Les fluctuations de courant : au delà du bruit

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Introduction:

what to measure ?

Averaged quantities

1) STATIC (thermodynamic or steady state) properties:

$\vec{M}(\vec{H})$	Magnetization vs. Magnetic field
$I(V)$	Current vs. Voltage
$\vec{n}(\vec{E})$	Molecular orientation vs. Electric field
$x(F)$	Displacement vs. Force
	X ray diffraction vs. V,P, etc…

Susceptibilities

2) DYNAMICAL properties: response to an oscillating field

$$H = H_{dc} + \delta H \cos \omega_0 t \qquad \chi(H_{dc}, \omega_0) = \frac{\partial M_{\omega_0}}{\partial H_{\omega_0}}$$

Magnetic susceptibility

$$V = V_{dc} + \delta V \cos \omega_0 t \qquad G(V_{dc}, \omega_0) = \frac{\partial I_{\omega_0}}{\partial V_{\omega_0}}$$

ac Conductance

Both are COMPLEX quantities: in-phase and out-of-phase response.
Interesting when:

$$\omega_0 \tau > 1$$

Aging of polymers or spin glasses, dielectric constant or magnetic permittivity, etc.

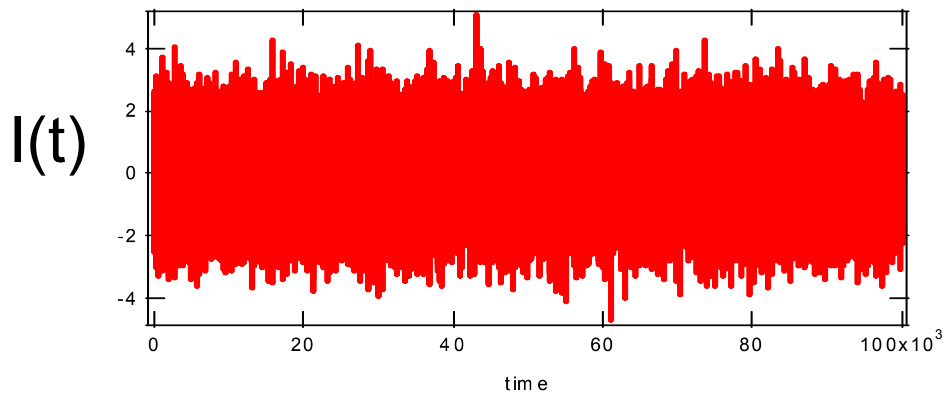
Fluctuations (noise)

3) Intrinsic FLUCTUATIONS (or noise):

Time average

Correlation function: $$C_2(V,\tau) = \langle \delta I(t+\tau)\delta I(t) \rangle$$

Noise spectral density: $$S_2(V,\omega) = \langle \delta I(\omega)\delta I(-\omega) \rangle$$



$$C_2(0) = \langle \delta I^2(t) \rangle$$

Magnetization noise (motion of domain walls), 1/f noise of electric dipoles in amorphous systems glasses, of voltage in transistors, electric noise associated with depinning of charge density waves, etc.

Summary

	Average $\langle \bullet \rangle$	Fluctuations $\langle \bullet \bullet \rangle$
	$I(V)$	$S_2(V,\omega) = \langle \delta I(\omega) \delta I(-\omega) \rangle$
Dynamical response $\frac{\partial \bullet}{\partial V_{\omega_0}}$	$G(V,\omega_0) = \frac{\partial I_{\omega_0}}{\partial V_{\omega_0}}$	

Beyond

	$\langle \bullet \rangle$	$\langle \bullet \bullet \rangle$	$\langle \bullet \bullet \bullet \rangle$
	$I(V)$	$\langle \delta I(\omega)\delta I(-\omega)\rangle$	$\langle \delta I(\omega)\delta I(\omega')\delta I(-\omega-\omega')\rangle$
$\dfrac{\partial \bullet}{\partial V_{\omega_0}}$	$\dfrac{\partial I_{\omega_0}}{\partial V_{\omega_0}}$	$\dfrac{\partial S_2(\omega)}{\partial V_{\omega_0}}$	
$\dfrac{\partial^2 \bullet}{\partial V_{\omega_0}\partial V_{\omega_1}}$	$\dfrac{\partial^2 I_{\omega_0 \pm \omega_1}}{\partial V_{\omega_0}\partial V_{\omega_1}}$		

THIRD CUMULANT

NOISE DYNAMICS

Frequency mixing (non linear transport)

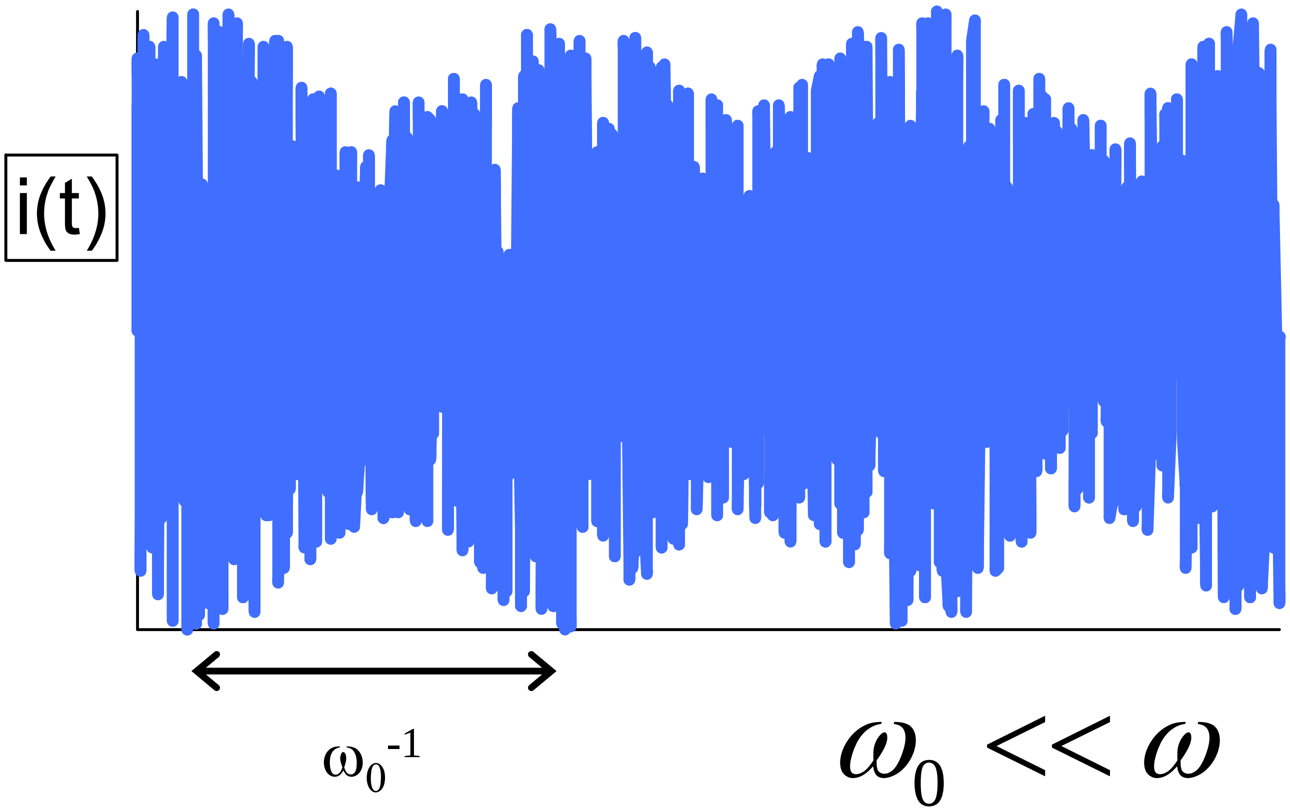
The Dynamics of noise:

The noise susceptibility in the classical regime

Noise susceptibility – How fast can one modulate noise ?

