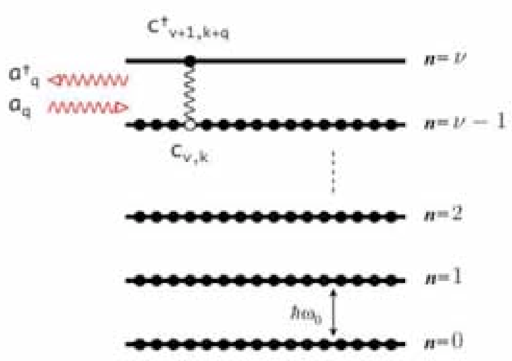
- 2D electron gas in semiconductor microcavities
CC , G. Bastard, I. Carusotto, PRB 72, 115303 (2005)

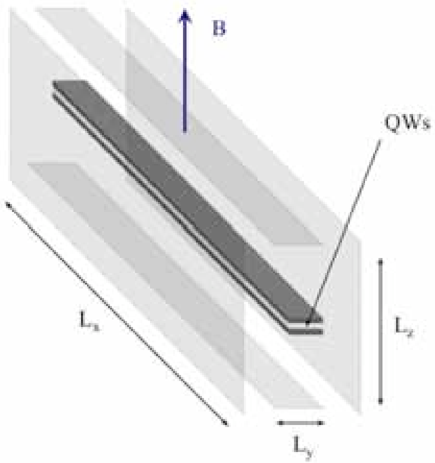




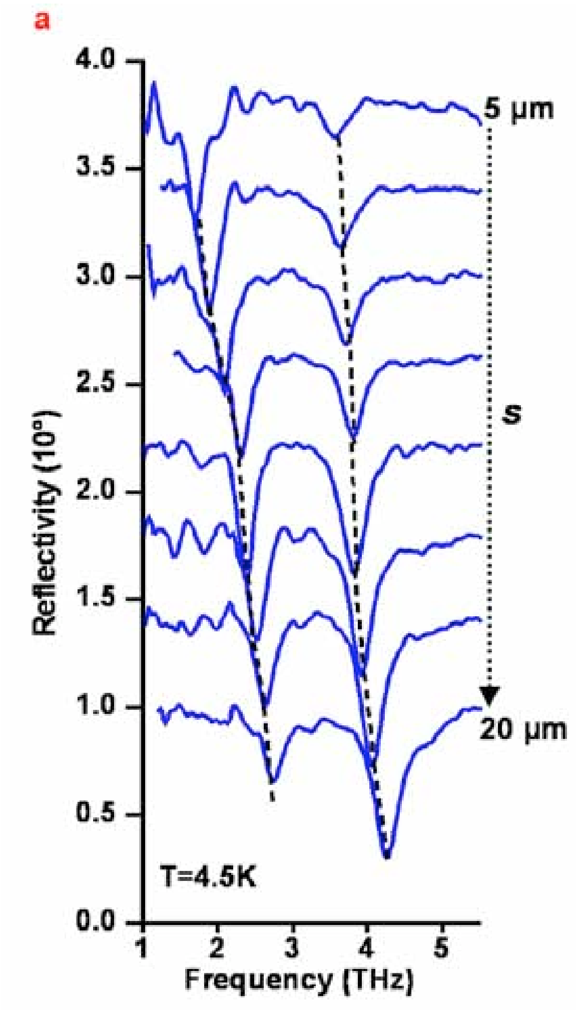
- 2D electron gas with magnetic field
D. Hagenmüller, S. De Liberato, CC, PRB 81, 235303 (2010)

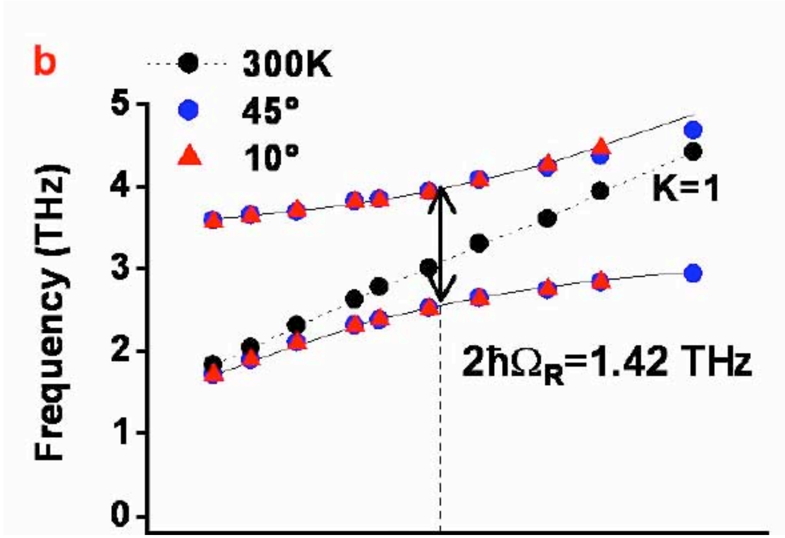
Cyclotron transition





State-of-the-art in semiconductors



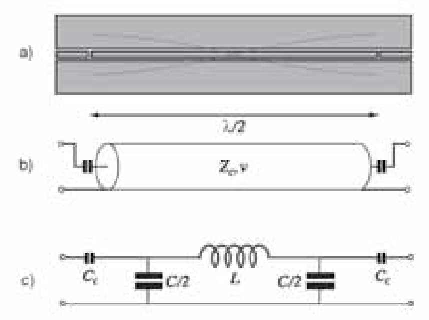


Y. Todorov et al., Phys. Rev. Lett. 102, 186402 (2009)

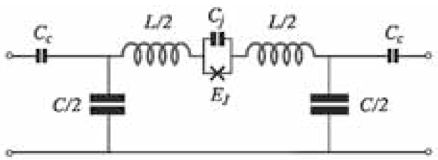
Y. Todorov, … C. Sirtori, unpublished (Lab MPQ, Univ. Paris Diderot)

How to reach the ultrastrong coupling in superconductors

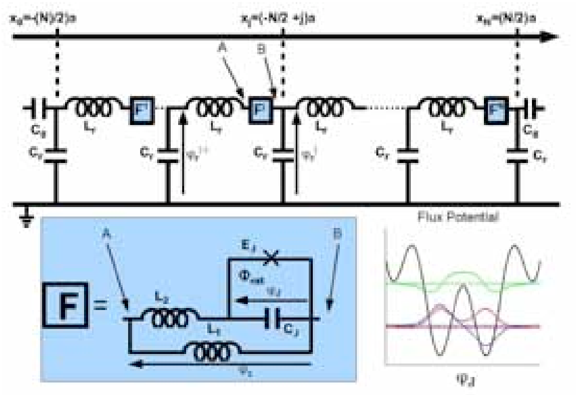
- Inductive coupling of a Josephson atom to a transmission line resonator :



M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Annalen der Physik 16, 767 (2007).



- Collective vacuum Rabi coupling of Josephson chains:



P. Nataf, CC, PRL 104 023601, (2010).

What happens in ultrastrong coupling cavity QED ?

More is different !

- Quantum vacuum (ground state) depends on interaction strength

- Antiresonant (non-rotating wave) terms of light-matter interaction becomes important

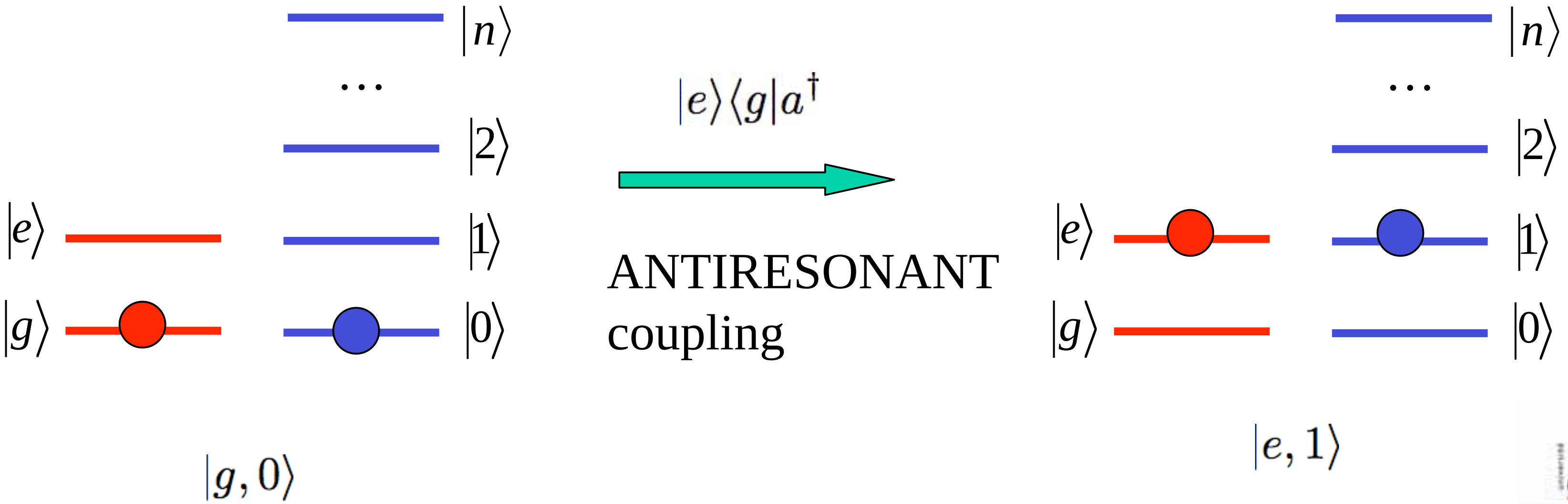
What are the antiresonant terms ?

Destruction of two excitations

$$H_{anti} = \hbar\Omega_0 \left(|e\rangle\langle g| a^\dagger + a |g\rangle\langle e| \right)$$

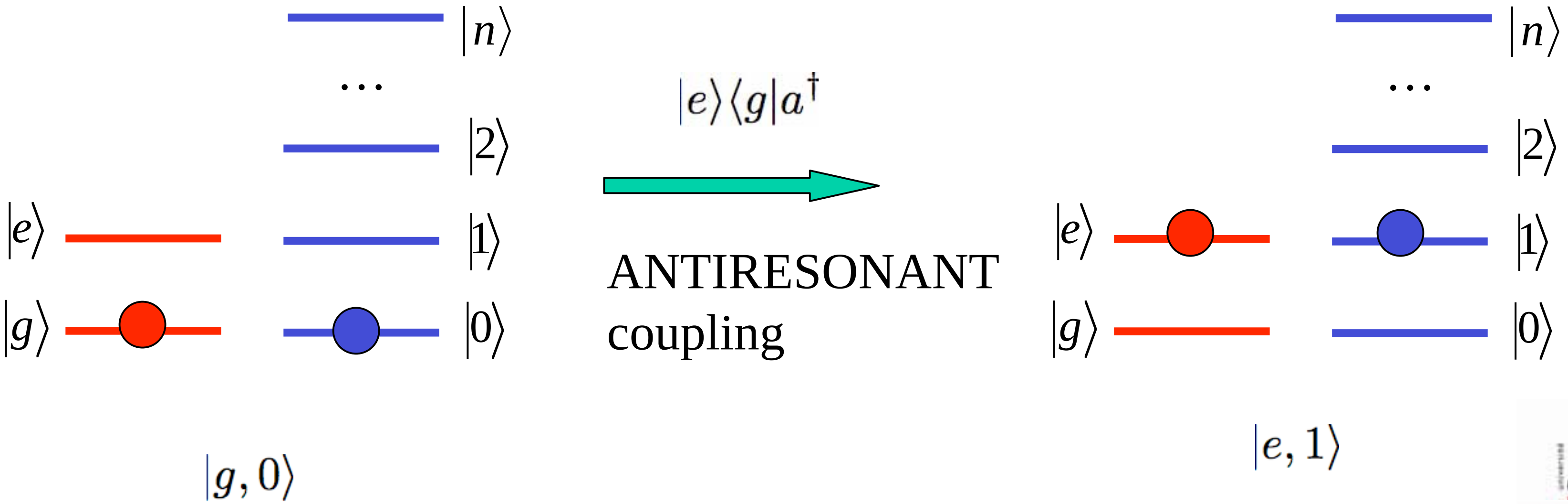
Creation of two excitations

NOTE: Antiresonant (non-rotating wave) terms are neglected in the Jaynes-Cummings model



