QEC when the noise is biased

- Biased noise qubit: coupling with the env is predominantly along one axis

Eg: in the case when the env couples mostly to z axis the errors are dominated by phase-flips while the prob of non-deph. errors is small.

We define bias, $\eta = \frac{P_z}{P_x + P_y}$ $\quad p \rightarrow$ prob of error

- Naively, EC should become more eff if only one type of error dominates.

- However, reality is more comp.
  Even if errors in the idle qubit are biased, implementing unitary operations which do not commute with the dominant error will unbias the channel.

- One example of such a gate is the CNOT gate, which is important for extracting error syndromes.

$$H_{cx} = V\left[\left(\frac{I+Z}{2}\right)\otimes I + \left(\frac{I-Z}{2}\right)\otimes X\right]$$

$$U(T) = \left(\frac{1+Z}{2}\right)\otimes I + \left(\frac{1-Z}{2}\right)\otimes X \qquad VT = \frac{\pi}{2}$$

If an phase-flip error happens at τ on the target qubit

$$\tilde{U}(T) = U(T-\tau)\, I\otimes Z\, U(\tau)$$

$$= I\otimes Z\,\underbrace{\left[e^{iV(T-\tau)(I-Z)\otimes X}\right]}_{\text{Error channel of target is unbiased!}} U(T)$$

So is error correction with biased noise hopeless? No!

A&P (Aliferis & Preskill PRA 78 052331 (2008)) came up with a scheme for ft QEC with biased noise qubits using gates that trivially preserve the bias.
(operations)

Physical level $\{ \text{CPHASE}, P_{|+\rangle}, M_x, \boxed{ZZ(\theta)}^{*} \} \Rightarrow$ Set is trivially biased *

$p_z \gg p_x, p_y$

$\Downarrow$ Rep code to correct for phase-flips

$\{ \overline{CX}, \overline{P}_{|0\rangle}, \overline{P}_{|+\rangle}, \overline{M}_x, \overline{M}_z, \overline{P}_{|+i\rangle}, \overline{P}_{|T\rangle} \}$

(symm. noise) (overall red noise strength)

$\Downarrow$

CSS code universal comp.

<u>Results</u>: Threshold improves by a factor of ~4 for bias 10^4 (rig. lower bound)

$ZZ(\theta)$ gate reduces overhead in magic state dist by a factor ~ 3-4

<u>Key issues</u> [1] The threshold is still small = 0.25% ($\eta = 10^4$)

[2] The uncorrected error grows rapidly, limiting the improvement in thresholds for realistic biases (~100)

* $P_{|+\rangle}$, $|+\rangle$ is eigenstate of X, so it is only effected by phase-flips Z (or $Y = iXZ$)

* M_x, it's a measurement and gives a classical output, and only Z errors effect these.

* $ZZ(\theta)$ gate for eff magic-state dist, Webster, Bartlett & Poulin PRA 92, 062309 (2015)

$ZZ(\theta) = e^{i\theta Z \otimes Z/2}$

$ZZ(\pi/2) = \text{CPHASE}$

Possible ways to overcome the challenges.

(i) Make the gadgets more compact

- The gadgets could be made more compact if a bias-preserving CNOT was available.
- We realized that even though a bias-preserving CNOT was impossible with strictly two dim system, we could implement it in CV systems (a particularly conv. system is the stabilized Kerr cat)

(ii) Make better codes that are tailored to the bias in the system.
(Phys. Rev. Applied 8, 064004 (2017); PRL 120, 050505 (2018))

Results & Future Goals (arXiv 1905.00450)

1. By using the bias preserving CNOT gates (possible with cats) the threshold improves to 0.43% for bias 10^4 (concat. scheme of AP) (rig. lower bound)
2. Bias of 10^4 could be reached with cats with mean photon number $\lesssim 10$
3. Overheads improved (Clifford gate counts as well as magic state dist → idea by Webster, Bartlett & Poulin, PRA 92 062309 (2015))
4. Numbers can be improved further using better decoding.
5. Implementation of bias preserving CCNOT gate with only 4 wave mixing (naive extention of CNOT idea would have req. 5 wave mixing)
6. Next step: Tailored surface codes with cats.

Non-linear oscillator

$$H = \omega_r a^\dagger a - K a^{\dagger 2} a^2$$

- Eigenstates → photon number Fock states
- Also the hamiltonian of a transmon (fourth-order expansion of the cosine) *
- Commutes with photon parity $e^{i\pi a^\dagger a}$ so the spectrum can be divided into even & odd parity eigenspace.

Lab frame　　　　Frame rotating at ω_r

- In the transmon $|0\rangle$ (or $|g\rangle$) and $|1\rangle$ (or $|e\rangle$) states define the qubit.
- The gap which sep. the qubit subspace from the rest of the Hilbert space $= 2K = \omega_{gap}$
- Single photon drive $\varepsilon(a^\dagger + a)$ at ω_r leads to rotations $|0\rangle \rightleftharpoons |1\rangle$
- Because of large ω_{gap}, $|1\rangle \to |2\rangle$ transition is not possible.