$$\frac{\partial S_2(\omega)}{\partial V_{\omega_0}}$$

$$V(t) = V_{dc} + \delta V \cos \omega_0 t$$



$$\omega_0 << \omega$$

Noise susceptibility – the case of a macroscopic conductor

Fluctuation-dissipation theorem, at equilibrium and low frequency $\hbar\omega << k_B T$:

$$S_2 = 4k_B TG$$

NOISE = electron THERMOMETER

$$\delta V(t) \Rightarrow \delta P_{Joule}(t) = I\delta V(t) \Rightarrow \delta T(t) \Rightarrow \delta S_2(t)$$

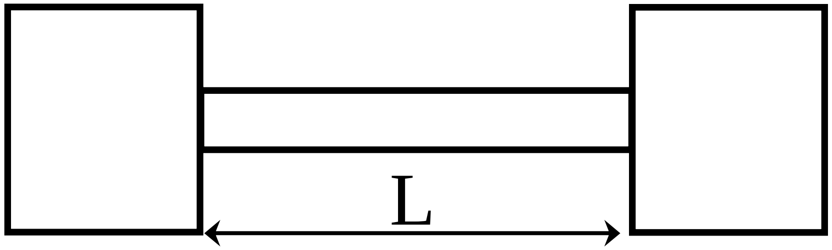
$$\chi_{\omega_0}(\omega) = \frac{\partial S_2(\omega)}{\partial V_{\omega_0}} \propto Z_{thermal}^{-1}(\omega_0)$$

$$\omega >> \omega_0$$

Its frequency dependence gives ENERGY RELAXATION (i.e. INELASTIC) time

Noise susceptibility – from macro- to mesoscopic conductor

diffusive metallic wire: length L >> mean free path



$$L^2 = D\tau_D$$

* long wire or SNS: phonon cooling

The noise suceptibility gives the ELECTRON-PHONON time

* intermediate wire: diffusion cooling

The noise suceptibility gives the DIFFUSION time

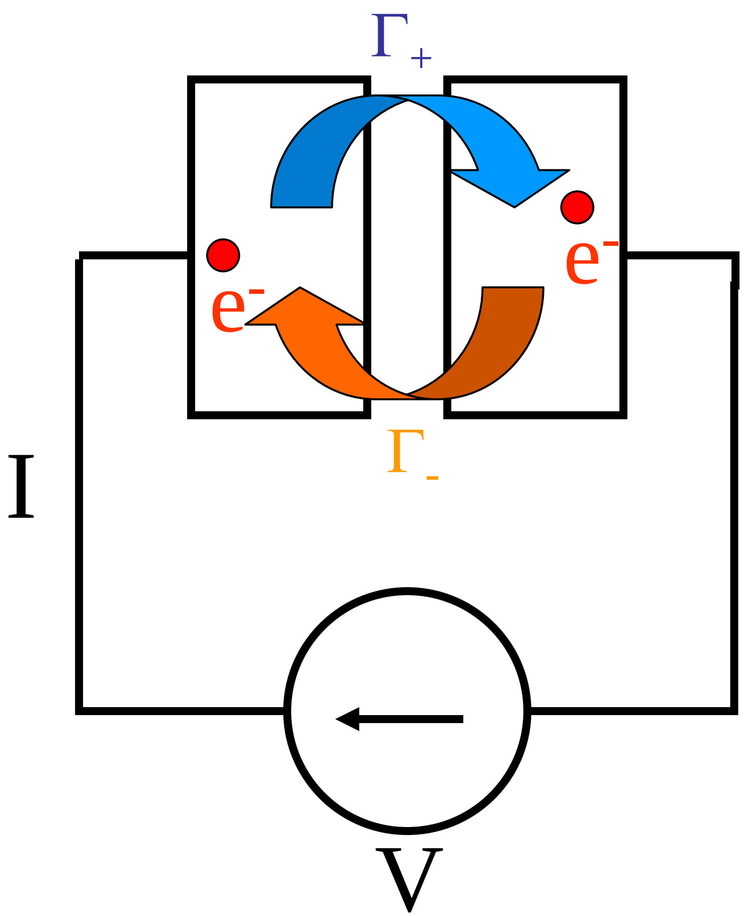
* short wire: elastic transport (independent electrons)

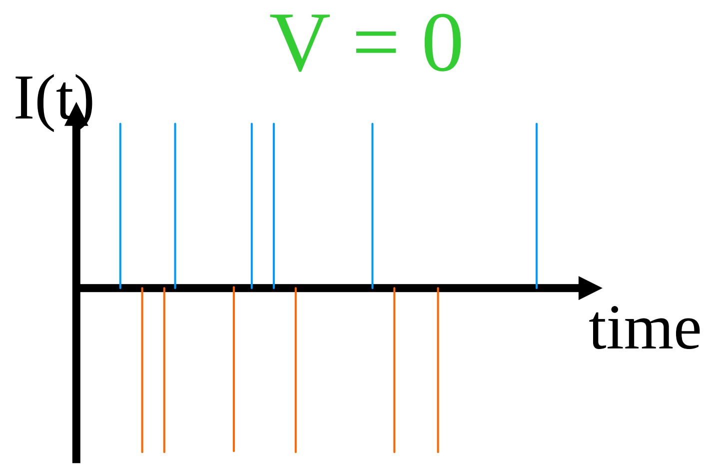
The noise suceptibility gives the ELECTRON-ELECTRON time

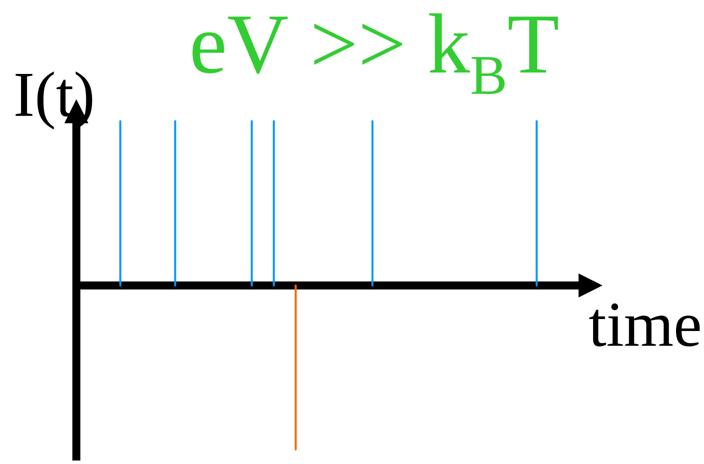
* ballistic wire (nanotube), quasi-crystals (sub-diffusive), … ??