Changing the vacuum

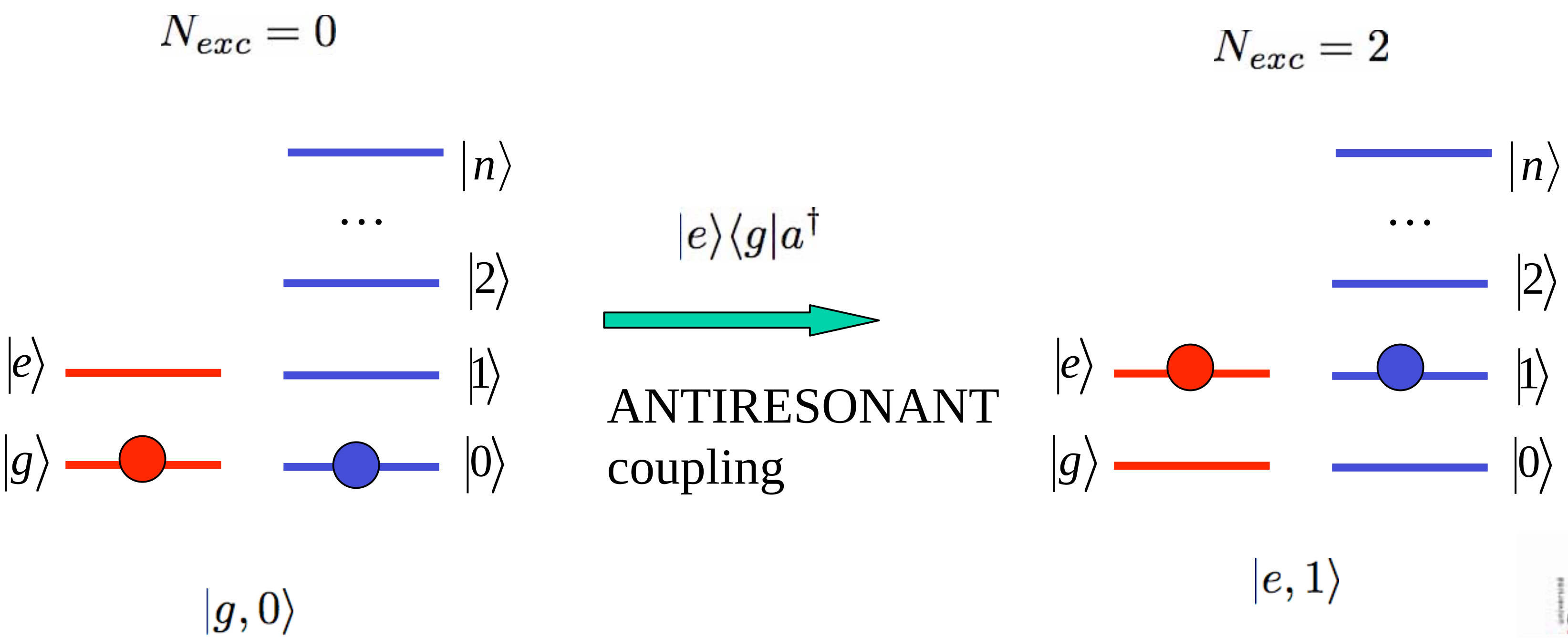
The standard vacuum $|g,0\rangle$ is no longer the quantum ground state !



Total excitation number is no longer conserved

$$\hat{N}_{exc} = a^{\dagger}a + |e\rangle\langle e|$$

$$[\hat{N}_{exc}, H_{anti}] \neq 0$$

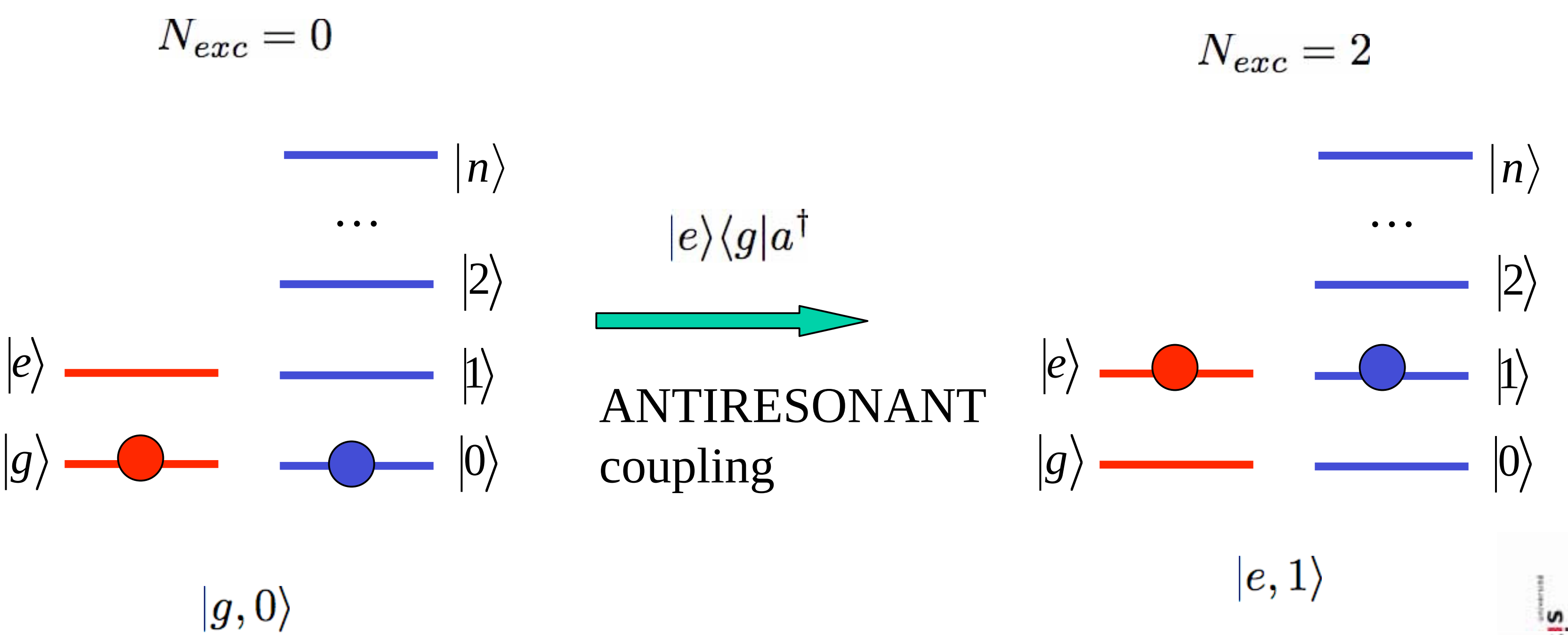


Parity is conserved

Parity operator

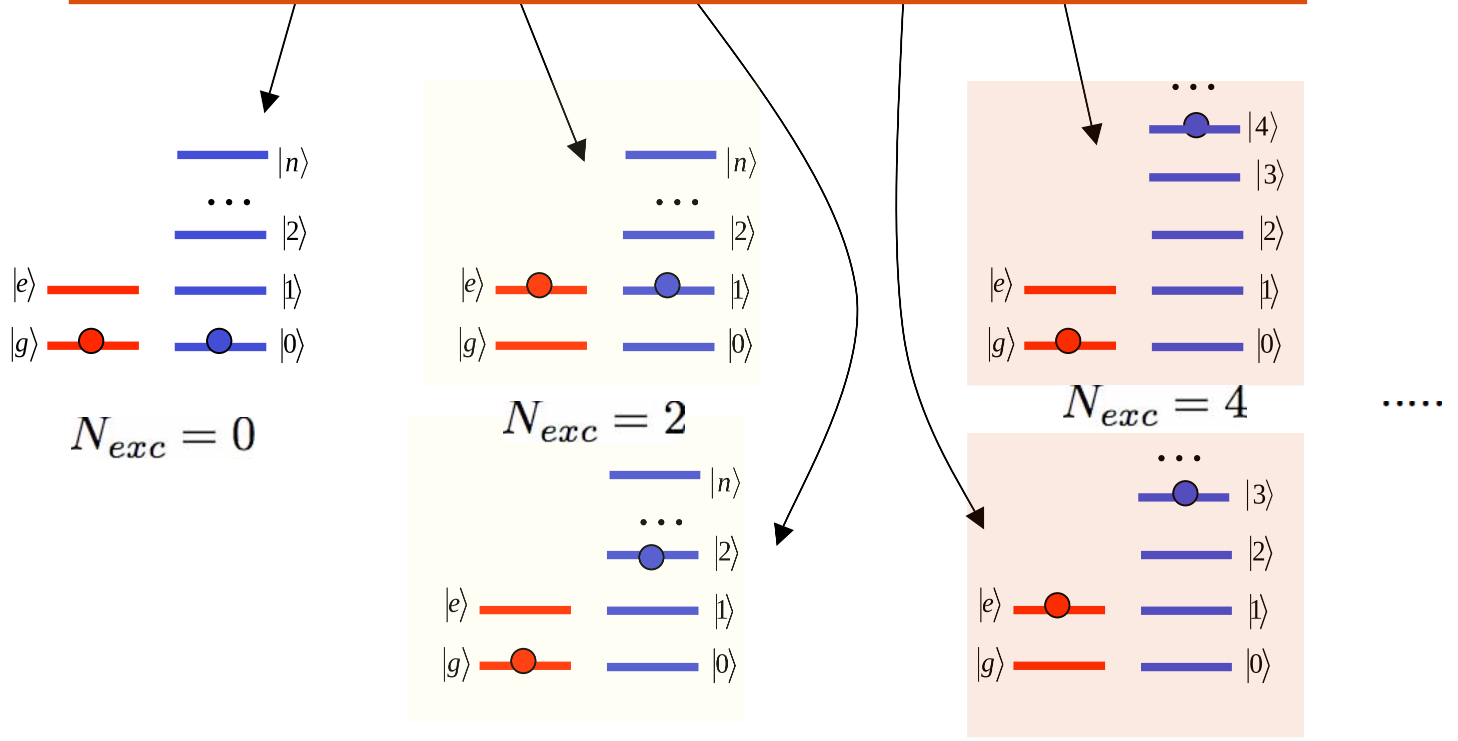
$$\hat{P}_{exc} = \exp(i\pi \hat{N}_{exc})$$

$$[\hat{P}_{exc}, H_{anti}] = 0$$



The form of the ground state with finite antiresonant interactions

$$|G\rangle = ...|g,0\rangle + ...|e,1\rangle + ..|g,2\rangle + ...|e,3\rangle + ...|g,4\rangle + ...$$



The ground state contains photons !! $\langle G|a^\dagger a|G\rangle > 0$

The total number of excitations (matter + photon) is even $\langle G|\hat{P}_{exc}|G\rangle = 1$

(unless a symmetry breaking)

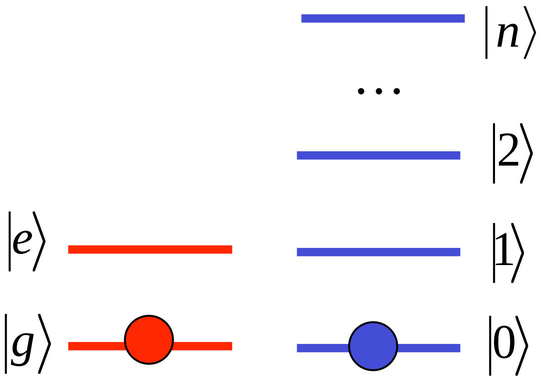
When are the antiresonant terms negligible ?