Nonlinear Oscillator + Two Photon Drive

$$H = \omega_r a^\dagger a - K a^{\dagger 2} a^2 + P\left(a^{\dagger 2} e^{-2i\omega_r t + i\phi} + a^2 e^{2i\omega_r t - \phi i}\right)_{(RWA)}$$

- Oscillator exchanges two photons at a time with the drive
- $\phi \rightarrow$ phase of the drive w.r.t. a ref. frame
- In the rotating frame,

$$H = -K a^{\dagger 2} a^2 + P\left(a^{\dagger 2} e^{i\phi} + a^2 e^{-i\phi}\right) \quad -①$$

$$= -K\left(a^{\dagger 2} - \frac{P}{K} e^{-i\phi}\right)\left(a^2 - \frac{P}{K} e^{i\phi}\right) + \frac{P^2}{K}$$

$$= -K\left(a^{\dagger 2} - \alpha^{*2}\right)\left(a^2 - \alpha^2\right) + \frac{P^2}{K}$$

$$\alpha = \sqrt{\frac{P}{K} e^{i\phi}}$$

__Obs__ : $(a^2 - \alpha^2)|\pm\alpha\rangle = 0\,|\pm\alpha\rangle \Rightarrow H|\pm\alpha\rangle = \frac{P^2}{K}|\pm\alpha\rangle$

$\Rightarrow |\pm\alpha\rangle$ are deg. eigenstates of H

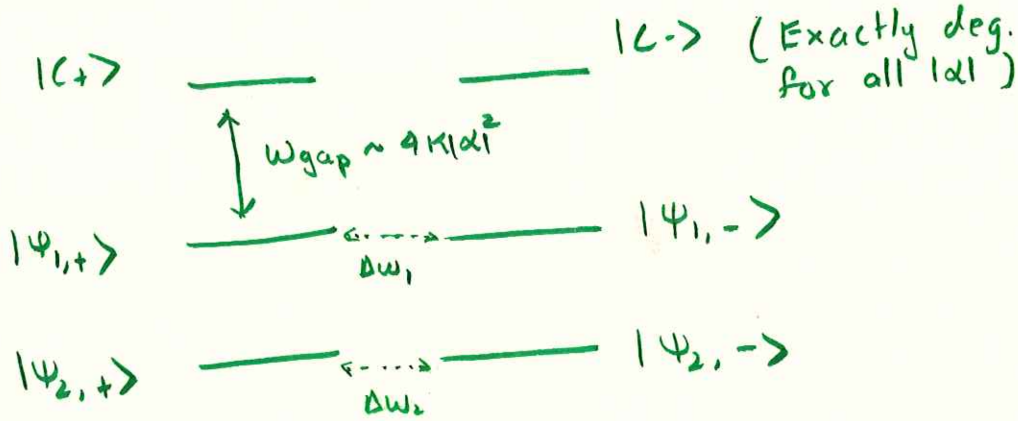
or

$$\boxed{|C_\pm\rangle = N_\pm\left(|\alpha\rangle \pm |-\alpha\rangle\right) \; ; \; N_\pm = \frac{1}{\sqrt{2(1 \pm e^{-2|\alpha|^2})}}}$$

are the deg. eigenstates of H!

__Notes__ :
- As $\alpha \rightarrow 0$, $|C_\pm\rangle \longrightarrow |0\rangle, |1\rangle$
- For α large $N_\pm \sim 1/\sqrt{2}$
- $|C_\pm\rangle$ are even & odd parity states resp.
- H commutes with parity operator.

Eigenstates & Energies



- Cat subspace sep. from the rest of the Hilbert space by a large energy gap

$$\left.\begin{aligned} \omega_{gap} &\sim 4K|\alpha|^2 \\ &= 4P \end{aligned}\right\} \text{For large } |\alpha|$$

For $|\alpha| \to 0$, $\omega_{gap} \to 2K$.

(In superconducting ckts, $\omega_{gap} \sim 10-100\,\text{MHz}$)

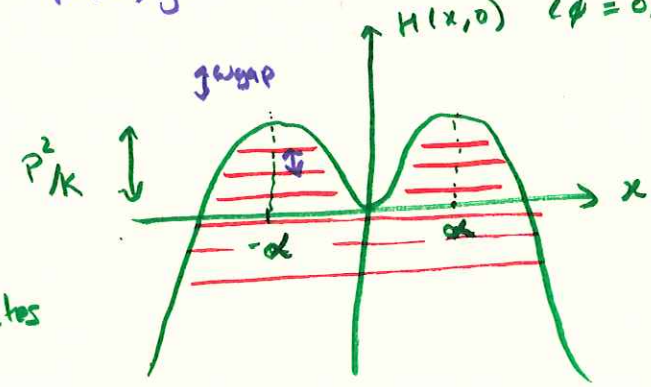
- $N \sim \frac{|\alpha|^2}{4}$ pairs of quasi-deg excited st.

$$\Delta\omega_i \sim E_i e^{-2|\alpha|^2}$$
$$(i = 1 - N)$$

Semi-classical interpretation for large $|\alpha|$

In ① replace $a, a^\dagger$ by $x+iy$; $x-iy$

$H(x,y) \to$ inverted double well



No. of confined states $= \dfrac{P^2}{K\omega_{gap}} \sim \alpha^2/4$

→ Two disp. Harmonic osc. for large α

→ States with energy <0 are not in the wells & are delocalized.