The noise susceptibility in the Quantum regime

Current in a tunnel junction







Current fluctuations in a tunnel junction at low frequency

$$\langle \delta I^2 \rangle = 2eIB\coth\left(\frac{eV}{2k_B T}\right)$$

B=bandwidth

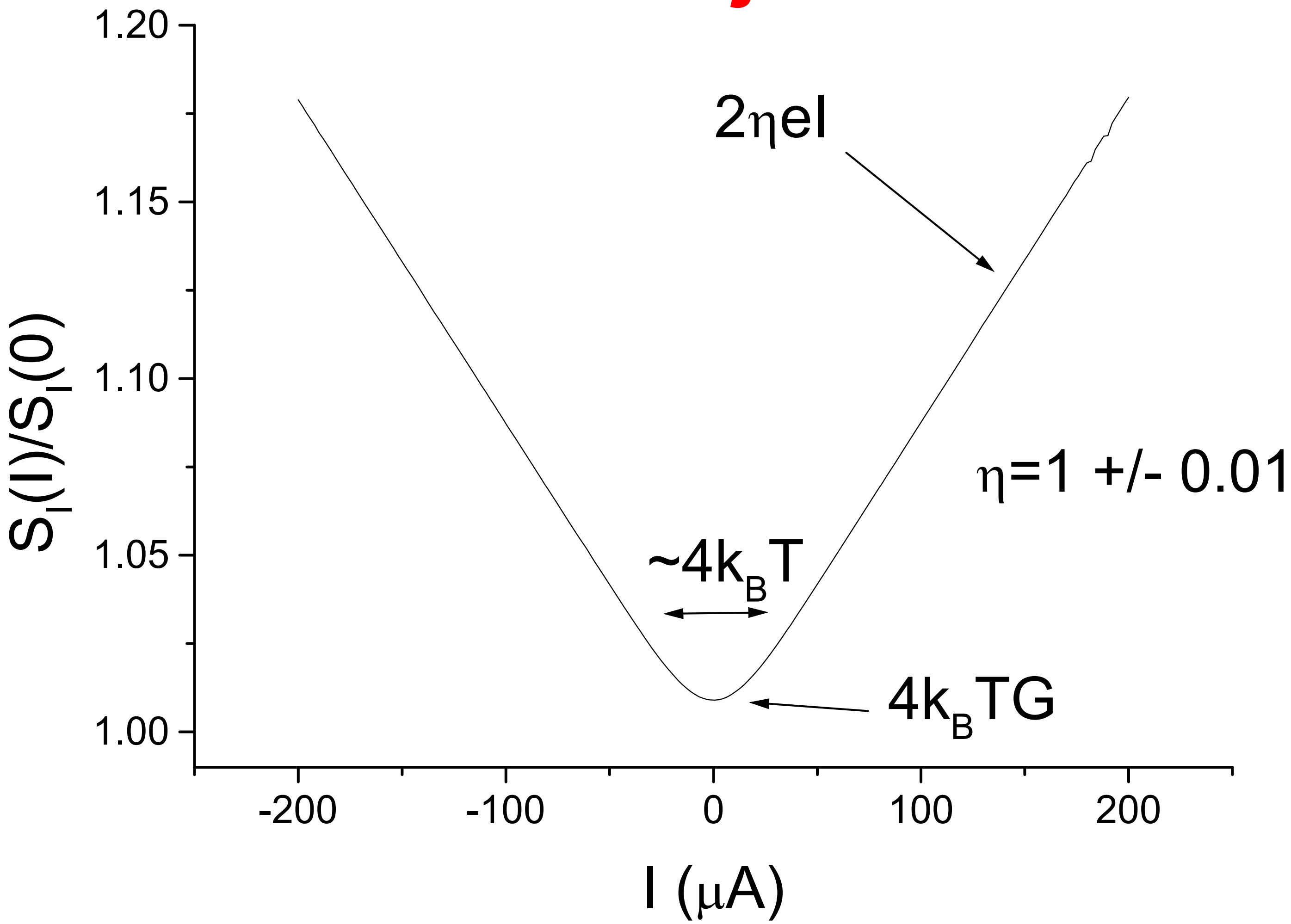
$$S_2 = \begin{cases} 4k_B TG & \text{if } eV << k_B T \\ 2eI & \text{if } eV >> k_B T \end{cases}$$

Equilibrium (Johnson) noise: macroscopic, fluctuation-dissipation theorem

Shot noise: discreteness of charge

Noise spectral density in A^2/Hz

Measurement of S_2 on a tunnel junction



Quantum mechanics: ordering of operators?

Average current: $I_{DC} = \langle \hat{I} \rangle$

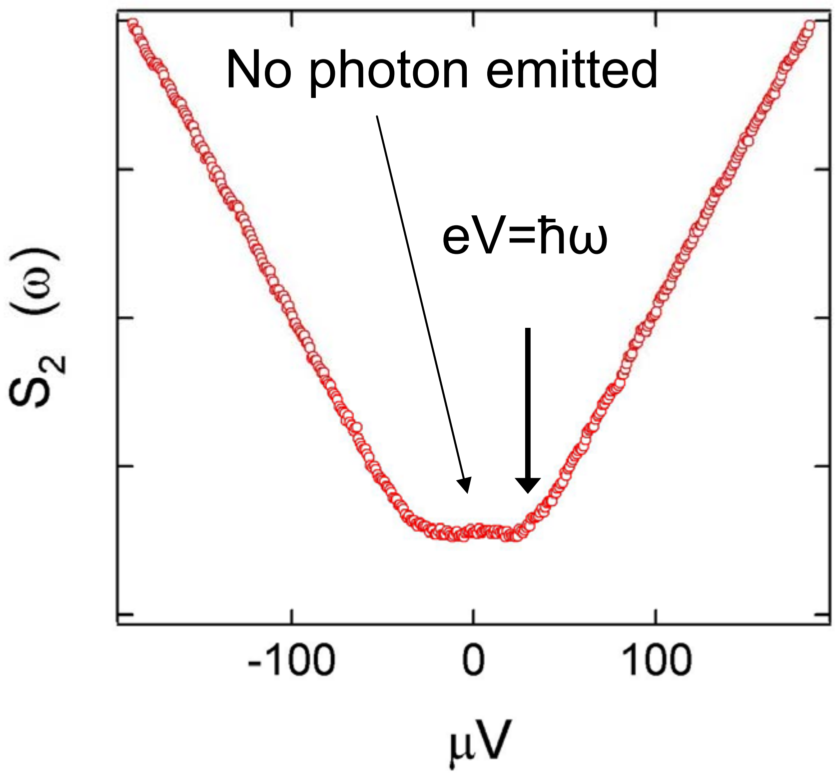
Noise S_2:

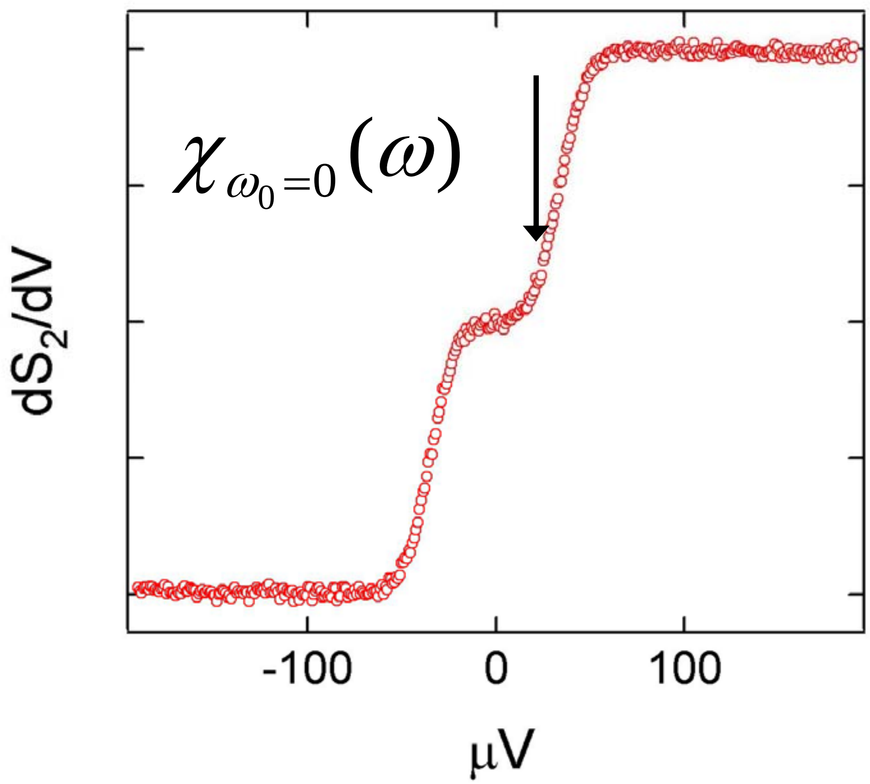
$$S_2(\omega) = \int dt e^{i\omega t} \begin{cases} \left\langle \hat{I}(0)\hat{I}(t) \right\rangle & \text{Absorption} \\ \left\langle \hat{I}(t)\hat{I}(0) \right\rangle & \text{Emission} \\ \frac{1}{2}\left(\left\langle \hat{I}(0)\hat{I}(t) \right\rangle + \left\langle \hat{I}(t)\hat{I}(0) \right\rangle \right) & \text{Classical} \end{cases}$$

$$S_2^{abs}(\omega) = S_2^{em}(-\omega)$$

$$S_2^{sym}(\omega) = S_2^{em}(\omega) + \frac{1}{2} G\hbar\omega$$

Reminder: noise in the quantum regime $\hbar\omega > k_B T, eV$ for a tunnel junction





Measured on a tunnel junction at T=35mK, f=6 GHz, $\hbar\omega/k_B T \sim 8.5$

Noise susceptibility – beyond the classical regime: theory

What if $\omega_0 > \omega$?
What if $\hbar\omega > k_B T$?

$$\chi_{\omega_0}(\omega) \propto \left\langle \delta I(\omega)\,\delta I(\omega_0 - \omega) \right\rangle$$

Calculation:
* Landauer-Büttiker formalism
* SYMMETRIZATION of the operators and $\omega \to -\omega$
The symmetrization rule depends on the experimental setup !