Antiresonant terms negligible only if $\frac{\Omega_0}{\omega} \ll 1$

Perturbative theory argument:

$\langle e, 1|H_0|e, 1\rangle - \langle g, 0|H_0|g, 0\rangle \approx 2\hbar\omega$ Difference between bare energies

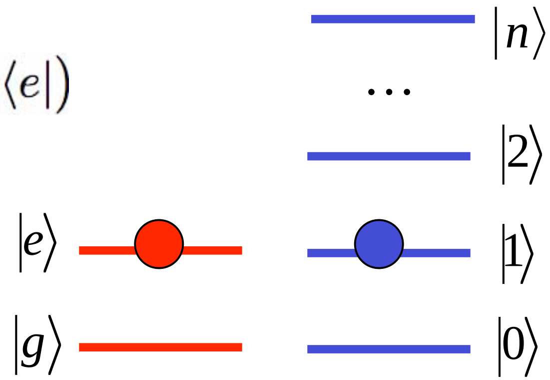
$\langle g, 0|H_{anti}|e, 1\rangle = \hbar\Omega_0$ Coupling energy



$|g, 0\rangle$

$$H_{anti} = \hbar\Omega_0 \left(|e\rangle\langle g|a^\dagger + a|g\rangle\langle e|\right)$$

ANTIRESONANT
coupling



$|e, 1\rangle$

The ‘virtuality’ of photons in the ground state (vacuum)

$$|G\rangle = ...|g,0\rangle + ...|e,1\rangle + ..|g,2\rangle + ...|e,3\rangle + ...|g,4\rangle + ...$$

The photons in the ground state canNOT escape the cavity !

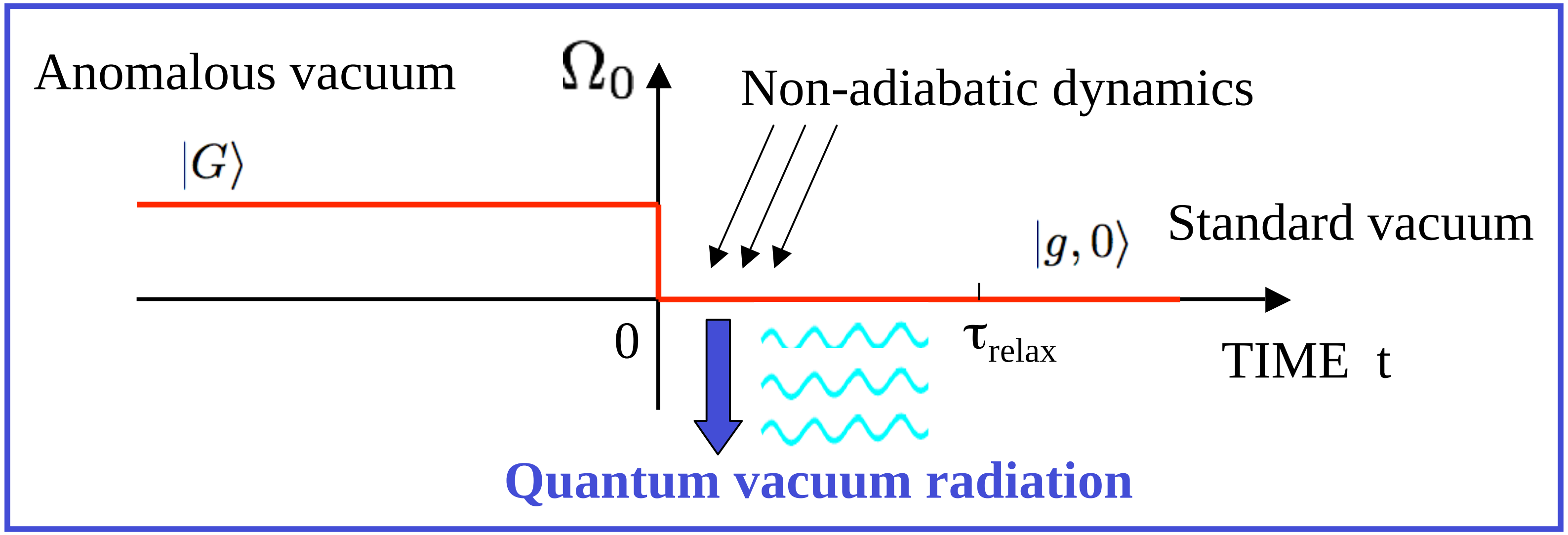
The ground state is the lowest energy state !

Non-adiabatic release of quantum vacuum photons

External Control Bias → Time-dependent vacuum Rabi frequency

$$\Omega_0(t) = \bar{\Omega}_0 + \Omega_0^{mod}(t)$$

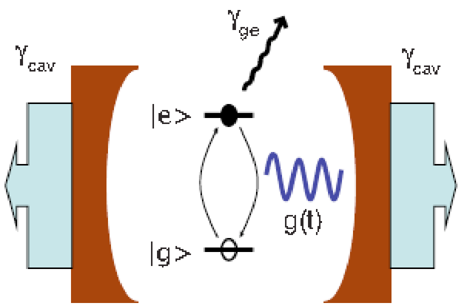
A *gedanken* experiment: Sudden switch-off

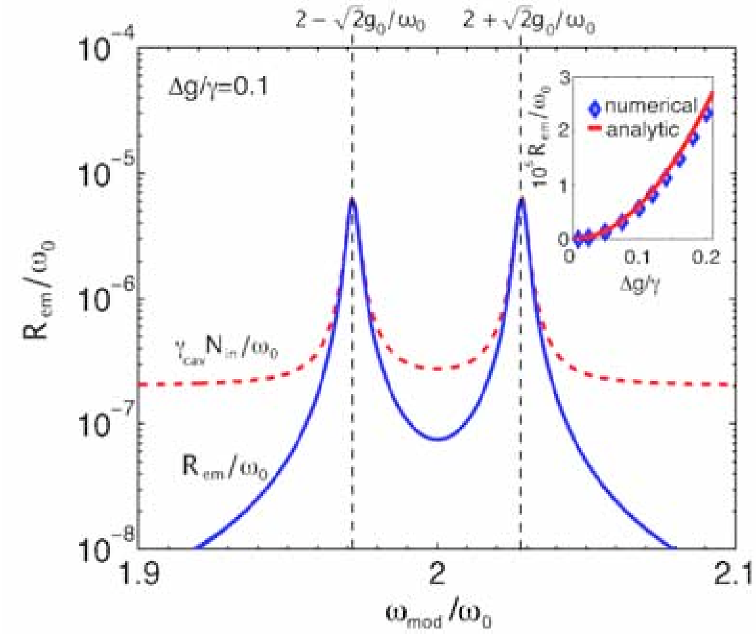


CC , G. Bastard, I. Carusotto, PRB 72, 115303 (2005)
CC, I. Carusotto, PRA 74, 033811 (2006).

Quantum vacuum radiation by modulating vacuum Rabi coupling

$$\Omega_0(t) = g_0 + \Delta g \sin(\omega_{mod} t)$$





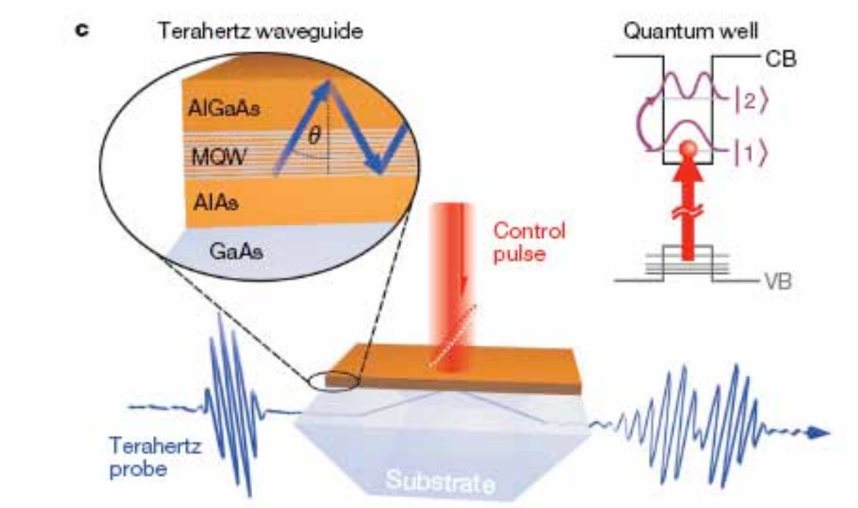
- For a dissipative *two-level* (qubit) system:
S. De Liberato, D. Gerace, I. Carusotto, CC, PRA 80, 053810 (2009).
- For a dissipative *bosonic* (polaritons) system:
S. De Liberato, CC, I. Carusotto, Phys. Rev. Lett. 98, 103602 (2007).

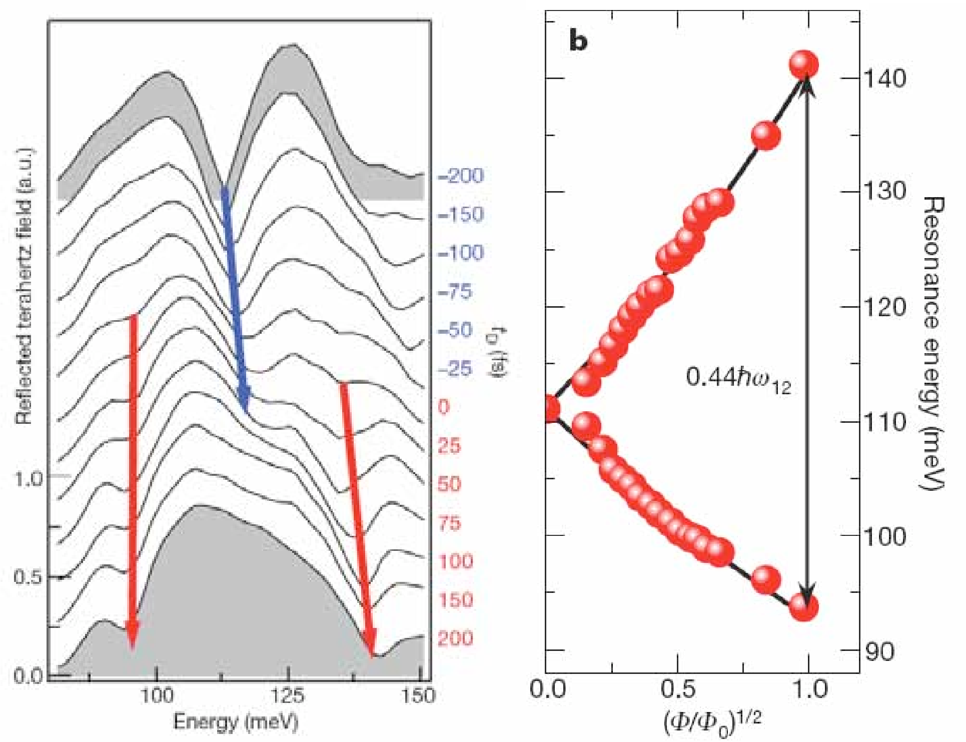
Emerging field: non-adiabatic cavity QED

nature Vol 458|12 March 2009|doi:10.1038/nature07838

Sub-cycle switch-on of ultrastrong light–matter interaction

G. Günter[1], A. A. Anappara[1,2], J. Hees[1], A. Sell[1], G. Biasiol[3], L. Sorba[2,3], S. De Liberato[4,5], C. Ciuti[4], A. Tredicucci[2], A. Leitenstorfer[1] & R. Huber[1]

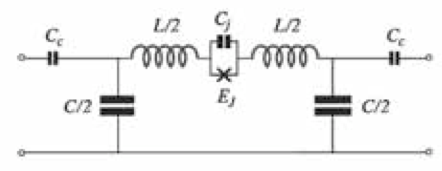


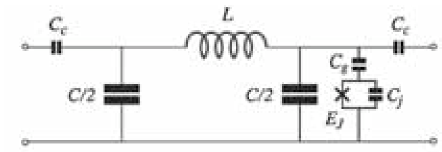


Problems in ultrastrong coupling circuit QED

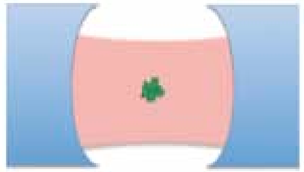
- What happens when *N* artificial atom are embedded in a transmission line resonator ?? Quantum phase transitions ??

