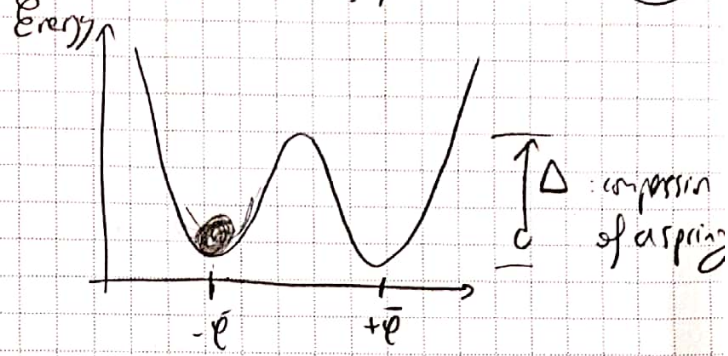
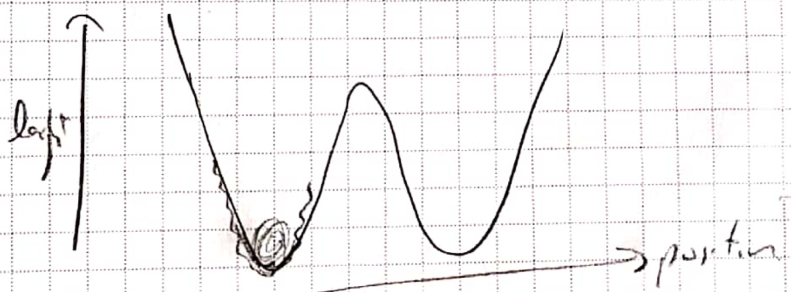
[Why is calassical information so stable?]

"on" = 0 "OFF" = 1.



Maxwell 1868.



$|0\rangle + |1\rangle \rightsquigarrow |0\rangle$ or $|1\rangle$

Lindblad equation:

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + D[L]\rho$$

$$D[L]\rho = L\rho L^{\dagger} + \frac{1}{2}L^{\dagger}L\rho - \frac{1}{2}\rho L^{\dagger}L \qquad L = \sqrt{\kappa}\, c$$

$L_2 = \sqrt{\kappa_2}(a^2 - \alpha^2) \qquad \alpha \in \mathbb{C}$

$$D[L_2]\rho = \kappa_2 \Big[(a^2-\alpha^2)\rho(a^{\dagger 2} - \alpha^{*2}) - \tfrac{1}{2}(a^{\dagger 2} - \alpha^{*2})(a^2-\alpha^2)\rho - \tfrac{1}{2}\rho(a^{\dagger 2} - \alpha^{*2})(a^2-\alpha^2) \Big]$$

$$= D[\sqrt{\kappa_2}\, a^2]\rho + \kappa_2 \Big(-\alpha^{*2} a^2 \rho - \alpha^2 \rho a^{\dagger 2} + \tfrac{1}{2} a^{\dagger 2}\alpha^2 \rho + \tfrac{1}{2}\alpha^{*2} a^2 \rho + \tfrac{1}{2}\rho a^{\dagger 2}\alpha^2 + \tfrac{1}{2}\alpha^{*2}\rho a^2 \Big)$$

$$= D[\sqrt{\kappa_2}\, a^2]\rho + \Big(-\tfrac{1}{2}\alpha^{*2} a^2 \rho - \tfrac{1}{2}\alpha^2 \rho a^{\dagger 2} + \tfrac{1}{2}\alpha^{*2}\rho a^2 + \tfrac{1}{2}\alpha^2 a^{\dagger 2}\rho \Big)\kappa_2$$

$$= D[\sqrt{\kappa_2}\, a^2]\rho + \kappa_2 \Big[\tfrac{1}{2}\alpha^2 a^{\dagger 2} - \tfrac{1}{2}\alpha^{*2} a^2 , \rho \Big]$$

$$\alpha^2 = \frac{2\epsilon_2}{\kappa_2}$$

$$= D[\sqrt{\kappa_2}\, a^2]\rho - \frac{i}{\hbar}[H, \rho].$$

$$\dot{\rho} = D\Big[\underbrace{\sqrt{\kappa_2}\,(a^2 - \alpha^2)}_{L_2}\Big]\rho$$