In particular: $\omega \sim \omega_0$

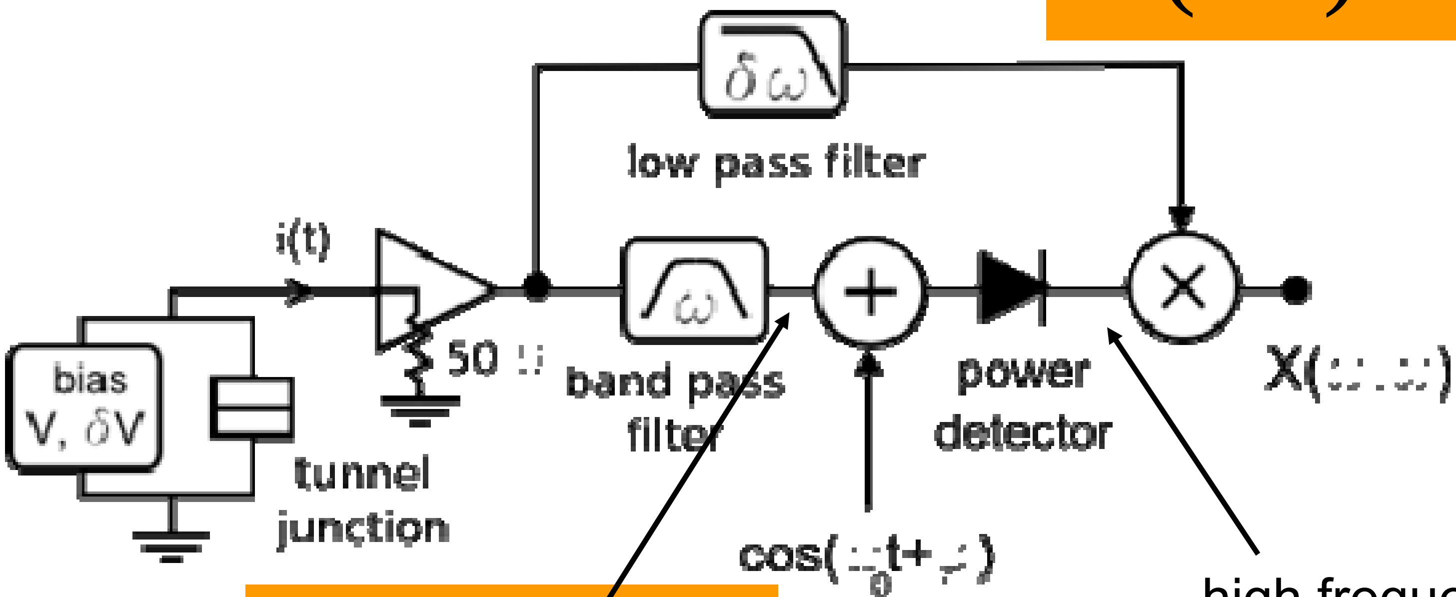
$$\chi_{\omega}(\omega) \propto \left\langle \delta I(\omega)\,\delta I(0) \right\rangle$$

Noise susceptibility – the quantum regime: experiment

$\omega_0 \sim \omega \sim 6$ GHz
$\hbar\omega/k_B T \sim 8.5$
$\delta\omega \sim 100$ MHz

low frequency current

$$\delta I(\pm\varepsilon)e^{\pm i\varepsilon t}$$

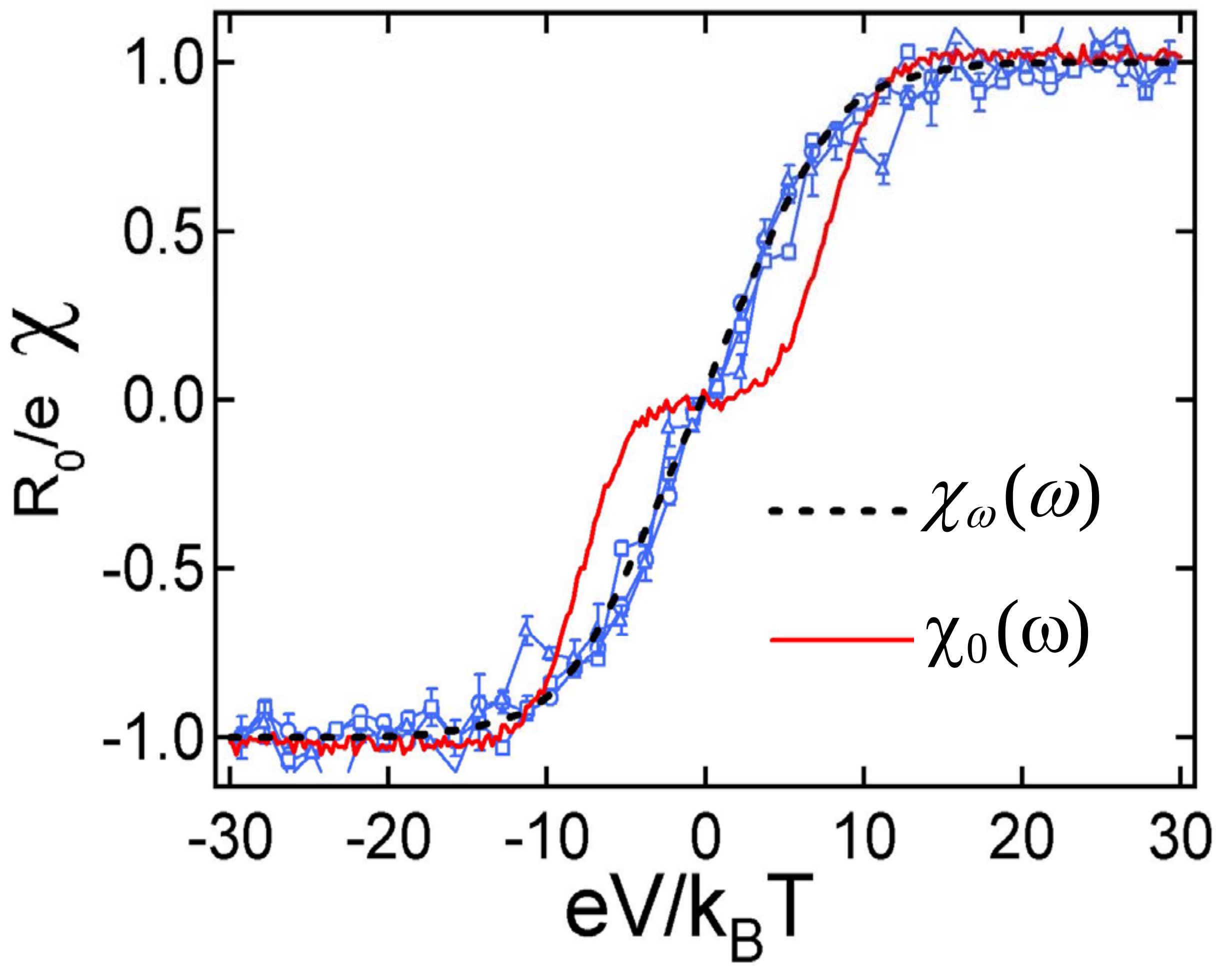


$$\delta I(\pm\omega)e^{\pm i\omega t}$$

high frequency current shifted to low freq.