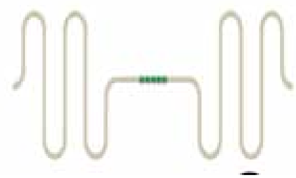
- What happens with different types of coupling ?





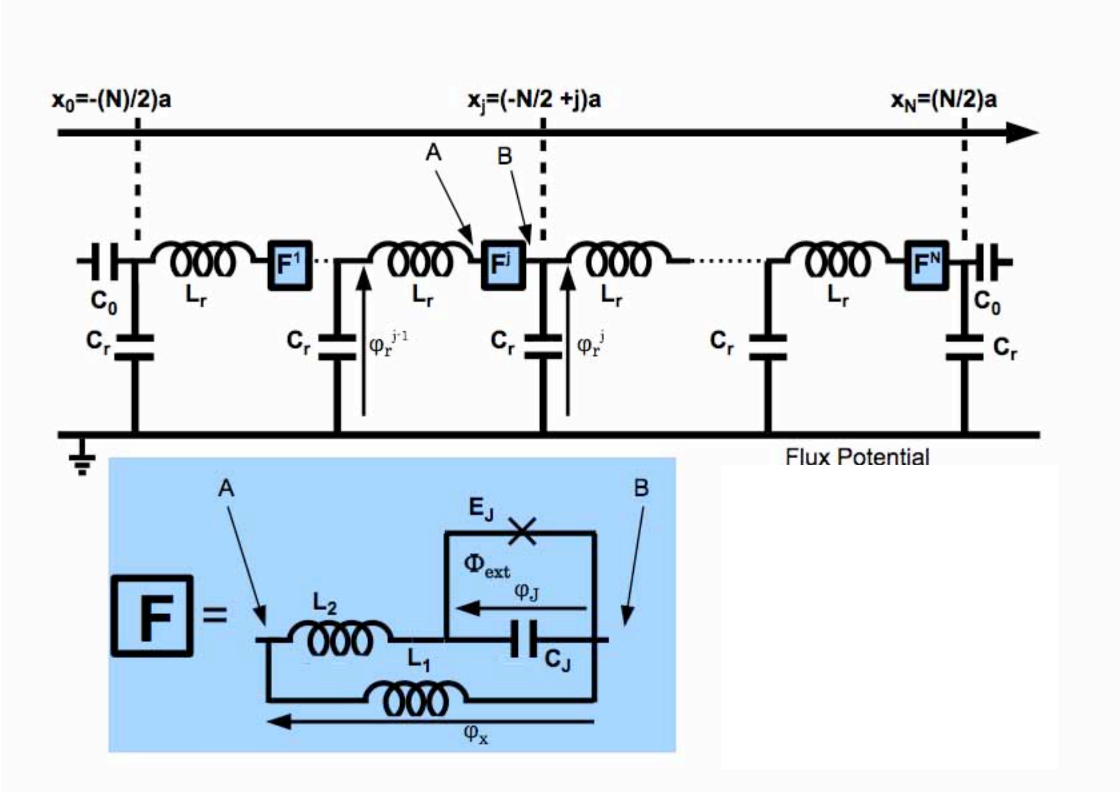
- Differences/analogies with cavity QED ?





- P. Nataf, CC, Phys. Rev. Lett. 104, 023601 (2010)
- P. Nataf, CC, submitted; preprint arXiv:1006.1801

N Josephson atoms inductively coupled to a resonator



P. Nataf, CC, Phys. Rev. Lett. 104, 023601 (2010)

The strength of inductive coupling in circuit QED

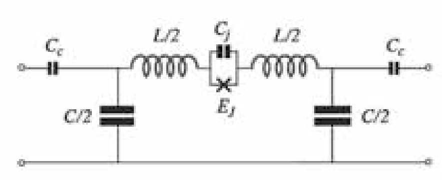
Ann. Phys. (Leipzig) **16**, No. 10–11, 767 – 779 (2007) / **DOI** 10.1002/andp.200710261

Circuit-QED: How strong can the coupling between a Josephson junction atom and a transmission line resonator be?

Michel Devoret[1,2,*], **Steven Girvin**[1], and **Robert Schoelkopf**[1]

[1] Applied Physics Department, Yale University, New Haven, CT 06520-8284, USA
[2] Collège de France, 75231 Paris cedex 05, France

After reviewing the limitation by the fine structure constant α of the dimensionless coupling constant of an hydrogenic atom with a mode of the electromagnetic field in a cavity, we show that the situation presents itself differently for an artificial Josephson atom coupled to a transmission line resonator. Whereas the coupling constant for the case where such an atom is placed inside the dielectric of the resonator is proportional to $\alpha^{1/2}$, the coupling of the Josephson atom when it is placed in series with the conducting elements of the resonator is proportional to $\alpha^{-1/2}$ and can reach values greater than 1.



Giant coupling: $\frac{\Omega_0}{\omega} \gg 1$ even with a single Josephson atom !!

Circuit quantum Hamiltonian

$$H = H_{res} + H_F + H_{coupling}$$

Resonator part

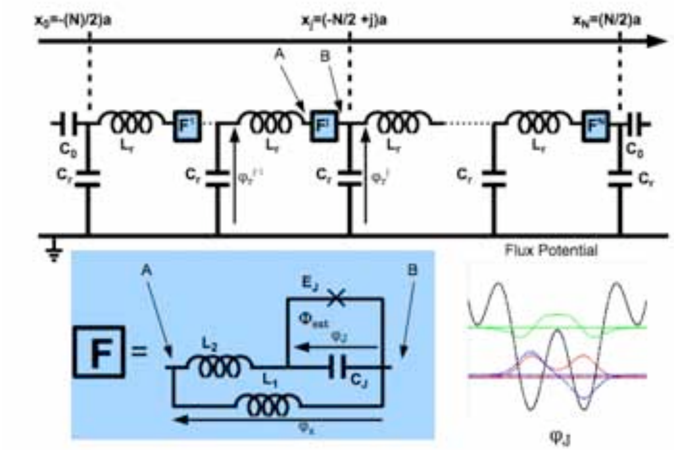
$$H_{res} = \sum_{j=1}^{N} 4E_{C_r}(\hat{N}_r{}^j)^2 + E_{L_r}\frac{(\hat{\varphi}_r^j - \hat{\varphi}_r^{j-1})^2}{2} \ ,$$

Josephson atomic part

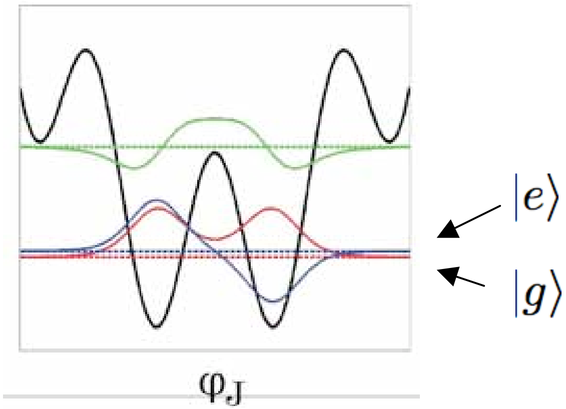
$$H_F = \sum_{j=1}^{N} 4E_{C_J}(\hat{N}_J^j)^2 + E_{L_J}\frac{(\hat{\varphi}_J^j)^2}{2} - E_J \cos(\hat{\varphi}_J^j + \frac{2e}{\hbar}\Phi_{ext}) \ ,$$

Inductive coupling between
Resonator and Josephson atoms

$$H_{coupling} = \sum_{j=1}^{N} G(\hat{\varphi}_r^j - \hat{\varphi}_r^{j-1})\hat{\varphi}_J^j,$$



Artificial atom Hamiltonian in terms of pseudospins



2-level system

$$\hat{\sigma}_{+,j} = |g\rangle\langle e|_j$$

Site-dependent Pauli matrices

$$\hat{\sigma}_{x,j} = \hat{\sigma}_{+,j} + \hat{\sigma}_{+,j}^{\dagger}$$

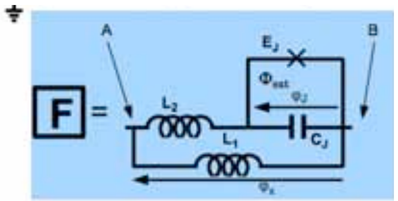
$$\hat{\sigma}_{y,j} = -i(\hat{\sigma}_{+,j} - \hat{\sigma}_{+,j}^{\dagger})$$

$$\hat{\sigma}_{z,j} = 2\hat{\sigma}_{+,j}\hat{\sigma}_{+,j}^{\dagger} - 1$$

Artificial atom bare energy:

$$H_{F,J} \simeq \hbar\omega_F \frac{1}{2}\hat{\sigma}_{z,j}$$

Fluxonium* atom



Josephson atom flux field

$$\hat{\varphi}_J \simeq -\varphi_{01}\hat{\sigma}_{x,j}$$

*V. E. Manucharyan, J. Koch, L. I. Glazman, and M. H. Devoret, Science 326, 113 (2009).