→ Each well is approx by $H' = 4K\alpha^2 a^\dagger a$

→ states with energy >0 are localized in the wells.

States confined in left well → displaced Fock states

→ $\mathcal{D}(-\alpha)|0\rangle, \mathcal{D}(-\alpha)|1\rangle, \ldots$

States confined in right well → $\mathcal{D}(\alpha)|0\rangle, \mathcal{D}(\alpha)|1\rangle, \ldots$

Parity eigenstates of the Hamilt.
(ignore normal. for conv.)

$$\left[\mathcal{D}(\alpha) \pm \mathcal{D}(-\alpha)\right]|0\rangle = |C_\pm\rangle$$
$$\left[\mathcal{D}(\alpha) \mp \mathcal{D}(-\alpha)\right]|1\rangle = |\psi_{1,\pm}\rangle$$
$$\left[\mathcal{D}(\alpha) \pm \mathcal{D}(-\alpha)\right]|2\rangle = |\psi_{2,\pm}\rangle$$

Tunnel splitting between $|\psi_{i,\pm}\rangle$ can be estimated with WKB approx.

$$\Delta\omega_i \propto \langle i|\mathcal{D}^\dagger(\alpha)\,\mathcal{D}(-\alpha)|i\rangle$$
$$\Downarrow$$
exp. small.

Consequences

- Because of large ω_{gap}, it can be used to encode bosonic cat-qubit.
- The bosonic qubit is realized <u>in the nonlinear mode</u>
- Larger the gap or larger the nonlinearity, faster gate operations.
- Kerr is feature not a bug!
- ϕ is a continuous deg. of freedom and decides orientation of cat in phase space
- Hence, each oscillator encodes a cont. infinitely large family of cat qubits.

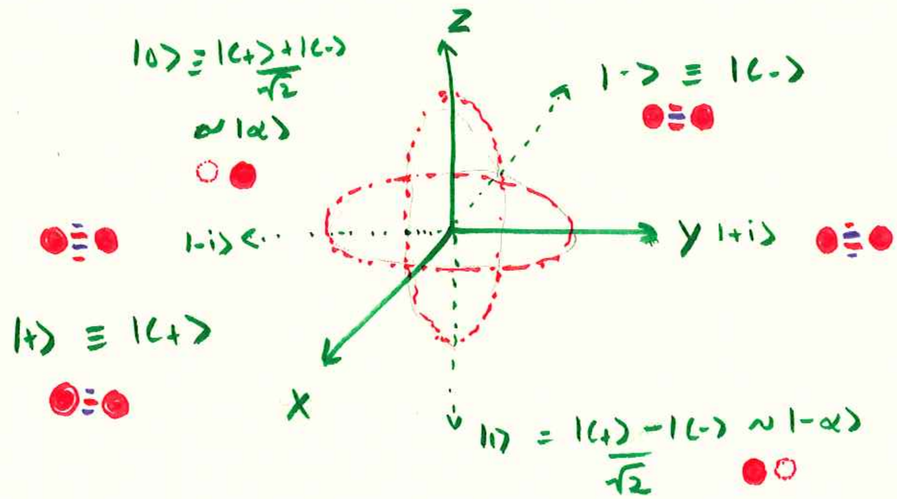
} Imp for bias-pres CX gate

- $|\alpha|$ is also a cont. parameter → Imp for low-weight CCX gate
- Pairwise deg. subspace imp to preserve bias inspite of leakage.

Bloch sphere

Qubit defined with $\phi = 0$ & amp. α

$$|C_\pm\rangle = N_\pm \left(|\alpha\rangle \pm |-\alpha\rangle\right)$$



$$X = |C_+\rangle\langle C_+| - |C_-\rangle\langle C_-|$$

$$Z = |C_+\rangle\langle C_-| + |C_-\rangle\langle C_+|$$

Box 1: Physical realization of the Hamiltonian in CQED

- Josephson-parametric amp.
  ($JJ \Rightarrow$ Kerr, flux pumping $\Rightarrow$ two phot. drive)
- Single junction in a 3D cavity
  ($JJ \Rightarrow$ Kerr, two equally red & blue det. drives undergo four wave mixing via the Kerr term to give the two-photon drive)
  (Issues $\rightarrow$ Stark shifts, limited drive strengths)
- **SNAIL** $\rightarrow$ superconducting, nonlinear asymmetric inductive element
  $\rightarrow$ Both third & fourth order nonlin.
  $$\omega_r a^\dagger a + g_4 (a^\dagger + a)^4 + g_3 (a^\dagger + a)^3$$
  A single drive at $2\omega_r$ gives the two photon drive due to the $g_3 (a^\dagger + a)^3$ term.