Noise susceptibility – the quantum regime: experiment

ω_0~ ω~ 6 GHz
T=35 mK
$\hbar\omega/k_BT$~8.5

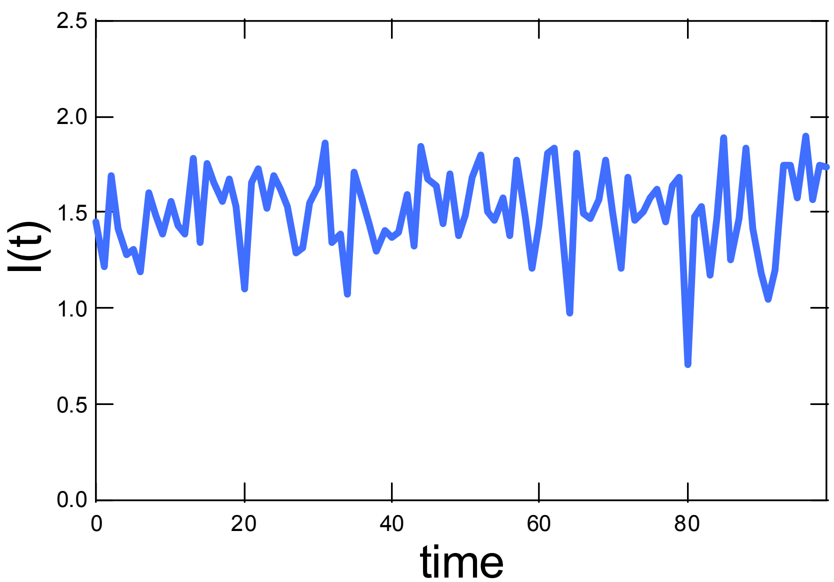


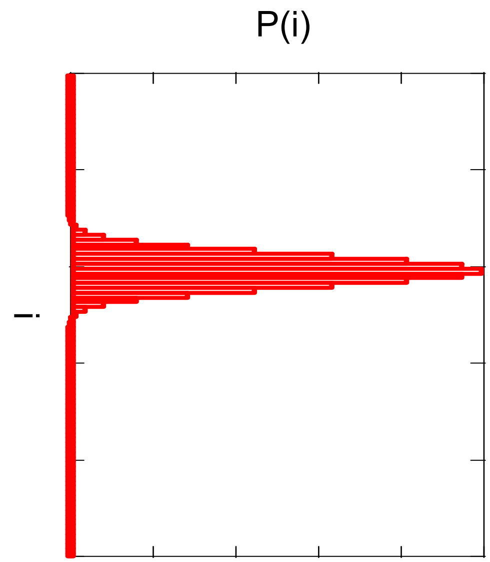
The third cumulant of current fluctuations in the classical regime

The third cumulant of noise

$$S_3(\omega,\omega') = \langle \delta I(\omega)\delta I(\omega')\delta I(-\omega-\omega')\rangle$$

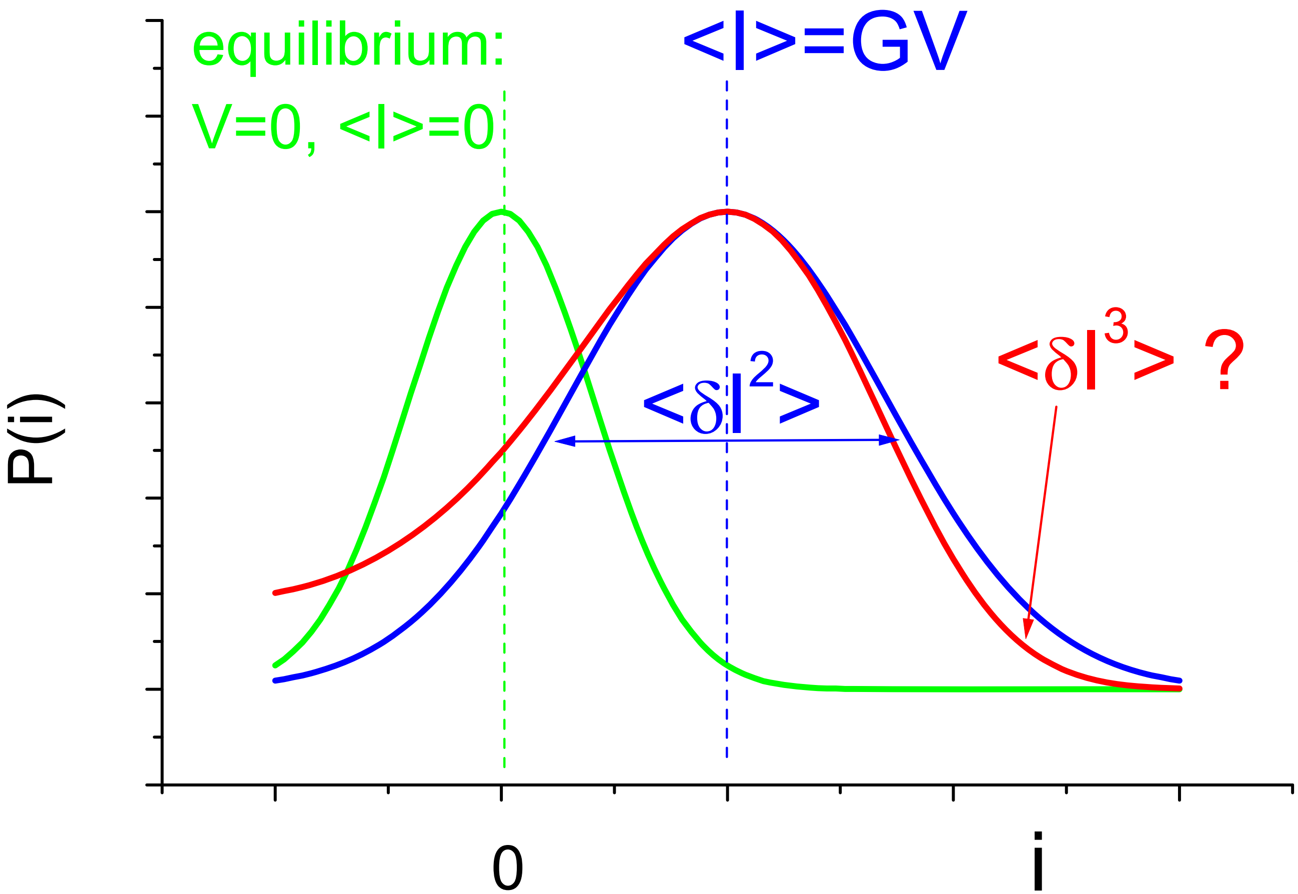
At low frequency: S_3 = SKEWNESS of the probability distribution of current fluctuations P(I): zero for gaussian noise