Mode expansion for transmission line resonator

Resonator flux field operator

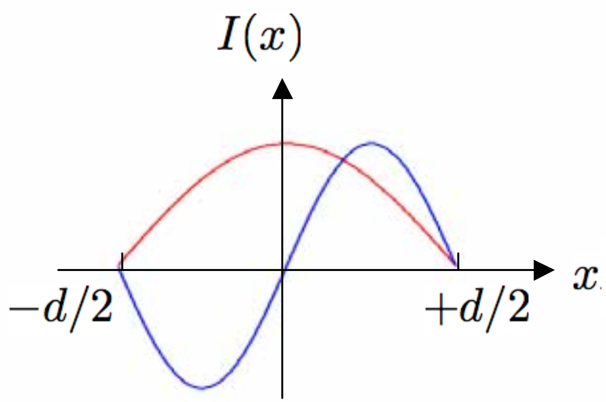
$$\hat{\phi}(x) = i \sum_{k \geq 1} \frac{1}{\omega_k} \sqrt{\frac{\hbar\omega_k}{2c_r}} f_k(x) \left(\hat{a}_k - \hat{a}_k^\dagger\right)$$

Mode frequencies

$$\omega_k = k\frac{\pi}{d}\frac{1}{\sqrt{l_r c_r}} \quad \text{with}\, k = 1, 2, 3, \ldots$$

$$f_k(x) = -\sqrt{2/d}\sin\left(\frac{k\pi x}{d}\right) \text{ for } k \text{ odd}$$

$$f_k(x) = \sqrt{2/d}\cos\left(\frac{k\pi x}{d}\right) \text{ for } k \text{ even}$$



For the resonator quantization, see, e.g.,A. Blais, R-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, PRA 69, 062320 (2004))

Multimode Spin-boson Hamiltonian

Multimode spin-boson Hamiltonian:

$$H \simeq constant + \sum_{k=1..N_m} \hbar\omega_k a_k^\dagger a_k + \sum_J \frac{\hbar\omega_F}{2}\hat{\sigma}_{z,j} +$$

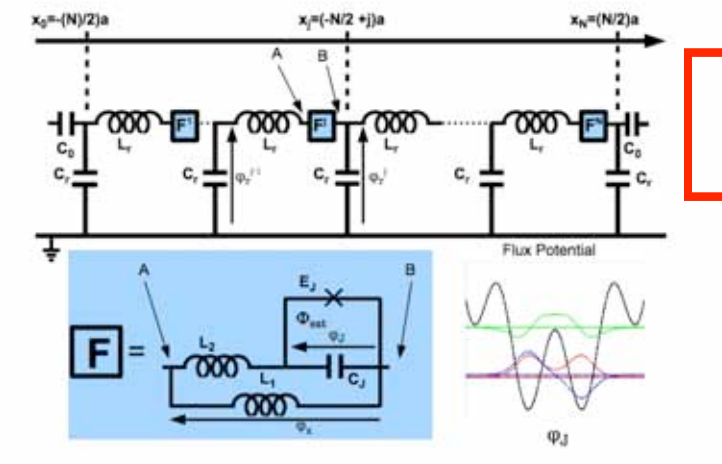
$$\sum_{k=1..N_m} \sum_{j=1}^{N} i\Omega_k \sqrt{\frac{2}{N}} \Delta f_k(x_j) (a_k - a_k^\dagger)\hat{\sigma}_{x,j}$$

Note: reminiscent of Dicke model Hamiltonian

Note: Analogous to the MAGNETIC COUPLING of real Spins to a cavity field

$$H_{int} = \sum_j \vec{\mu}_j \cdot \vec{B}_{cav}$$

Normalized vacuum Rabi frequency



$$\frac{\Omega_{k=1}}{\omega_{k=1}} = g\sqrt{N} = \sqrt{\frac{Z_{vac}}{2Z_r\alpha}}\mu\nu\chi\sqrt{N} \sim 5.7\chi\sqrt{N}$$

Branching ratio (it allows to tune the coupling)

$$\chi = \left(\frac{L_r}{L_1L_r + L_1L_2 + L_2L_r}\right)^{\frac{1}{4}} \frac{L_1}{(L_1+L_2)^{\frac{3}{4}}}$$

$$0 \leq \chi \leq 1$$

Parameters and constants:

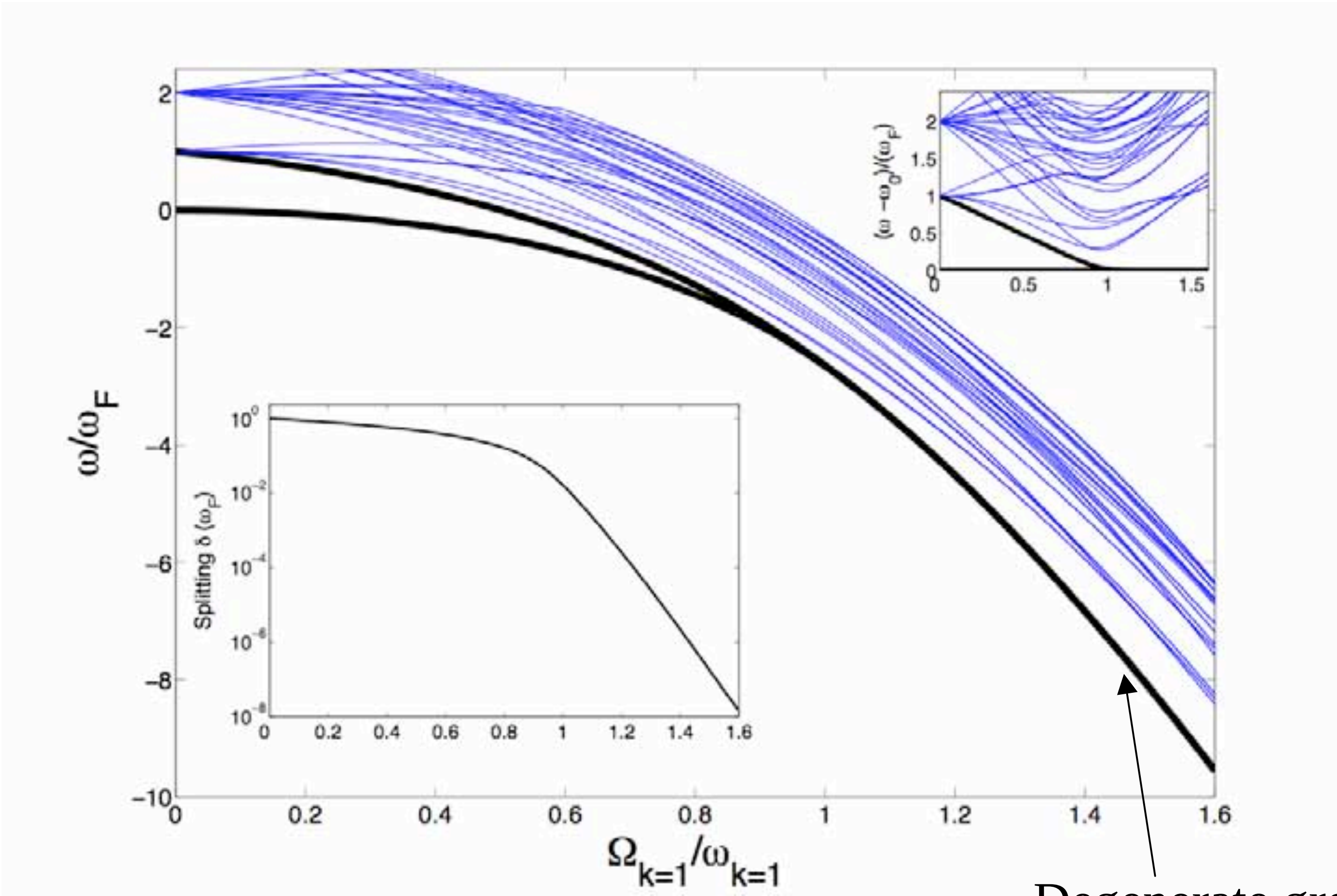
$$Z_r = \sqrt{\frac{L_r}{C_r}} = 50\Omega$$

$$\frac{Z_{vac}}{2\alpha} = \frac{h}{e^2} = R_k \sim 25.8k\Omega$$

$$\mu = \frac{\sin(\frac{\pi a}{2d})}{\frac{\pi a}{2d}}$$

$$\nu = \frac{1}{4\pi}\varphi_{01} \sim \frac{1}{4}$$

Energy spectrum for a finite-size system



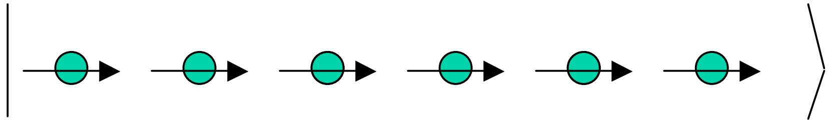
Degenerate ground state

5 artificial Josephson atoms

P. Nataf, CC, Phys. Rev. Lett. 104, 023601 (2010)

2 Degenerate vacua in the ultrastrong coupling limit

$$|G_+\rangle = C_G \Pi_j |+\rangle_j \otimes \Pi_{k_o} e^{+\left(\frac{g\sqrt{2}\; i^{k_o}}{k_o^{1.5}\,\mathrm{sin}(\frac{\pi}{2N})} a^\dagger_{k_o}\right)} |0\rangle_{k_o} \otimes \Pi_{k_e} |0\rangle_{k_e}$$



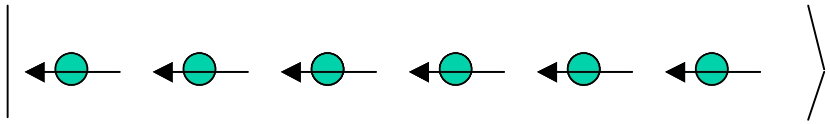
$\otimes \; |\alpha\rangle$

« **Ferromagnetic** » **state** for Josephson atoms

Coherent state for resonator field

$$\hat{\sigma}_{j,x} |\pm\rangle_j = \pm |\pm\rangle_j$$

$$|G_-\rangle = C_G \Pi_j |-\rangle_j \otimes \Pi_{k_o} e^{-\left(\frac{g\sqrt{2}\; i^{k_o}}{k_o^{1.5}\,\mathrm{sin}(\frac{\pi}{2N})} a^\dagger_{k_o}\right)} |0\rangle_{k_o} \otimes \Pi_{k_e} |0\rangle_{k_e}$$



$\otimes \; |-\alpha\rangle$

Degeneracy splitting for finite coupling/size

Finite-size scaling (analytically calculated)

$$\delta = \omega_F \exp\left(-g^2 \beta(N)\right)$$

Frequency splitting

Josephson
Atom
transition
frequency

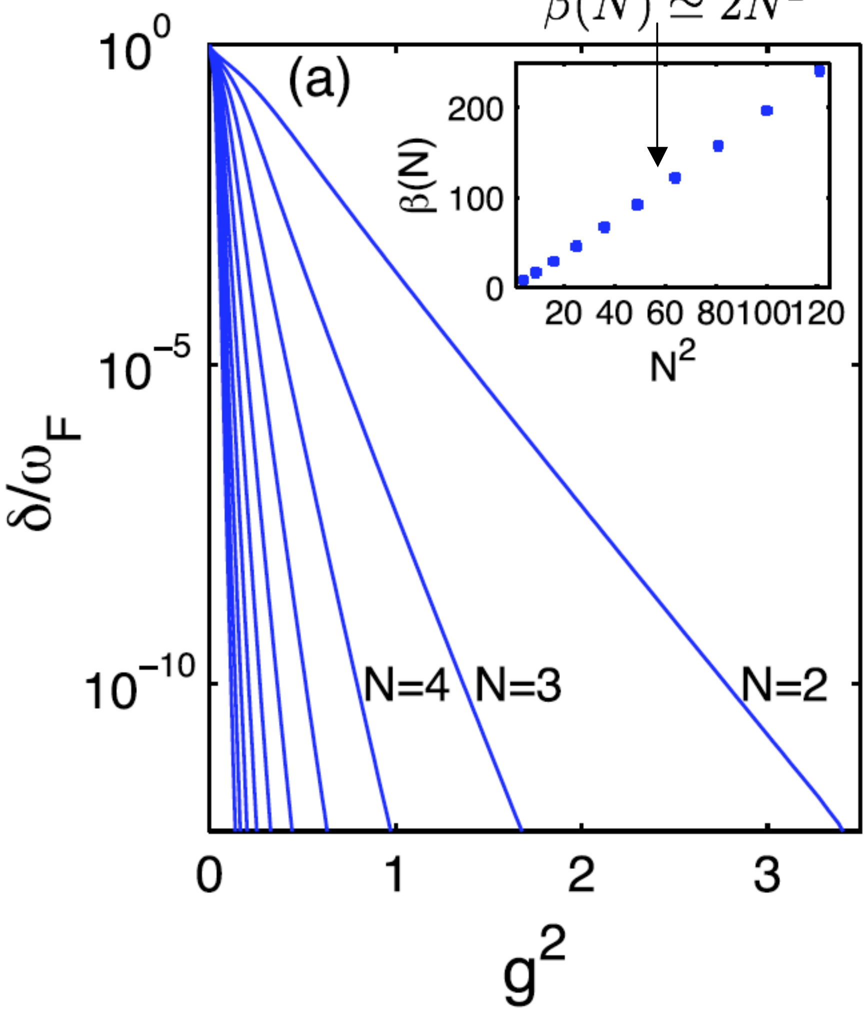
$g =$ Normalized
Vacuum Rabi coupling
(per atom)