$a|\alpha\rangle = \alpha|\alpha\rangle.$

$a^2|\alpha\rangle = \alpha^2|\alpha\rangle$

$a^2|-\alpha\rangle = (-\alpha)^2|-\alpha\rangle = \alpha^2|\alpha\rangle$

$L_2|\alpha\rangle = 0$

$L_2|-\alpha\rangle = 0.$

$|\alpha\rangle, |-\alpha\rangle$ by linearity all superpositions of $|\alpha\rangle, |-\alpha\rangle$ are steady states

$$\frac{H(t)}{\hbar} = \omega_0 b^\dagger b + \varepsilon^*(t) b + \varepsilon(t) b^\dagger + f(b, b^\dagger) \qquad L = \sqrt{\kappa_b}\, b$$

Quantum Langevin equation

$$\dot{b}(t) = -\frac{i}{\hbar}[b, H] - \frac{\kappa_b}{2} b \qquad [b, H(b,b^\dagger)] = \frac{\partial H(b,b^\dagger)}{\partial b^\dagger}$$

$$= -i\left(\omega_0 b + \varepsilon(t)\right) - i\left[b, f(b,b^\dagger)\right] - \frac{\kappa_b}{2} b$$

1) Define $\tilde{b}$ such that $b = \tilde{b} + \varsigma(t)$ $\qquad \varsigma : t \in \mathbb{R} \longrightarrow \varsigma(t) \in \mathbb{C}$

$$\dot{\tilde{b}}(t) + \dot{\varsigma}(t) = -i\left(\omega_0 \tilde{b} + \omega_0 \varsigma(t) + \varepsilon(t)\right) - i\left[\tilde{b}, f(\tilde{b} + \varsigma(t), \tilde{b}^\dagger + \varsigma^*(t))\right] - \frac{\kappa_b}{2}\tilde{b} - \frac{\kappa_b}{2}$$

chose $t \mapsto \varsigma(t)$ s.t $\quad \| \; \dot{\varsigma}(t) = -i\omega_0 \varsigma(t) - i\varepsilon(t) - \frac{\kappa_b}{2}\varsigma(t)$

then $$\dot{\tilde{b}}(t) = -i\omega_0 \tilde{b} - i\left[\tilde{b}, f(\tilde{b} + \varsigma(t), \tilde{b}^\dagger + \varsigma^*(t))\right] - \frac{\kappa_b}{2}\tilde{b}$$

2) Define $\tilde{\tilde{b}}$ s.t. $\tilde{b} = \tilde{\tilde{b}} e^{-i\omega_0 t}$

$$\cancel{\dot{\tilde{\tilde{b}}} e^{-i\omega_0 t}} \; \underline{- i\omega_0 \tilde{\tilde{b}} e^{-i\omega_0 t}} = \underline{-i\omega_0 \tilde{\tilde{b}} e^{-i\omega_0 t}} - i\left[\cancel{\tilde{\tilde{b}} e^{-i\omega_0 t}}, f(\tilde{\tilde{b}} e^{-i\omega_0 t} + \varsigma(t), \tilde{\tilde{b}}^\dagger e^{+i\omega_0 t} + \varsigma^*(t))\right] - \frac{\kappa_b}{2}\cancel{\tilde{\tilde{b}} e^{-i\omega_0 t}}$$

$$\dot{\tilde{\tilde{b}}} = -i\left[\tilde{\tilde{b}}, f(\tilde{\tilde{b}} e^{-i\omega_0 t} + \varsigma(t), \tilde{\tilde{b}}^\dagger e^{+i\omega_0 t} + \varsigma^*(t))\right] - \frac{\kappa_b}{2}\tilde{\tilde{b}}$$

Displace + rotate $\quad \tilde{\tilde{H}} = f\left(\tilde{\tilde{b}} e^{-i\omega_0 t} + \varsigma(t), \tilde{\tilde{b}}^\dagger e^{i\omega_0 t} + \varsigma^*(t)\right)$

Rq: if $\varepsilon(t) = \varepsilon_p e^{-i\omega_p t}$ $\qquad \varsigma(t) = \varsigma_p e^{-i\omega_p t}$

$$-i\omega_p \varsigma_p = -i\omega_0 \varsigma_p - i\varepsilon_p - \frac{\kappa_b}{2}\varsigma_p$$

$$\Big\| \; \varsigma_p = \frac{-i\varepsilon_p}{i(\omega_0 - \omega_p) + \frac{\kappa_b}{2}}$$