Current expt. setup @ Devoret

Initialization of the Qubit

$$-Ka^{\dagger 2}a^2 \xrightarrow{P\,\text{ramp vs. } t} -Ka^{\dagger 2}a^2 + P(a^{\dagger 2}+a^2)$$

Adiabatically ramp up the two-photon drive

Undriven oscillator in vacuum $|0\rangle$ evolves to $|C_+\rangle$

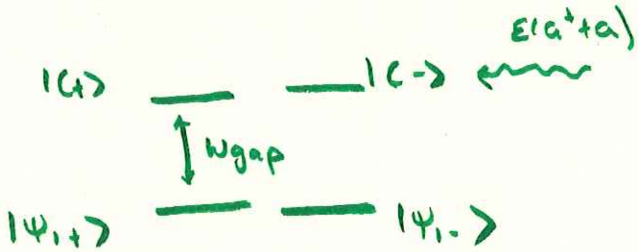
undriven oscillator in Fock state $|1\rangle$ evolves to $|C^-\rangle$

For adiabaticity

Ramp-up time $\gg 1/K$

(Larger K is better)

Coupling to the cat-qubit



- Single photon drive at ω_r

$$a|C_+\rangle = \alpha\sqrt{\frac{1-e^{-2\alpha^2}}{1+e^{-2\alpha^2}}}\,|C_-\rangle = \alpha r|C_-\rangle$$

$$a|C_-\rangle = \frac{\alpha}{r}|C_+\rangle \;;\; r = \sqrt{\frac{1-e^{-2\alpha^2}}{1+e^{-2\alpha^2}}}$$

- a, restricts transitions within the cat subspace

$$a^\dagger|C_\pm\rangle = \alpha r^{\mp}|C_\mp\rangle + \boxed{|\psi_{1,\mp}\rangle}$$

(note the change in parity)

- $a^\dagger$ can cause transitions to excited states
- However, the energy req. to go from $|C_+\rangle$ to $|\psi_{1,-}\rangle$ or $|C_-\rangle$ to $|\psi_{1,+}\rangle$ is ω_{gap} (in rotating frame) or $\omega_r-\omega_{gap}$ (in lab frame)
- Amount of virtual excitation $\left(\frac{\varepsilon}{\omega_{gap}}\right)^2$

- A drive at ω_r <u>cannot</u> therefore cause out-of-cat-subspace transitions.
- Consequently, it is possible to simply project the operators a, a^{+} in the cat subspace

$$a = \alpha\left(\frac{r+r^{-1}}{2}\right)Z + i\alpha\left(\frac{r-r^{-1}}{2}\right)Y$$

$$a^{+} = \alpha\left(\frac{r+r^{-1}}{2}\right)Z - i\alpha\left(\frac{r-r^{-1}}{2}\right)Y$$

$r \to 1$ for large α & so,

$$\boxed{a, a^{+} \sim \alpha Z \mp i\alpha e^{-2\alpha^2} Y}$$

for small α, $\alpha \to 0$

$$a, a^{+} \sim (Z \mp iY)/2$$

- Action of a single photon drive applied with a phase θ w.r.t. the two photon drive

$$H = -Ka^{+2}a^{2} + P(a^{+2}+a^{2}) + J(ae^{-i\theta} + h.c)$$

$$\equiv \alpha\cos\theta\, J(r+r^{-1})Z + \alpha\sin\theta\, J(r-r^{-1})Y$$

& for large α

$$\boxed{H \sim 2\alpha\cos\theta\, J Z - 2\alpha\sin\theta\, J e^{-2\alpha^2} Y}$$

& For small α

$$\sim (2JZ\cos\theta - 2\alpha\sin\theta\, JY)/2$$

For large α, the single photon drive couples pref. with Z, whereas for small α it is symm. This cont. transition has been obs. in experiments @ Devoret Lab

$Z(\theta)$, $ZZ(\theta)$ gates

$Z(\theta)$: $H = -K a^{\dagger 2} a^2 + P(a^{\dagger 2} + a^2) + J(a^\dagger + a)$

$$\equiv \alpha J(\gamma + \gamma^{-1}) Z \qquad (J \ll \omega_{gap})$$

Evolve for time τ

$$U = e^{-i\theta Z}$$

$$\theta = \alpha J(\gamma + \gamma^{-1})$$

$$\to 2\alpha J \quad (\text{large } \alpha)$$

$ZZ(\theta)$: $H = -K a^{\dagger 2} a^2 + P(a^{\dagger 2} + a^2) - K b^{\dagger 2} b^2 + P(b^{\dagger 2} + b^2)$
$+ J(a^\dagger b + a b^\dagger)$

$$\equiv J\alpha^2(\gamma^2 + \gamma^{-2}) Z_1 Z_2 + J\alpha^2(\gamma^2 - \gamma^{-2}) Y_1 Y_2$$

$$\sim 2J\alpha^2 Z_1 Z_2 \qquad (\text{large } \alpha)$$

gate speed depends on α & J

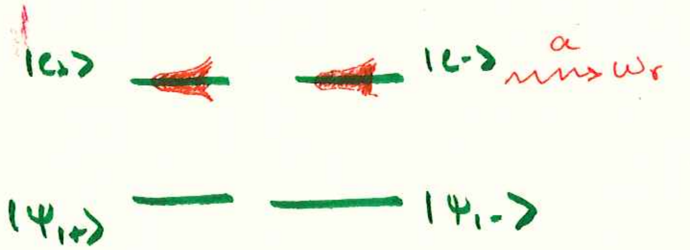
and is limited by ω_{gap}

Kerr gate ($e^{i\frac{\pi}{4}X}$)

- stabilization must be turned off to do $e^{i\pi X/4}$
- $H = -K a^{\dagger 2} a^2 \underbrace{- K a^\dagger a}_{\text{fixes ref frame.}}$, $\frac{|C^+\rangle \pm i|C^-\rangle}{\sqrt{2}} \longleftrightarrow \frac{|C^+\rangle \mp i|C^-\rangle}{\sqrt{2}}$

Effect of noise

1. 1 Photon loss – Single photon loss is the dominant error channel.