P. Nataf, CC, Phys. Rev. Lett. 104, 023601 (2010)

Degeneracy splitting: numerically calculated finite-size scaling

$$\delta = \omega_F \exp\left(-g^2 \beta(N)\right)$$

$$\beta(N) \simeq 2N^2$$



Protection with respect to some kind of local noise sources

Degeneracy protected with respect to the following kind of perturbation:

$$H_{pert} = \sum_j \frac{\hbar \Delta_j}{2} \hat{\sigma}_{z,j} + \frac{\hbar \Lambda_j}{2} \hat{\sigma}_{y,j}$$

No effect up to N-th order perturbation theory:

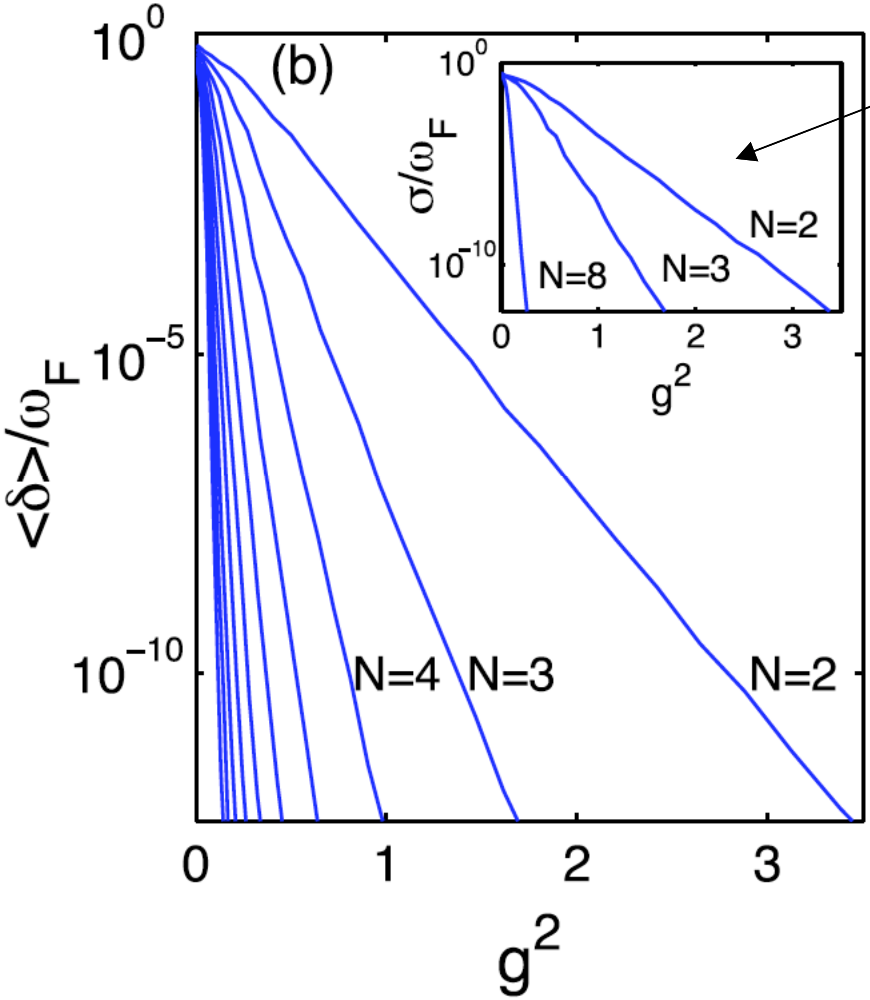
$$\langle G_\pm | H^m_{pert} | G_\pm \rangle = \langle G_\pm | H^m_{pert} | G_\mp \rangle = 0 \text{ with } m \leq N-1$$

(Partial) analogy with protected systems:

B. Douçot, M. V. Feigel'man, L. B. Ioffe, and A. S. Ioselevich, PRB 71, 024505 (2005)

Effect of noise: numerical study

$$\omega_{j,F} = \omega_F + \Delta_j = \omega_F(1 + 0.5\xi_j)$$



Average splitting

Still Exponential !

Variance of splitting

Degenerate vacua as a valuable qubit ???

- Encode quantum information in degenerate vacuum subspace ??

$$|\Psi\rangle = c_+|G_+\rangle + c_-|G_-\rangle$$

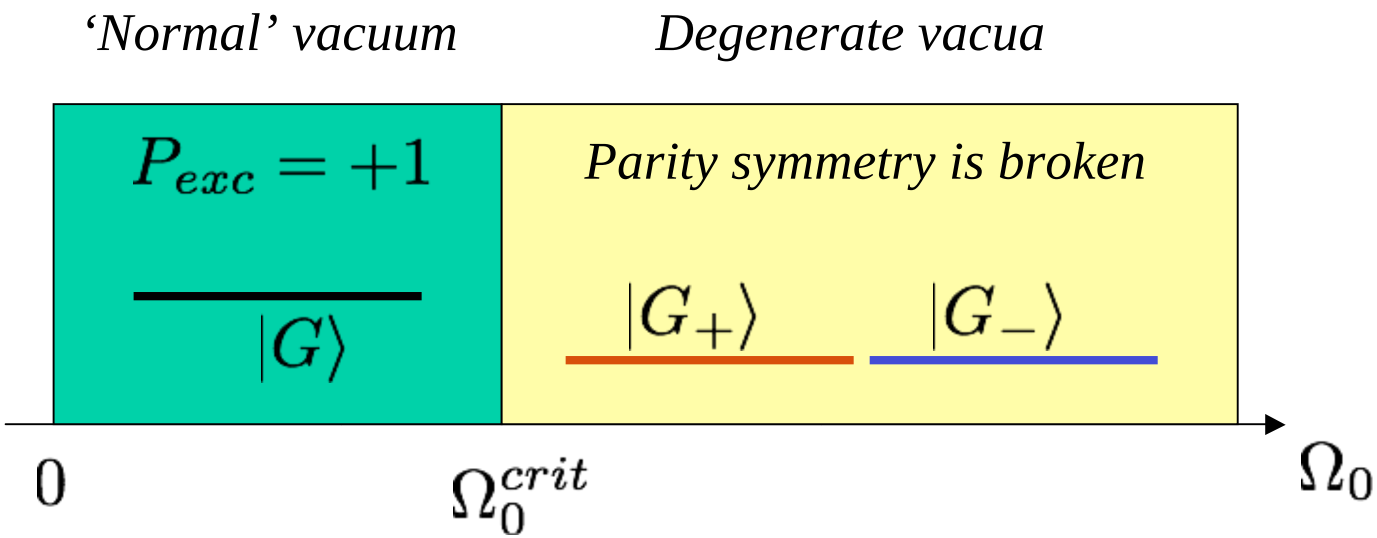
- Protection with respect to some kind of noise

- In the ultrastrong coupling limit, insensitivity to variation of Josephson elements

- Unprotected channel can be used to perform operations and readout

Quantum phase transition in the 'thermodynamic' limit

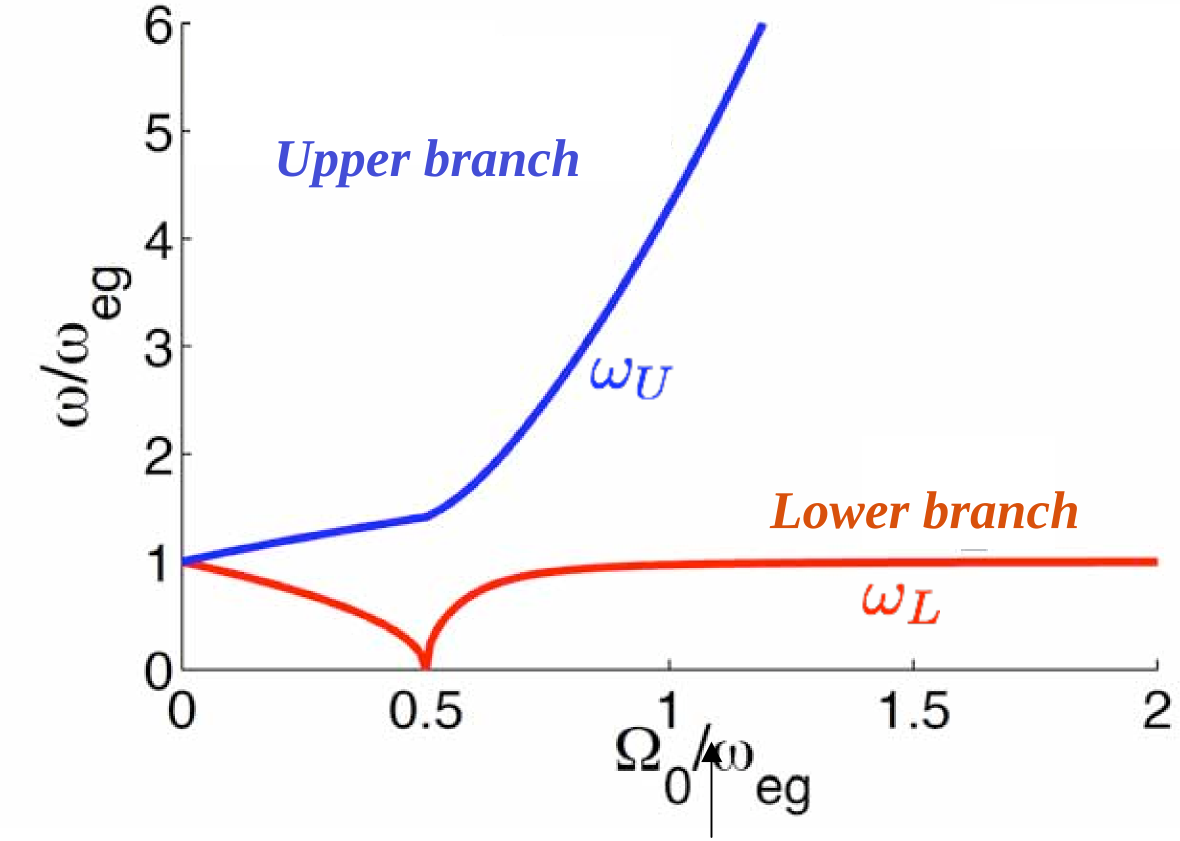
$N \gg 1$ (thermodynamical limit)

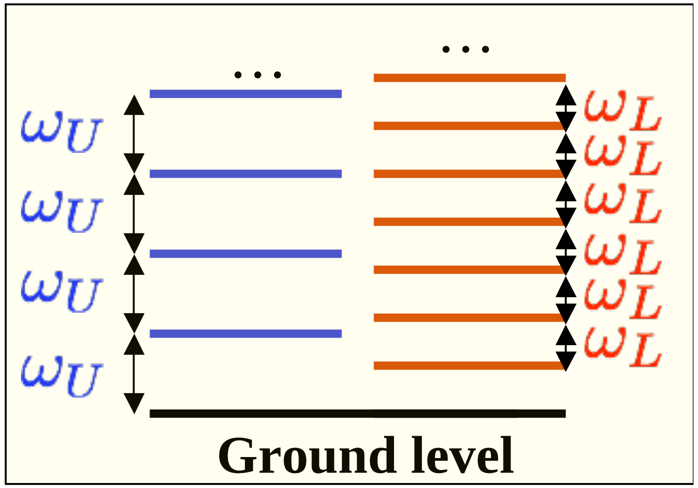


Critical vacuum Rabi frequency: $\Omega_0^{crit} = \frac{1}{2}\sqrt{\omega_{eg}\omega_{cav}}$