3) Adiabatic elimination

$$\frac{H}{\hbar} = \left(g A b^\dagger + g^* A^\dagger b\right) \qquad L_b = \sqrt{\kappa_b}\, b \qquad A = f(a, a^\dagger)$$

$$\frac{d\rho}{dt} = -i\left[g A b^\dagger + g^* A^\dagger b, \rho\right] + \kappa_b\left(b\rho b^\dagger - \frac{1}{2} b^\dagger b \rho - \frac{1}{2}\rho b^\dagger b\right)$$

$$\frac{1}{\kappa_b}\frac{d\rho}{dt} = -i\frac{1}{\kappa_b}\left[g A b^\dagger + g^* A^\dagger b, \rho\right] + \left(b\rho b^\dagger - \frac{1}{2} b^\dagger b\rho - \frac{1}{2}\rho b^\dagger b\right) \quad (*)$$

$$= \frac{-i}{\kappa_b}\left(g A b^\dagger \rho + g^* A^\dagger b \rho - g\rho A b^\dagger - g^* \rho A^\dagger b\right) +$$

$$\frac{|g|}{\kappa_b} \sim \delta$$

Ansatz $$\rho(t) = \rho_{00} \otimes |0\rangle\langle 0| + \delta\left(\rho_{10}|1\rangle\langle 0| + \rho_{01}|0\rangle\langle 1|\right) + \delta^2\left(\rho_{11}|1\rangle\langle 1| + \rho_{20}|2\rangle\langle 0| + \rho_{02}|0\rangle\langle 2|\right) + O(\delta^3).$$

mode a mode b

$b|0\rangle = 0$ $\langle 0|b = \langle 1|$

$b^\dagger|0\rangle = |1\rangle$ $\langle 0|b^\dagger = 0$

${}_b\langle 0|(*)|0\rangle_b$ $$\frac{1}{\kappa_b}\frac{d}{dt}\rho_{00} = -i\frac{1}{\kappa_b}\left(\delta g^* A^\dagger \rho_{10} - \delta g \rho_{01} A\right) + \delta^2 \rho_{11}$$

$$\frac{1}{\kappa_b}\frac{d}{dt}\rho_{00} = -i\delta^2\left(\bar{A}^\dagger \rho_{10} - \rho_{01}\bar{A}\right) + \delta^2 \rho_{11} + O(\delta^3). \quad \boxed{1}$$

$\bar{A} = \frac{1}{\delta\kappa_b} g A$ $\|\tilde{A}\|$ order 1

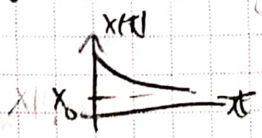
slowly varying $\left|\frac{1}{\kappa_b}\frac{d\rho_{00}}{dt}\right| = O(\delta^2)$

idem:

${}_b\langle 0|(*)|1\rangle_b$ $$\frac{1}{\kappa_b}\frac{d}{dt}\rho_{01} = i\rho_{00}\bar{A}^\dagger - \frac{1}{2}\rho_{01} + O(\delta) \quad \boxed{2}$$ damping $O(1)$.

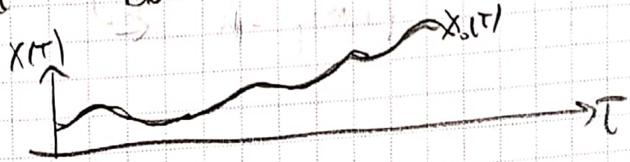
$\langle 1|(*)|1\rangle$ $$\frac{1}{\kappa_b}\frac{d}{dt}\rho_{11} = i\rho_{10}\bar{A}^\dagger - i\hat{A}\rho_{01} - \rho_{11} + O(\delta) \quad \boxed{3}$$

$$\frac{dx}{dt} = -(x - x_0)$$



slow variation

$$\frac{dx}{d\tau} = -(x - x_0(\tau))$$



Adiabatic elimination

$$\boxed{2} \Rightarrow \rho_{01} = 2i\,\rho_{00}\bar{A}^{\dagger} \qquad \rho_{10} = -2i\,\bar{A}\rho_{00}$$