Single photon loss at rate k

$k \ll \omega_{gap}$

photons are lost at freq ω_r &

$$a|C_\pm\rangle = \alpha r^{\pm 1}|C_\mp\rangle$$

$$a = \alpha\left(\underbrace{\frac{r+r^{-1}}{2}}_{\approx 1}\right)Z + i\alpha\left(\underbrace{\frac{r-r^{-1}}{2}}_{\sim e^{-2\alpha^2}}\right)Y$$

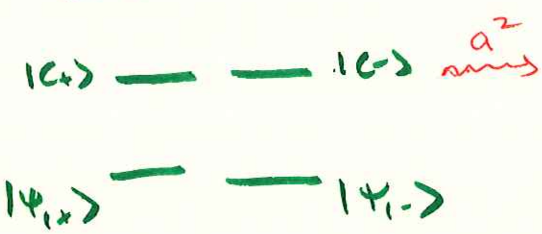
Error channel: $\boxed{\mathcal{E}(\rho) = \sum_{i=1,2} M_i \rho M_i^\dagger, \quad M_1 = (\lambda_1 I + \lambda_x X), \; M_2 = \lambda_z Z - i\lambda_y Y}$

$$\lambda_1 \sim \sqrt{1-p}, \quad \lambda_z \sim \sqrt{p}, \quad \lambda_x \sim p e^{-2\alpha^2}, \quad \lambda_y \sim \sqrt{p}\, e^{-2\alpha^2}$$

p = prob. of phot. loss

$= k\alpha^2 t$

2. 2-photon loss Photons lost in pairs



$$a^2|C_\pm\rangle = \alpha^2|C_\pm\rangle$$

$$\boxed{\mathcal{E}(\rho) = I\rho I} \quad (*)$$

$\boxed{\text{Small amount of photon dissipation useful to autonomously correct leakage errors}}$

(*)

$$\dot{\rho} = -i\,[H, \rho] + \kappa_2 \mathcal{D}[a^2]\rho$$

$$H = -K a^{\dagger 2} a^2 + P(a^{\dagger 2} e^{i\phi} + a^2 e^{-i\phi})$$

Effective Hamilt in the q. jump approach

$$H_{eff} = H - \frac{i}{2}\kappa_2 a^{\dagger 2} a^2$$

$$\& \quad \dot{\rho} = -i\left[H_{eff}\,\rho - \rho H_{eff}^{\dagger}\right] + a^2 \rho a^{\dagger 2} \kappa_2$$

$$H_{eff} = -\left(K + i\frac{\kappa_2}{2}\right) a^{\dagger 2} a^2 + P(a^{\dagger 2} e^{i\phi} + a^2 e^{-i\phi})$$

The eff. hamiltonian stabilizes the states $|C_{\pm}, \phi\rangle$

$$H_{eff}|C_{\pm}, \phi\rangle = \frac{P^2}{K + i\kappa_2/2}|C_{\pm}, \phi\rangle$$

$$|C_{\pm}, \phi\rangle = N_{\pm}\left(|\beta\rangle \pm |-\beta\rangle\right), \quad \beta = \sqrt{\frac{P e^{i\phi}}{K + i\kappa_2/2}}$$

The evolution under the eff. hamiltonian is interupted by q. jumps $\quad a^2|C_{\pm}, \phi\rangle = \beta^2 |C_{\pm}, \phi\rangle$

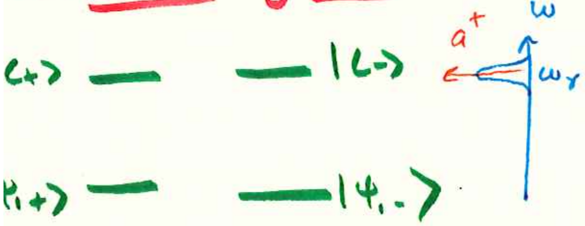
$\Rightarrow |C_{\pm}, \phi\rangle$ in invariant to two-phot. loss

Suppose the two-phot. diss $= 0$ & the cat qubit is defined with the cat oriented along the position axis of the phase space ($\phi = 0$). Now we want to turn κ_2 on so as to correct for leakage. In this case if we don't want to rotate the cat, we choose

$$\phi = \tan^{-1}\frac{\kappa_2}{2K} \quad \& \quad P = \alpha^2\sqrt{K^2 + \frac{\kappa_2^2}{4}}$$

(α defines the qubit $|C_{\pm}\rangle = N_{\pm}(|\alpha\rangle \pm |-\alpha\rangle)$)