Energy of elementary bosonic excitations for *N* >> 1

Single-mode Dicke Model

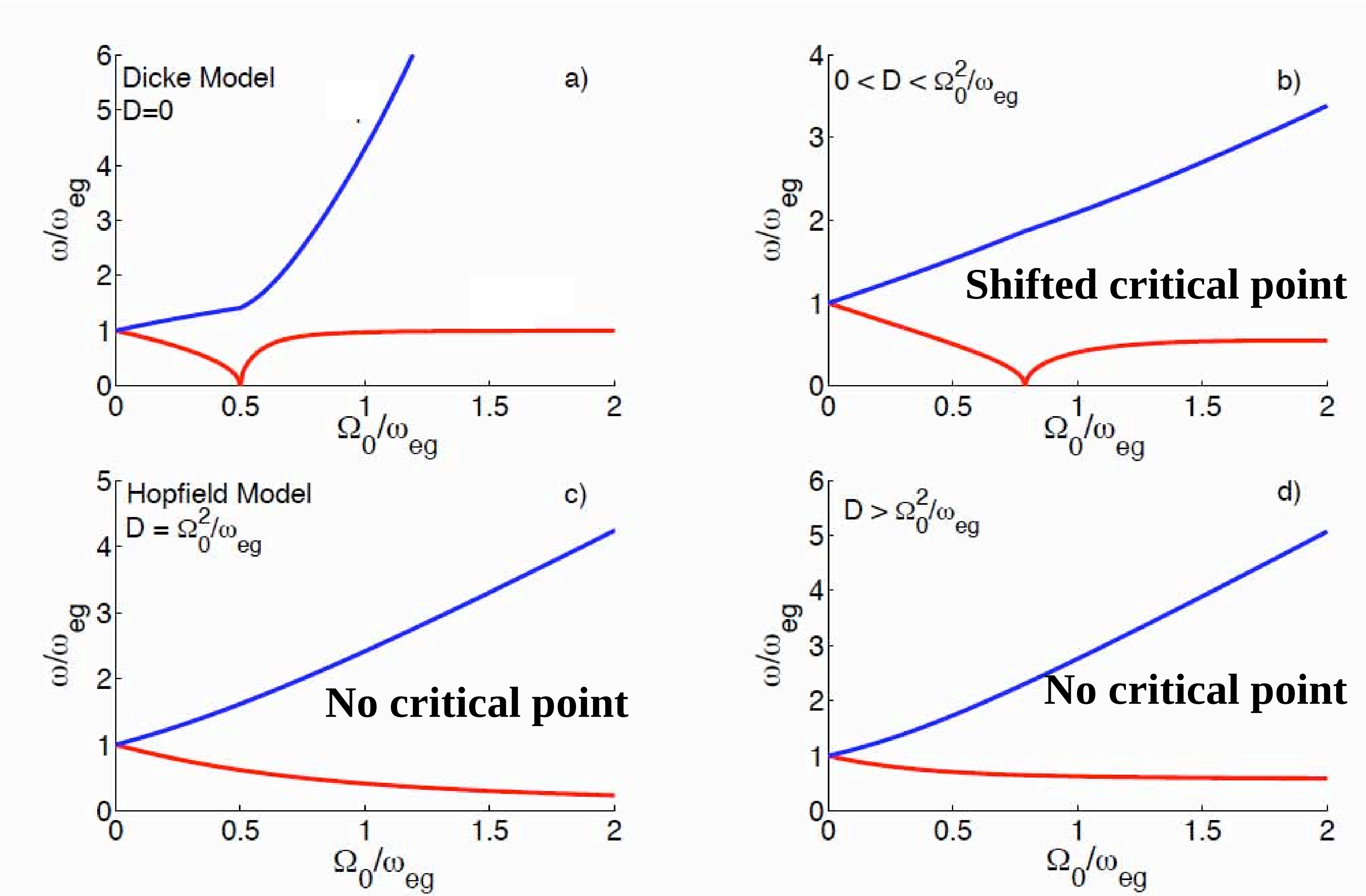




Quantum Critical Point

C. Emary and T. Brandes, Phys. Rev. E 67, 066203 (2003)

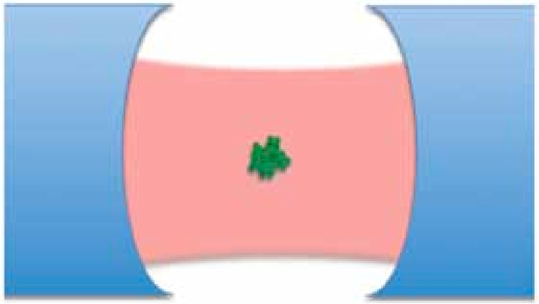
Possible scenarios …



P. Nataf, CC, submitted; preprint arXiv:1006.1801

Electric dipole coupling in cavity QED

$$H = \sum_j \frac{1}{2m}\left(\mathbf{p}_j - q_j \hat{\mathbf{A}}(\mathbf{r}_j)\right)^2 + V_j + H_{cav}$$



Electrical dipole approximation

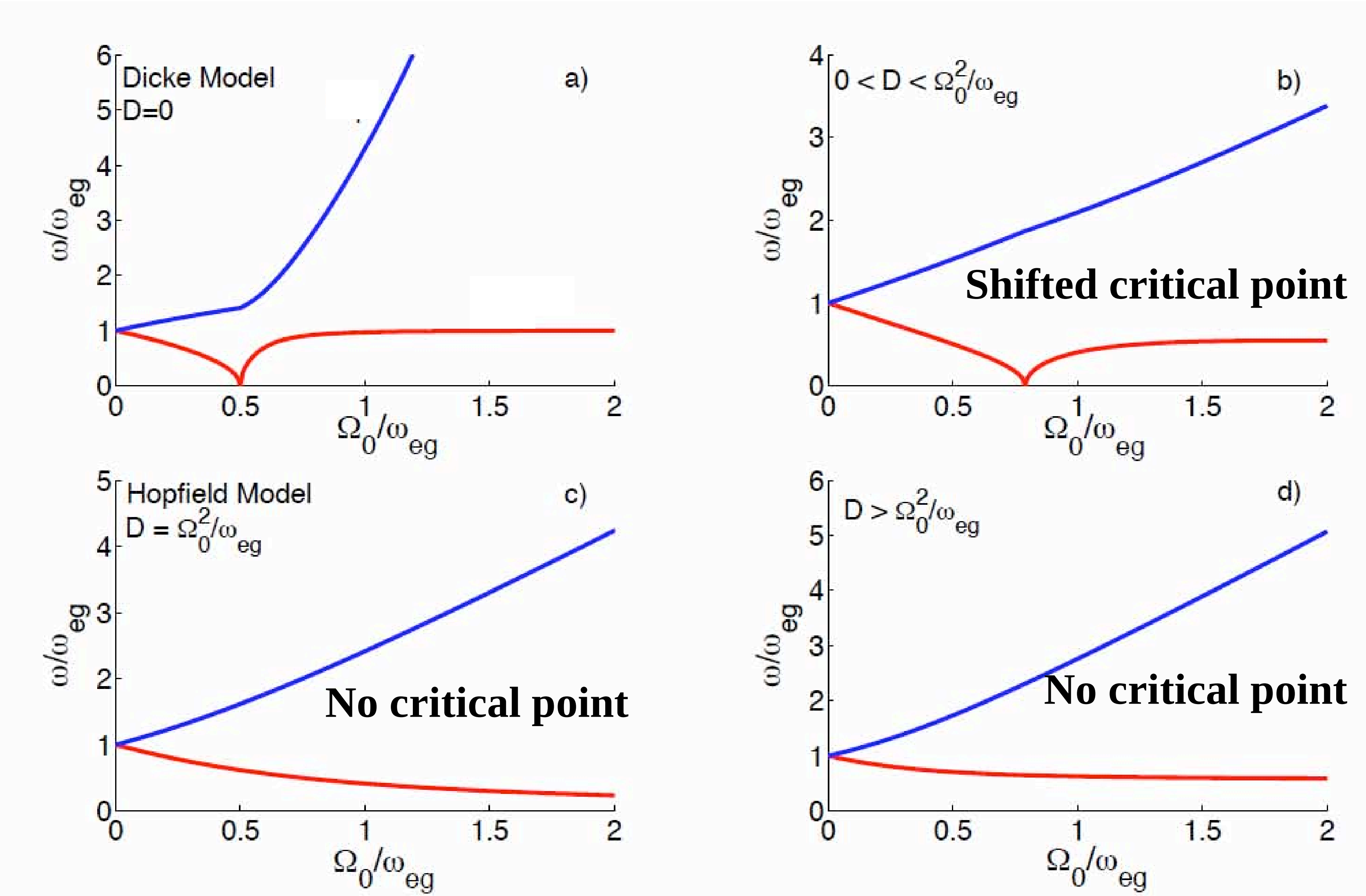
$$\hat{\mathbf{A}}(\mathbf{r}_j) \simeq \hat{\mathbf{A}}_0$$

$$H_{int} = -\sum_j \frac{q_j}{m}\mathbf{p}_j \cdot \hat{\mathbf{A}}_0 \rightarrow \Omega_0(a + a^\dagger)\frac{1}{\sqrt{N}}\sum_j \hat{\sigma}_{x,j} = \Omega_0(a + a^\dagger)(b + b^\dagger)$$

Bosonic excitation

$$H_{A^2} = \sum_j \frac{q_j^2}{2m}\hat{\mathbf{A}}_0^2 \rightarrow D(a + a^\dagger)^2$$

The value of the A^2 term is crucial !



Excitation spectra

Within the Hopfield-Bogoliubov (or Holstein-Primakoff) approaches,
the frequency spectrum of the normal phase is obtained by diagonalizing:

$$\mathcal{M} = \begin{pmatrix} \omega_{cav} + 2D & -i\Omega_0 & -2D & -i\Omega_0 \\ i\Omega_0 & \omega_{\rm eg} & -i\Omega_0 & 0 \\ 2D & -i\Omega_0 & -(\omega_{cav} + 2D) & -i\Omega_0 \\ -i\Omega_0 & 0 & i\Omega_0 & -\omega_{\rm eg} \end{pmatrix}.$$

$$Det(\mathcal{M}) = \omega_{\rm eg}\omega_{cav}(\omega_{\rm eg}(4D + \omega_{cav}) - 4\Omega_0^2).$$