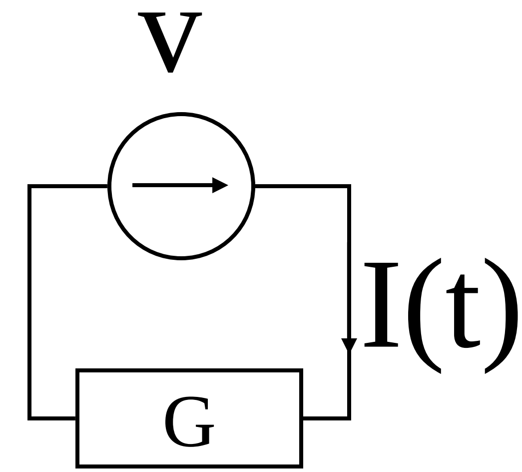




$$S_3(0,0) \approx \langle \delta I^3 \rangle$$

Statistics of the current



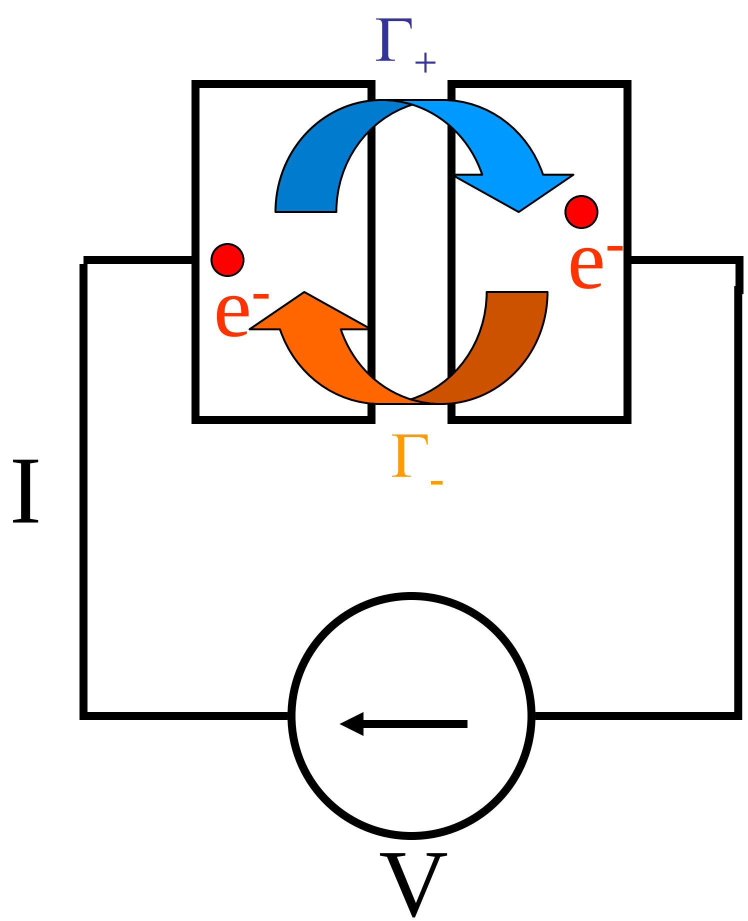


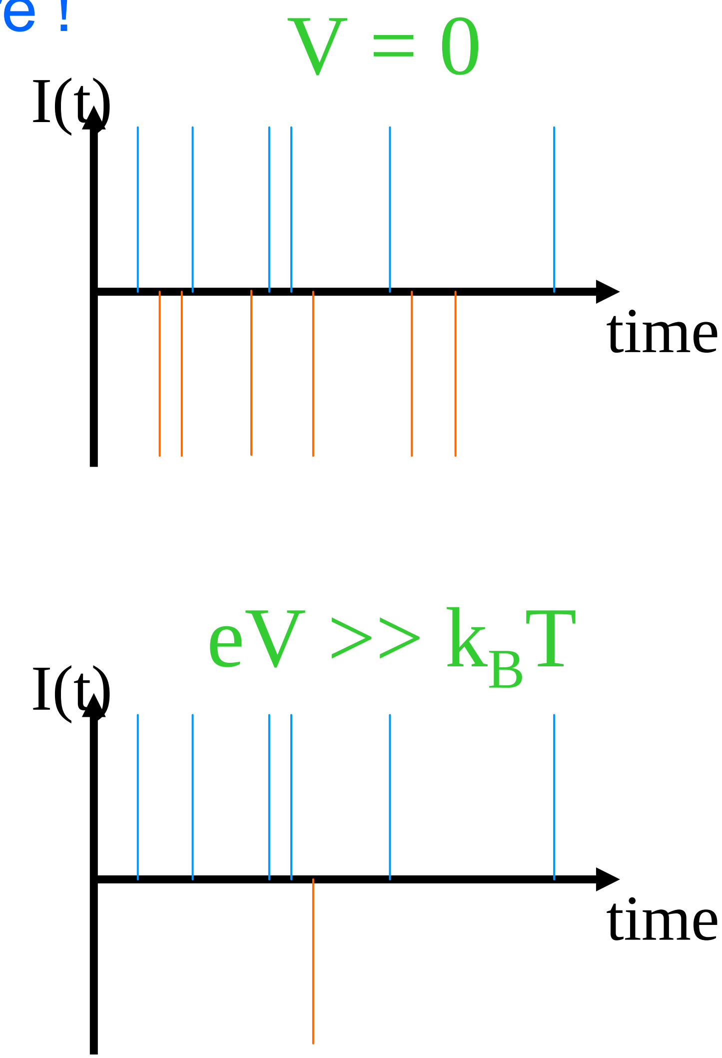
$$I(t) = \langle I \rangle + \delta I(t)$$

At equilibrium: Johnson noise: $<\delta I^2>=4k_BTGB$, (B=bandwidth)

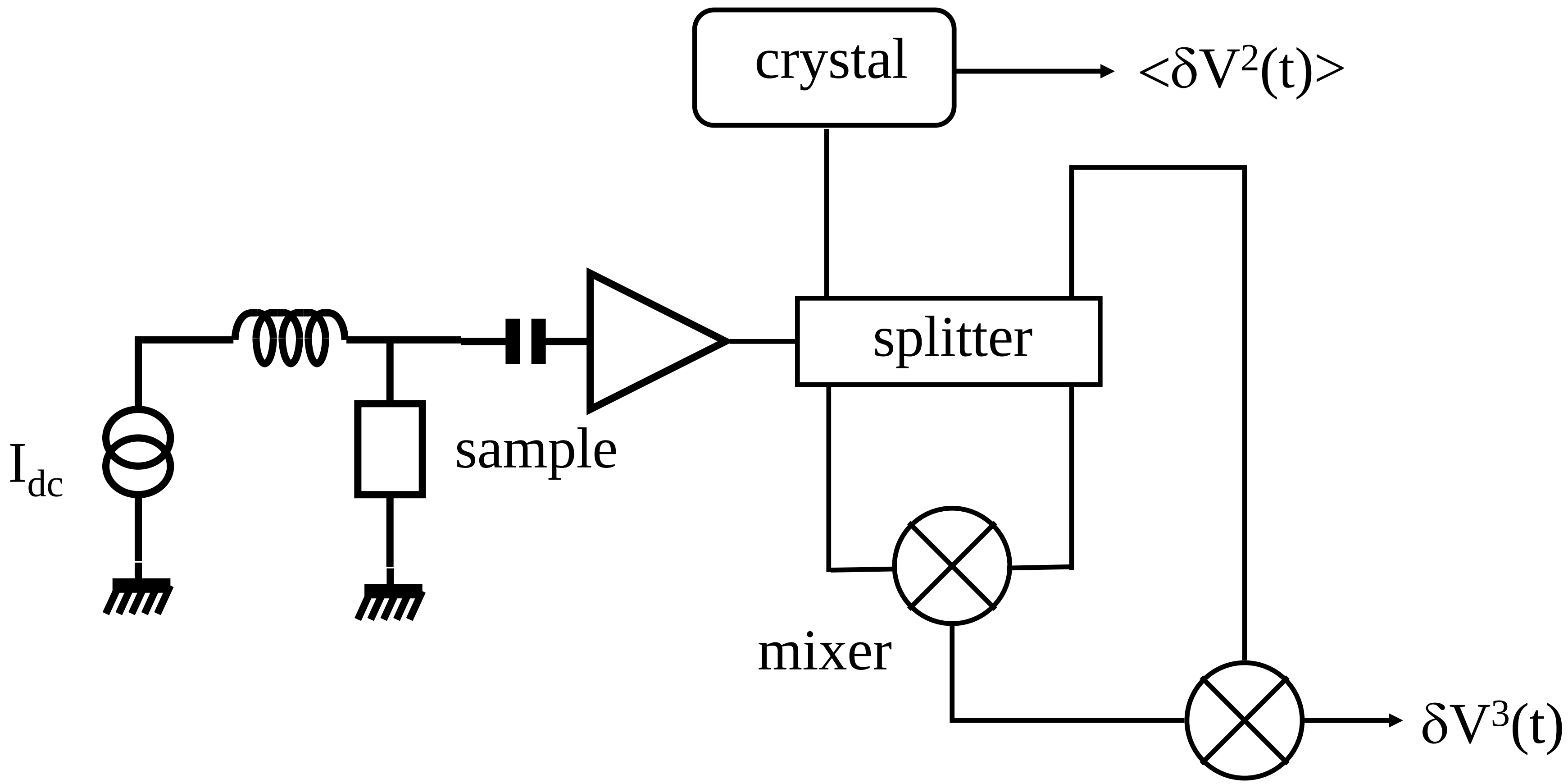
Current in a tunnel junction

$S_3 = e^2 I$ independent of temperature !