3. Photon Gain



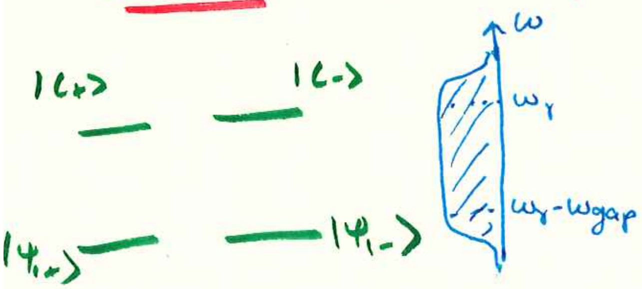
Case 1: Freq. of thermal phot $\sim \omega_r$ (narrow spectral density cent. around ω_r)

$$a^\dagger|C_\pm\rangle = \alpha r^{\pm}|C_\mp\rangle + \cancel{|\psi_{1\mp}\rangle}$$

$$\Rightarrow a^\dagger = \alpha\underbrace{\left(\frac{r+r^{-1}}{2}\right)}_{\approx 1} Z - i\alpha \underbrace{\left(\frac{r-r^{-1}}{2}\right)}_{\approx e^{-2\alpha^2}} Y$$

Non-deph. errors are exponentially supp.!

Case 2: Thermal photons with broad spectral density.



$$a^\dagger|C_\pm\rangle = \alpha r^{\pm1}|C_\mp\rangle + |\psi_{1\mp}\rangle \checkmark$$

$\langle\psi_{1\mp}|a^\dagger|C_\pm\rangle = 1$ (Leakage rate $= \kappa_\uparrow = \kappa n_{th}$)

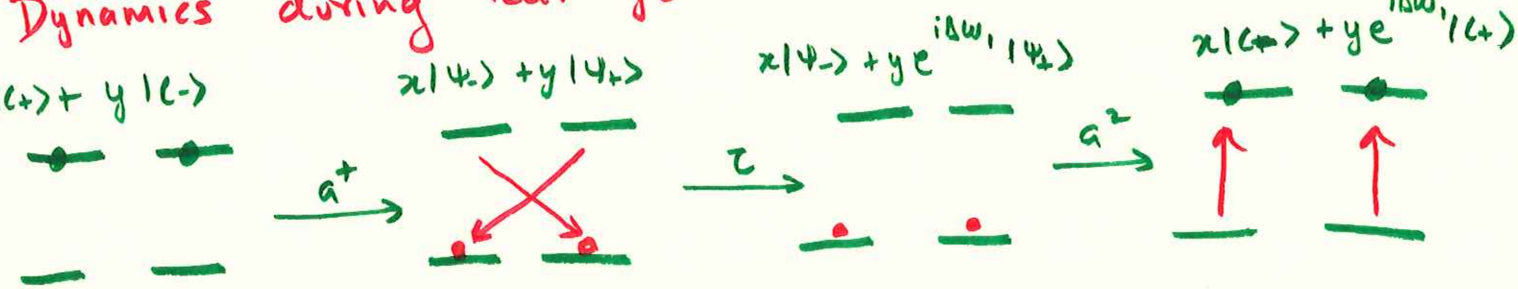
Photon loss will correct for leakage: $\langle C_\pm|a|\psi_{1\mp}\rangle = 1$

But single photon loss will also cause phase-flips. Hence, it's better to introduce two-photon loss:

$$\langle C_\pm|a^2|\psi_\pm\rangle = 2\alpha$$

$$\left(\kappa_\downarrow = 4\kappa_2\alpha^2 \text{ and for } \kappa_\downarrow \gg \kappa_\uparrow,\ \kappa_2 \gg \frac{\kappa n_{th}}{4\alpha^2}\right)$$

Dynamics during leakage & aut. correction with a^2



That is,

$$x|C_+\rangle + y|C_-\rangle \longrightarrow x|C_-\rangle + y e^{i\Delta\omega_1 \tau}|C_+\rangle$$

$$= Z \underbrace{e^{i\frac{\Delta\omega_1}{2}\tau(1-x)}}_{\text{non-deph. error!}} \left(x|C_+\rangle + y|C_-\rangle\right)$$

However, $\Delta\omega_1 \sim e^{-2\alpha^2}$ (large α limit)

$\therefore$ non-deph errors are supp!

4 Freq. fluctuations

Follows previous discussion.

$$a^\dagger a|C_\pm\rangle = \underbrace{r_\alpha^{\pm 2}{}^{2}|C_\pm\rangle}_{\text{non-dephase error } \propto e^{-2\alpha^2}} + \underbrace{\alpha|\Psi_{1\pm}\rangle}_{\text{Leakage, corrected by two-phot. dissipation.}}$$

Generalization to noise of the form $X_{mn} a^{\dagger m} a^n$

For small perturbations $|X_{mn}\alpha^{m+n-1}| \ll \omega_{gap}$

($\Rightarrow$ Born approx, $\Rightarrow$ Eigenspectrum of the system is $\approx$ given by eigenspectrum of Kerr + two photon drive)

[1] If spectral density is narrow $\Rightarrow$ excitations out of cat-qubit subspace are supp.