$Det(\mathcal{M}) = 0 \Rightarrow$ Gapless excitation = Quantum Critical Point

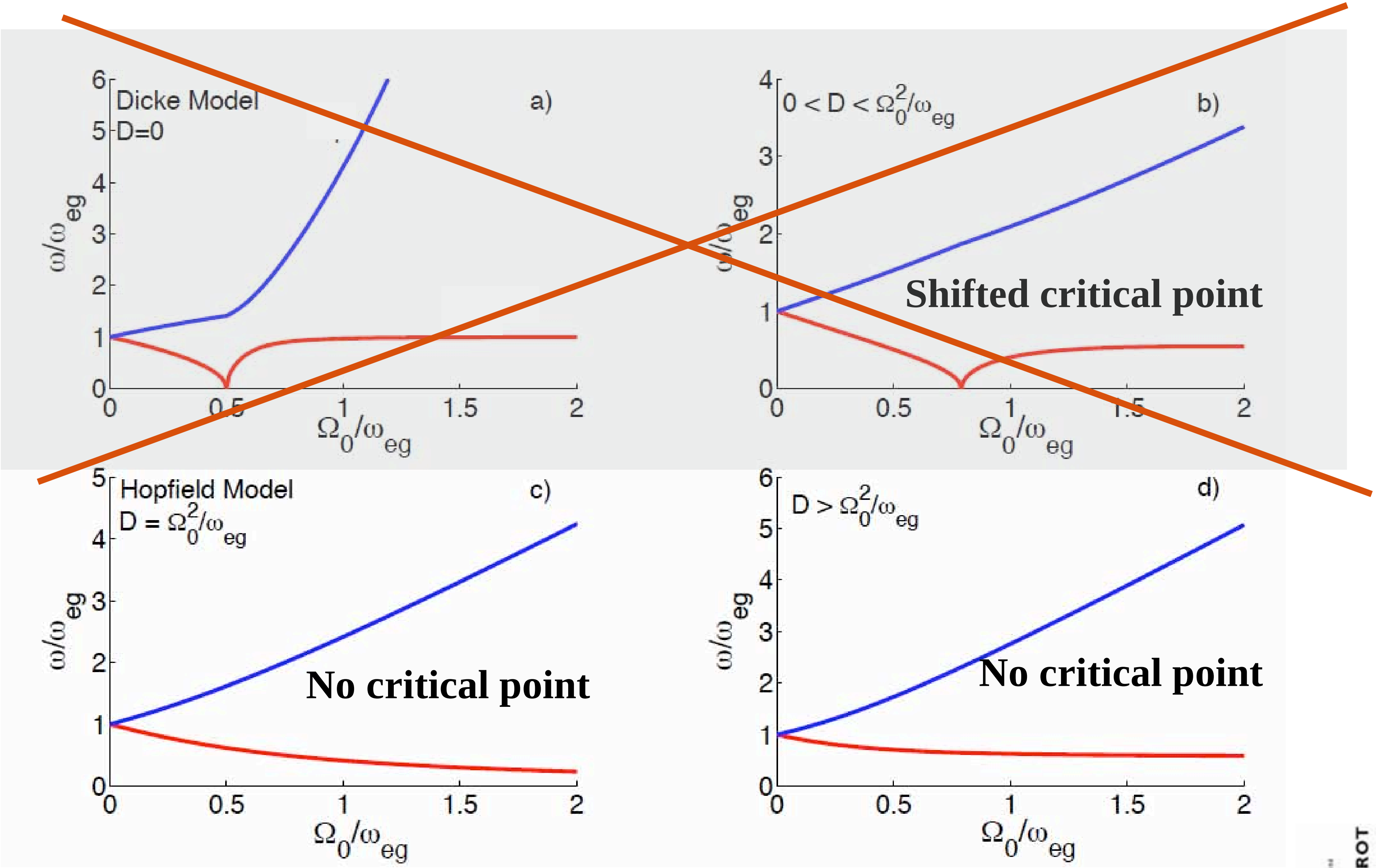
Cavity QED: constraints by oscillator strength sum rule

Thomas-Reiche-Kuhn oscillator strength sum rule

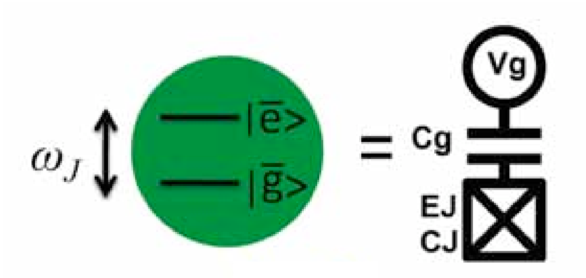
$$D \geq \Omega_0^2/\omega_{eg}$$

Note: for electric dipole transitions

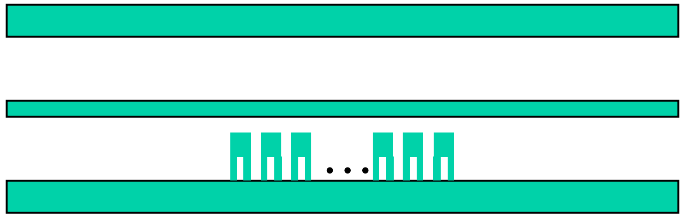
Diagrams for cavity QED (electric dipole coupling)



N Cooper pair boxes *capacitively* coupled to a resonator



$$n_g = \frac{C_g}{2e} V_g = \frac{1}{2}$$



$$H = \hbar\omega_{\rm res} a^\dagger a + \sum_{j=1}^{N} \left\{ 4E_c \sum_{n\in\mathbb{Z}} (n - (\hat{n}_{ext})_j)^2 |n\rangle\langle n|_j - \frac{E_J}{2} \sum_{n\in\mathbb{Z}} (|n+1\rangle\langle n| + |n\rangle\langle n+1|)_j \right\}$$

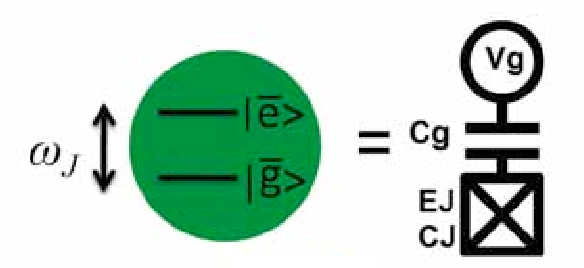
$$(\hat{n}_{ext})_j = \frac{C_g}{2e}(V_g + \hat{V})_j$$

Charge coupled to Cooper pair box

$$\hat{V} = \mathcal{V}(a + a^\dagger)$$

Resonator quantum voltage (single-mode)

Interaction terms and analogy with electric dipole in cavity QED



$$(\hat{n} - (\hat{n}_{ext})_j)^2 \Leftrightarrow \left(\mathbf{p}_j - q_j \hat{\mathbf{A}}(\mathbf{r}_j)\right)^2$$

$$H_{coupl} = -i4E_c \frac{C_g}{2e} \mathcal{V}(a + a^\dagger) \sum_{j=1}^{N} (|\bar{\mathrm{e}}\rangle \langle \bar{\mathrm{g}}|)_j \; + \; \mathrm{h.c.} = -i\hbar\bar{\Omega}_0 (a + a^\dagger)(b + b^\dagger)$$

$$H_{V^2} = \sum_{j=1}^{N} 4E_c (\frac{C_g}{2e})^2 \mathcal{V}^2 (a + a^\dagger)^2 = \hbar \bar{D} (a + a^\dagger)^2.$$

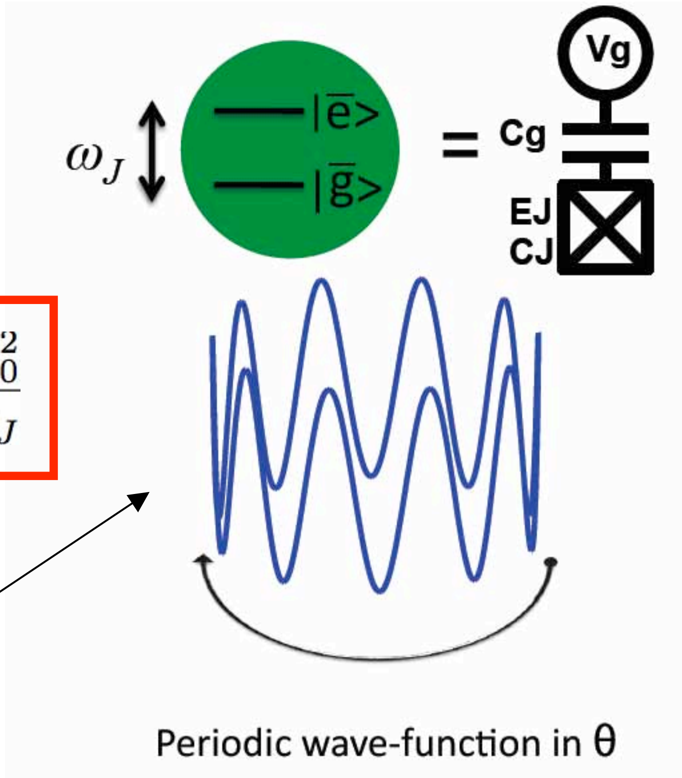
Analogous to A^2-term

Relation between V^2 term and vacuum Rabi frequency

For the considered system: $\bar{D} = \frac{\bar{\Omega}_0^2}{\omega_J} \frac{E_J}{4E_c}.$

Strong charging energy limit: $\frac{E_J}{4E_C} \ll 1 \Rightarrow \bar{D} \ll \frac{\Omega_0^2}{\omega_J}$

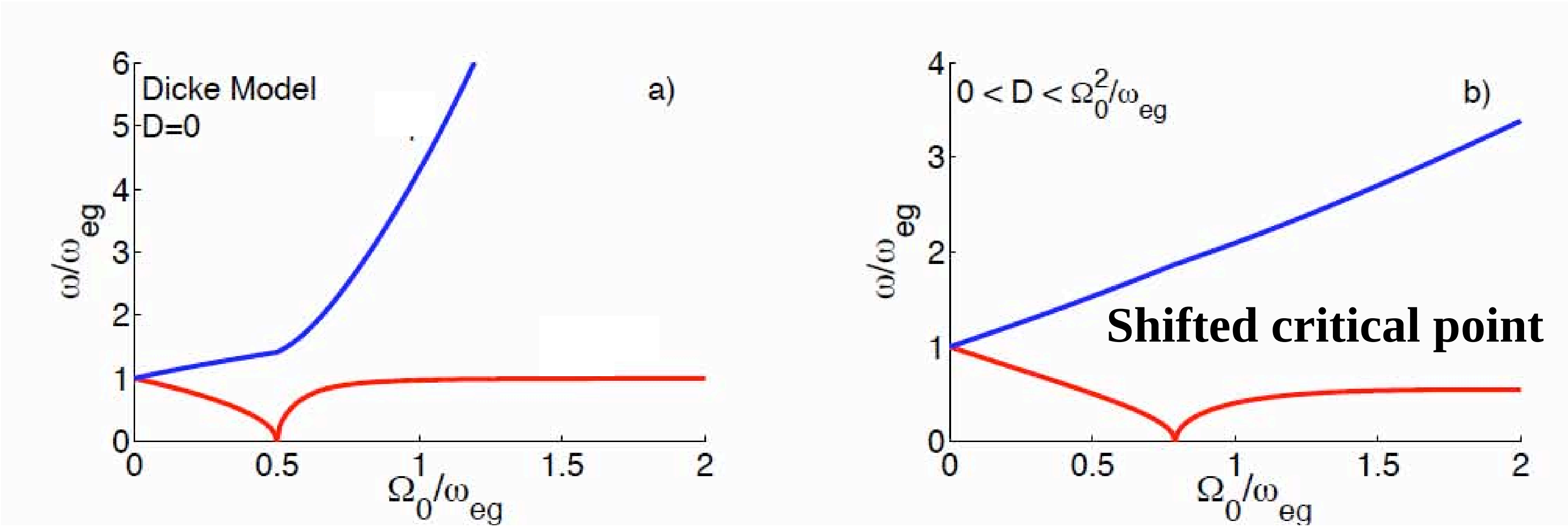
Analogous of TRK sum rule is violated due to compact 1D wavefunction topology !



Quantum critical coupling: $\bar{\Omega}^c = \frac{\sqrt{\omega_{res}\omega_J}}{2\sqrt{1 - \frac{E_J}{4E_c}}}.$

- P. Nataf, CC, submitted; preprint arXiv:1006.1801

Diagrams for Cooper pair boxes capacitively to resonator



Note: for other Josephson atoms *capacitively* coupled to a resonator the quantum phase transition can disappear ….

Conclusions

- Ultrastrong coupling regime: manipulating the QED vacuum

- Quantum phase transitions in circuit QED

- Vacuum degeneracy and finite-size scaling properties:
  qubits based on degenerate vacua ?

- Inductive and capacitive coupling

- Circuit QED is not only analogous to cavity QED: fundamental differences can occur !