$$\boxed{3} \Rightarrow \rho_{11} = i\,\rho_{10}\bar{A}^{\dagger} - i\,\bar{A}\rho_{01} = i\left(-2i\,\bar{A}\rho_{00}\bar{A}^{\dagger}\right) - i(2i)\,\bar{A}\rho_{00}\bar{A}^{\dagger}$$

$$= 4\,\bar{A}\rho_{00}\bar{A}^{\dagger}$$

$$\boxed{1} \Rightarrow \quad \frac{1}{\kappa_b}\frac{d}{dt}\rho_{00} = -i\,\delta^2\left(\bar{A}^{\dagger}(-2i)\bar{A}\rho_{00} - (2i)\,\rho_{00}\bar{A}^{\dagger}\bar{A}\right) + 4\delta^2\,\bar{A}\rho_{00}\bar{A}^{\dagger}$$

$$\frac{1}{\kappa_b}\frac{d}{dt}\rho_{00} = 2\delta^2\left(-\bar{A}^{\dagger}\bar{A}\rho_{00} - \rho_{00}\bar{A}^{\dagger}\bar{A}\right) + 4\delta^2\,\bar{A}\rho_{00}\bar{A}^{\dagger}$$

$$= 4\delta^2\left[\bar{A}\rho_{00}\bar{A}^{\dagger} - \frac{1}{2}\bar{A}^{\dagger}\bar{A}\rho_{00} - \frac{1}{2}\rho_{00}\bar{A}^{\dagger}\bar{A}\right]$$

$$= 4\cancel{\delta^2}\,\frac{|g|^2}{\cancel{\delta^2}\kappa_b^2}\left[A\rho_{00}A^{\dagger} - \frac{1}{2}A^{\dagger}A\rho_{00} - \frac{1}{2}\rho_{00}A^{\dagger}A\right]$$

$$\frac{d}{dt}\rho_{00} = \frac{4|g|^2}{\kappa_b}\left[\quad\right]$$

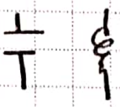
$$= D\left[\sqrt{\kappa_{eff}}\,A\right]\rho_{00} \qquad \boxed{\kappa_{eff} = 4|g|^2/\kappa_b}$$

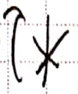
we want $A = a^2 - \alpha^2$

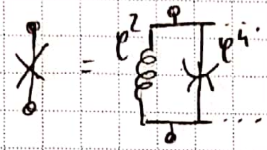
$$H = g(a^2 - \alpha^2)\,b^{\dagger} + g^*(a^{\dagger 2} - \alpha^{*2})\,b$$

$$= \underbrace{g\,a^2 b^{\dagger} + g^* a^{\dagger 2} b}_{\text{non-linear interaction}} - \underbrace{\left(g\,\alpha^2 b^{\dagger} + g^*\alpha^{*2}\,b\right)}_{\text{linear drive on } b}$$

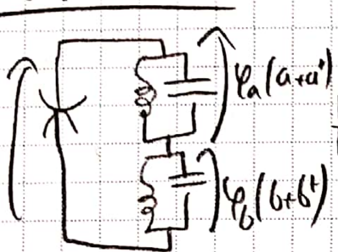
(How do we get $a^2 b^\dagger + a^{\dagger 2} b$?)

qubic $\Rightarrow$  won't do

φ  $-E_J \cos(\varphi)$

φ  φ^6 ...

Trivial circuit :



$$H = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b - \frac{E_J}{4!}\left(\varphi_a(a+a^\dagger) + \varphi_b(b+b^\dagger)\right)^4$$
$$+ \varepsilon(t) b^\dagger + \varepsilon^*(t) b$$

$$\varphi = \varphi_a(a+a^\dagger) + \varphi_b(b+b^\dagger)$$

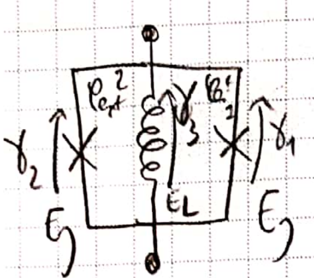
Displace rotate :