$$a^\dagger, a \sim \alpha(Z \pm i e^{-2\alpha^2} Y)$$

ie, no leakage & phase flips dominate

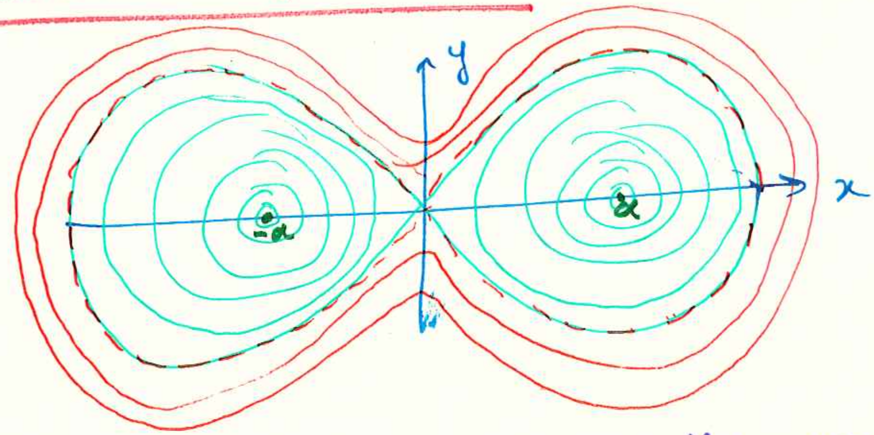
[2] If spectral density is broad, $a^{\dagger m} \Rightarrow$ will excite to $|\Psi_{m\pm}\rangle$

if $m < \dfrac{\alpha^2}{4}$

(i.e, leakage contained within pair-wise deg. manifold)

then phase flips will be supp. after dissipation corrects for leakage

Semiclassical interpretation



Meta potential or phase-space pot or semiclass paths of motion for $-Ka^{\dagger 2}a^2 + P(a^{\dagger 2} + a^2)$

- As long as perturbations keep the particle on the "closed/confined" green lines/paths, phase-errors will dominate while non dephasing errors will be supp.
- The "threshold path" passes through 0 as shown with green + red dashed lines.

Recap

- Cat qubit in a nonlinear, parametrically driven oscillator, which has a highly biased error channel.
- Adiabatic initialization ($P_{|C_+\rangle}$, $P_{|C_-\rangle}$) → Trivially bias preserving.
- Gates $\{\underbrace{Z(\theta), ZZ(\theta)}_{\text{Trivially bias preserving}}, \underbrace{e^{i\pi X/4}}_{\text{Does not preserve bias!}}\}$
- Readout : Single-shot QND along Z axis (basically homodyne measurement to diff $|\alpha\rangle$ & $|-\alpha\rangle$)
  - $= M_Z$
  - $e^{i\pi/4} + M_Z \Rightarrow M_X$ → Trivially bias preserving.

  (arxiv: 1807.09334)

- Next up: CNOT!

Towards ~~C~~NOT gate / CX gate

[1] Single-qubit X gate

$$X\left(x|C_+\rangle + y|C_-\rangle\right) = x|C^+\rangle - y|C^-\rangle$$

Recall : $H = -Ka^{\dagger 2}a^2 + P(a^{\dagger 2}e^{i\phi} + a^2 e^{-i\phi})$

The qubit is defined with $\phi = 0$

Rotate the phase of the two photon drive from 0 to 2π in time $\tau \gg 1/\omega_{gap}$.

$$|C_+\rangle = N_+\left(|\alpha\rangle + |-\alpha\rangle\right) \xrightarrow[\text{phase}]{\text{rot}} N_+\left(|\alpha e^{i\phi/2}\rangle + |-\alpha e^{-i\phi/2}\rangle\right)$$

$$\xrightarrow{\tau}$$

$$|C_+\rangle = N_+\left(|-\alpha\rangle + |\alpha\rangle\right)$$

$$|C_-\rangle = N_-\left(|\alpha\rangle - |-\alpha\rangle\right) \xrightarrow[\text{phase}]{\text{rot}} N_-\left(|\alpha e^{i\phi/2}\rangle - |-\alpha e^{-i\phi/2}\rangle\right)$$

$$\xrightarrow{\tau}$$

Top. phase (does not depend on the path in phase space, or how fast one moves in phase space)

$$-|C_-\rangle = N_-\left(|-\alpha\rangle - |\alpha\rangle\right)$$

$$\Rightarrow \underbrace{x|C_+\rangle + y|C_-\rangle \xrightarrow{\tau} x|C_+\rangle - y|C_-\rangle}_{X\text{-gate.}}$$

In reality, rotation in phase space also gives rise to geometric phase $\propto$ area of the path.

The area enclosed by the $|C_+\rangle$ state is slightly $(\exp \propto e^{-2|\alpha|^2})$ smaller than that enclosed by the $|C_-\rangle$ state (for large α).

$$\phi_g^+ - \phi_g^- = -\frac{4\pi\alpha^2 e^{-2\alpha^2}}{1 - e^{-4\alpha^2}}$$

For large α, this phase diff. can be neglected.

- The topological phase arises becaus the cat states $|C_\alpha^-\rangle$ is 2π periodic in the phase of α, while $|C_\alpha^+\rangle$ is π periodic.
- Geometric | Berry phase $\propto$ area under the path

  $\phi_g^+ - \phi_g^- \longrightarrow$ decreases exp with α for larg α

  $\phi_g^+ - \phi_g^- \longrightarrow = -\pi$ for $\alpha = 0$
- For bias-presering nature, the only source of phase diff must be the topology. And hence large α implies large bias in the gate.

This operation does not unbias the channel.

$x|C_+\rangle + y|C_-\rangle = |\psi\rangle$ = initial state

$x|\psi\rangle = x|C_+\rangle - y|C_-\rangle$ = final state which is desired.

$$x|C_+\rangle + y|C_-\rangle \equiv x|C_\alpha^+\rangle + y|C_\alpha^-\rangle$$

$$\Big\downarrow \text{phase rot for time } \tau'$$

$$x|C^+_{\alpha e^{i\phi(\tau')/2}}\rangle + y|C^-_{\alpha e^{i\phi(\tau')/2}}\rangle$$

$$\Big\downarrow Z \text{ (phase flips at } \tau')$$

$$x|C^-_{\alpha e^{i\phi(\tau')/2}}\rangle + y|C^+_{\alpha e^{i\phi(\tau')/2}}\rangle$$

$$\Big\downarrow \text{phase rot continues}$$

$$x|C^-_{\alpha e^{i\pi}}\rangle + y|C^+_{\alpha e^{i\pi}}\rangle$$

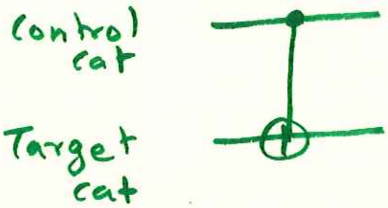
$$= -x|C^-\rangle + y|C^+\rangle$$

$$= -(x|C^-\rangle - y|C^+\rangle)$$

$$= -Z\,x|\psi\rangle$$

$\Rightarrow$ Z error $\frac{\text{during}}{\text{after}}$ the gate propagates as Z error the gate!

12] Extending the intuition of the X gate to build a CNOT / CX gate



$\rightarrow$ Rotate the target cat in phase space depending on the state of the control cat.

If control is $\frac{|C_+\rangle + |C_-\rangle}{\sqrt{2}} \sim |\alpha\rangle$, do nothing

If control is $\frac{|C_+\rangle - |C_-\rangle}{\sqrt{2}} \sim |-\alpha\rangle$ rotate in phase space.

- Error propagation

$\rightarrow$ phase-fips in control cat prop as phase-flips in the control cat itself

$\rightarrow$ Non-phase-flip errors in control will cause leakage in target cat $\Rightarrow$ but these are exp. supp. anyway

$\rightarrow$ Leakage in control cat (confined to the quas-deg. $d^2/4$ levels) does not spread

$\rightarrow$ Phase-flips in target cat can prop as phase-flips in the control cat $\Rightarrow$ but this does not destroy the bias

$\rightarrow$ Non-phase-flip errors in target can prop as phase-flips in the control.

$\rightarrow$ Leakage error in the target could at worse cause phase-flips in control

$\rightarrow$ Moreover, leakage can be corrected by adding photon dissipation

$\rightarrow$ control errors (small) are equiv to leakage. and can also be corrected by two-photon dissipation.

Hamiltonian for the CX gate

$$H = -K\,(a_c^{\dagger 2}-\beta^2)(a_c^2-\beta^2)$$
$$-K\left[a_t^{\dagger 2} - \alpha^2 e^{-i\phi(t)}\left(\frac{\beta - a_c^\dagger}{2\beta}\right) - \alpha^2\left(\frac{\beta + a_c^\dagger}{2\beta}\right)\right] \times$$
$$\left[a_t^2 - \alpha^2 e^{i\phi(t)}\left(\frac{\beta - a_c}{2\beta}\right) - \alpha^2\left(\frac{\beta + a_c}{2\beta}\right)\right]$$
$$-\frac{\dot\phi(t)}{4\beta}\,a_t^\dagger a_t\,(2\beta - a_c^\dagger - a_c)$$

Can be implemented with four-wave mixing capabilities of the Kerr cats itself! No external coupler req.

$a_c \rightarrow$ control , $a_t \rightarrow$ target.

when control is in state $|C_+\rangle + |C_-\rangle/\sqrt{2} \sim |\beta\rangle$

$$H = -K\,(a_c^{\dagger 2}-\beta^2)(a_c^2-\beta^2) \underbrace{- K\,(a_t^{\dagger 2}-\alpha^2)(a_t^2-\alpha^2)}_{\text{No rotation of target.}}$$

when control is in $|C_+\rangle - |C_-\rangle/\sqrt{2} \sim |-\beta\rangle$

$$H = -K\,(a_t^{\dagger 2}-\beta^2)(a_c^{\dagger 2}-\beta^2) \underbrace{-K\,(a_t^{\dagger 2} - \alpha^2 e^{-i\phi(t)})(a_t^2 - \alpha^2 e^{i\phi(t)})}_{\text{rotate the target}}$$
$$\underbrace{-\dot\phi(t)\,a_t^\dagger a_t}$$

dynamic phase ⇒ cancels geometric phase

⇓

not nec. as the geometric phase can be tracked in software.

For numerical estimates of the bias see 1905.00450

With reasonable expt. parameters we expect $\eta \sim 10000$ for $|\alpha|^2 \sim 10$.