$$\tilde{H} = -\frac{E_J}{4!}\left(\varphi_a(a e^{-i\omega_a t} + a^\dagger e^{i\omega_a t}) + \varphi_b(b e^{-i\omega_b t} + b^\dagger e^{i\omega_b t} + \xi_p e^{-i\omega_p t} + \xi_p^* e^{i\omega_p t})\right)^4$$

$$= -\frac{E_J}{4!}\varphi_a^2 \varphi_b^2 \binom{4}{2}\binom{2}{1} \underbrace{e^{-2i\omega_a t}\, e^{i\omega_b t}\, e^{i\omega_p t}}_{=1 \text{ if } \omega_p = 2\omega_a - \omega_b} a^2 b^\dagger \xi_p$$

$$\underbrace{-\frac{E_J}{4!}\varphi_a^2 \varphi_b^2\, 4!\, a^\dagger a b^\dagger b}_{\text{bad .}} + \dots$$

ATS Asymmetrically Threaded SQUID:



$$\gamma_1 - \gamma_3 = \varphi_{ext1}$$

$$\gamma_3 - \gamma_2 = \varphi_{ext2}$$

$$\gamma_1 - \gamma_2 = \varphi_{ext1} + \varphi_{ext2}$$

parametrization:

$$\gamma_1 = \gamma + \tfrac{1}{2}\varphi_{ext1} + \tfrac{1}{2}\varphi_{ext2} = \gamma + \varphi_\Sigma$$

$$\gamma_2 = \gamma - \tfrac{1}{2}\varphi_{ext1} - \tfrac{1}{2}\varphi_{ext2} = \gamma - \varphi_\Sigma$$

$$\gamma_3 = \gamma - \tfrac{1}{2}\varphi_{ext1} + \frac{\varphi_{ext2}}{2} = \gamma - \varphi_\Delta$$

$$\varphi_\Sigma = \frac{\varphi_{ext1} + \varphi_{ext2}}{2}$$

$$\varphi_\Delta = \frac{\varphi_{ext1} - \varphi_{ext2}}{2}$$

$$U(\gamma) = -E_J\left[\cos(\gamma + \varphi_\Sigma) + \cos(\gamma - \varphi_\Sigma)\right] + \tfrac{1}{2}E_L\,(\gamma - \varphi_\Delta)^2$$

$$= -2E_J\cos(\varphi_\Sigma)\cos(\gamma) + \tfrac{1}{2}E_L(\gamma - \varphi_\Delta)^2$$

$$\gamma \to \gamma + \varphi_\Delta$$

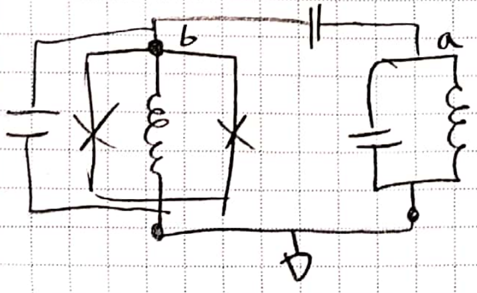
$$U(\gamma) = -2E_J\cos(\varphi_\Sigma)\cos(\gamma + \varphi_\Delta) + \tfrac{1}{2}E_L\gamma^2$$

take $\varphi_\Delta = \pi/2$

$\varphi_\Sigma = \pi/2 + \varepsilon(t)$ $\Longrightarrow$ $U(\gamma) = -2E_J\sin(\varepsilon(t))\sin(\gamma)$

$\varphi_{ext_1} = \pi + \varepsilon(t)$ ← AC pump

$\varphi_{ext_2} = 0 + \varepsilon(t)$



$$\gamma \to \gamma_b(b + b^\dagger) + \gamma_a(a + a^\dagger)$$

$$\gamma_b \sim 1 \quad \gamma_a \ll 1$$

$$|\varepsilon(t)| \ll 1 \qquad \sin(\varepsilon(t)) \sim \varepsilon(t) = \varepsilon_p\cos(\omega_p t)$$

$$\tilde{H}_{NL}(t) = -2E_J\,\varepsilon_p\cos(\omega_p t)\left(\gamma_b(b e^{-i\omega_b t} + b^\dagger e^{i\omega_b t}) + \gamma_a(a e^{-i\omega_a t} + a^\dagger e^{i\omega_a t})\right)^3 + \mathcal{O}(\gamma^5)$$