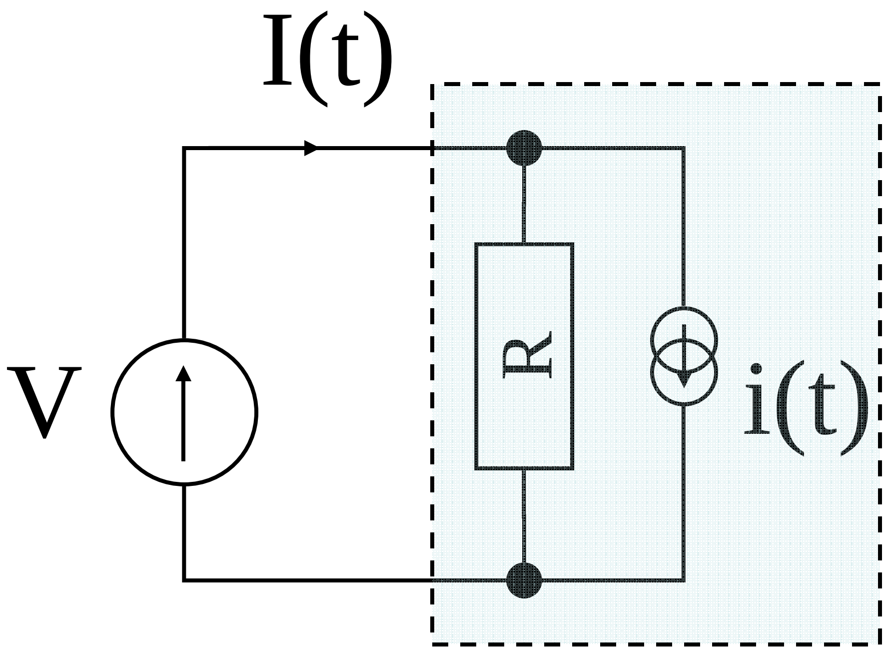


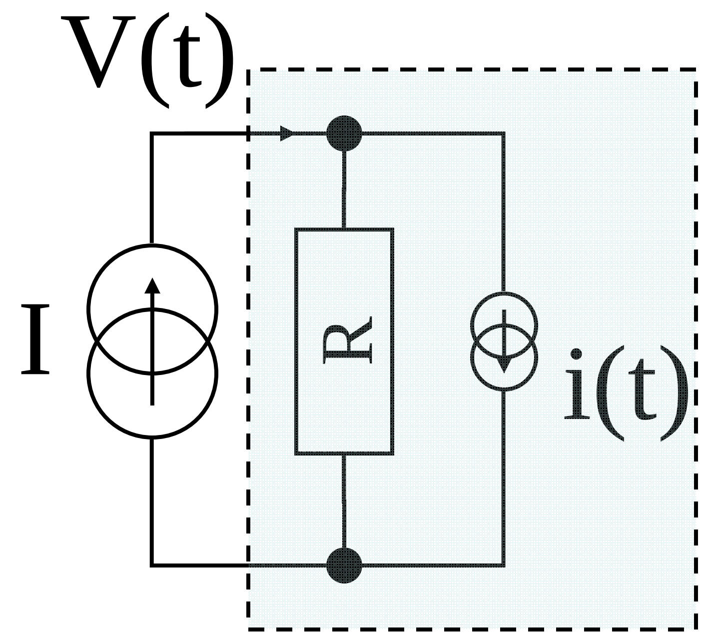
Measurement: method



Bandwidth: 10-1200 MHz

Voltage bias vs. Current bias





$$\delta I(t) = i(t)$$

$$\delta V(t) = -Ri(t)$$

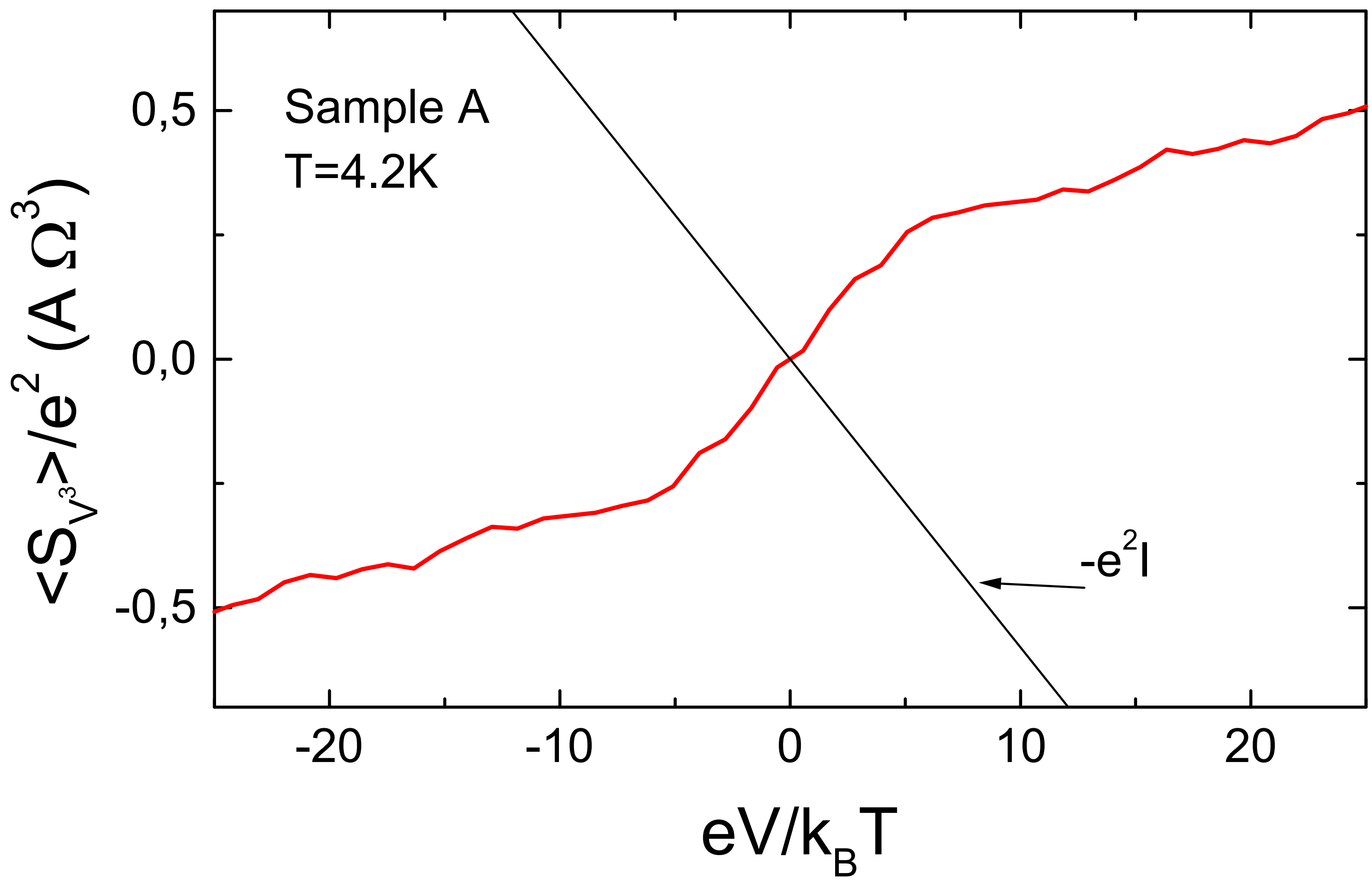
$$V = RI$$

$$\langle \delta V^2 \rangle = R^2 \langle \delta I^2 \rangle$$

$$\langle \delta V^3 \rangle = -R^3 \langle \delta I^3 \rangle ?$$

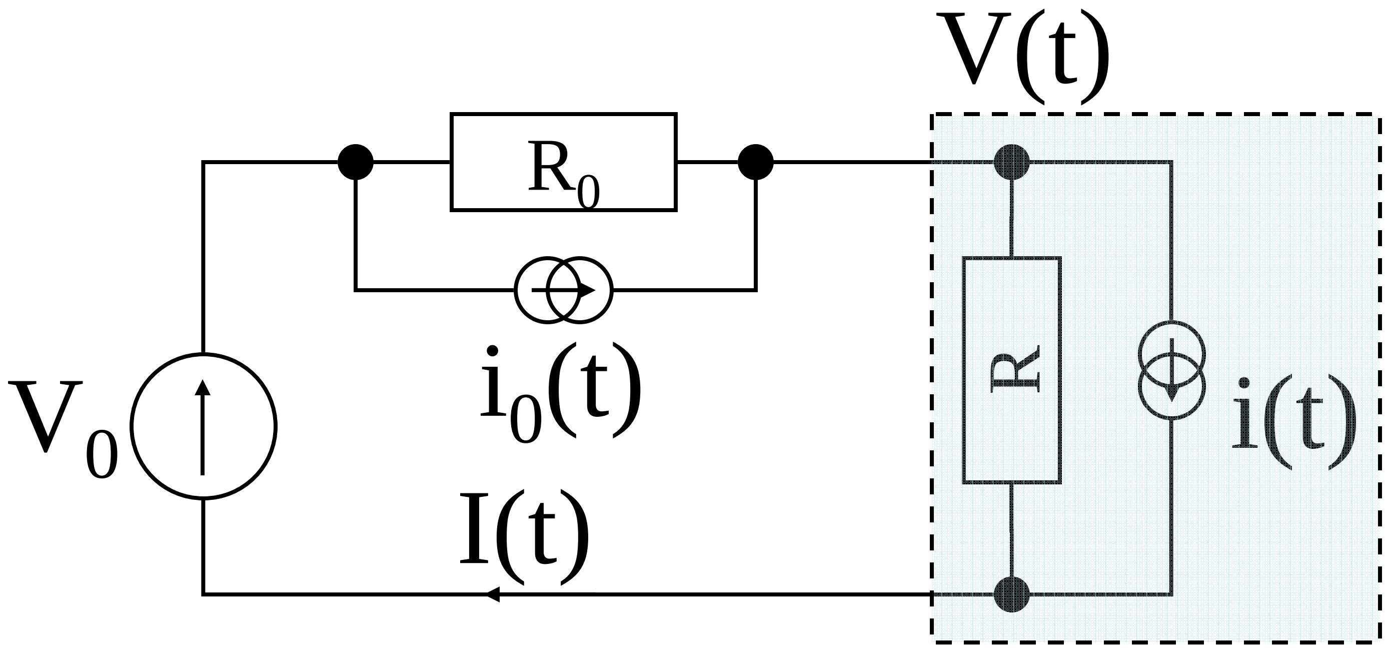
Result (T=4.2K)

(averaged over 12 days !)



Environmental effects

Imperfect voltage bias



$$\delta V(t) = (R // R_0)\left(i_0(t) - i(t)\right)$$

External noise Feedback of the environment

The probability distribution P(i) depends on V(t)

Effect of the environment on the probability distribution P(i)

$$P(i,V) = P_V(i, V - R_0 i) \cong P_V(i,V) - R_0 i \frac{\partial P_V(i,V)}{\partial V}$$

↑

Probability when voltage biased

Effect on the moments:

$$\langle i^n \rangle = \langle i^n \rangle_V - R_0 \frac{\partial \langle i^{n+1} \rangle_V}{\partial V}$$

Origin: e-e interactions

Application: n=3

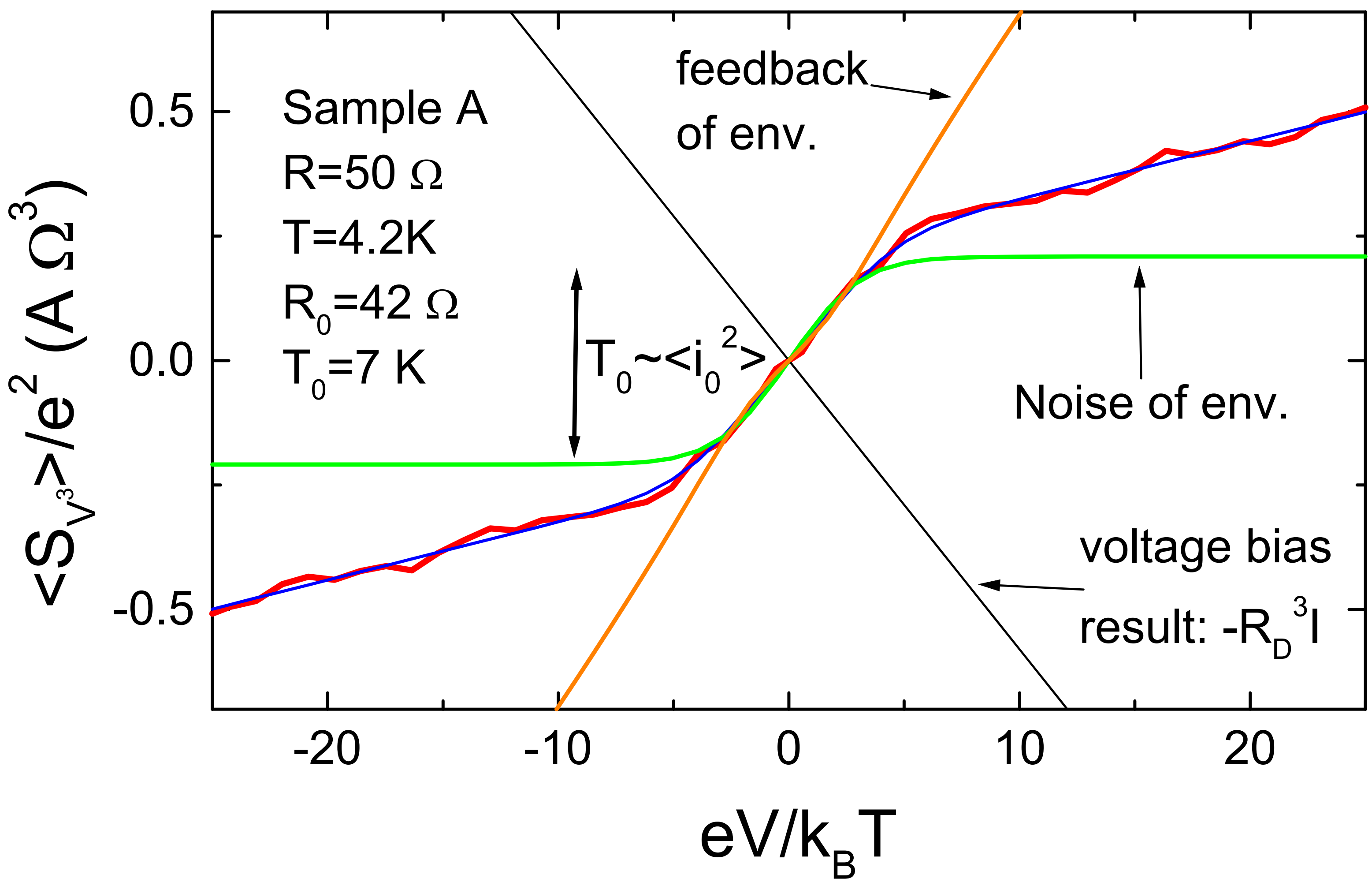
* Effect on the third moment:

$$\left\langle i^3 \right\rangle = e^2 I - R_0 \frac{\partial S_4}{\partial V} \approx e^2 I - 6 R_0 S_2 \frac{\partial S_2}{\partial V}$$

Noise Susceptibility
at $\omega_0 \sim 0$

$$\chi_{\omega_0 = 0}(\omega) = \frac{\partial S_2(\omega)}{\partial V}$$

Exp. Result + theory



Effect of the environment on the probability distribution P(i)

$$\langle i^n \rangle = \langle i^n \rangle_V - R_0 \frac{\partial \langle i^{n+1} \rangle_V}{\partial V}$$

Application: n=1

* Effect on the dc current:
dynamical Coulomb blockade

$$\langle i \rangle = I_{dc} - R_0 \frac{\partial S_2}{\partial V}$$

Noise Susceptibility at $\omega_0 \sim \omega$

$$\chi_\omega(\omega)$$

Link between Dynamical Coulomb Blockade and Noise Susceptibility

$$\delta I = \frac{\hbar}{e^2}\int K(-\omega)\,\omega\,\chi_{\omega}(\omega)\,d\omega$$

Correlation function of the environment

Noise susceptibility for $\omega_0=\omega$

cf Lamb shift…

For small environmental impedance:

$$\delta I = -\int \mathrm{Re}\,Z_0(\omega)\,\chi_{\omega}(\omega)\,d\omega$$

There is nothing quantum …

The third cumulant of current fluctuations in the quantum regime

S_3 and Q mechanics: ordering ???

$$S_3(\omega,\omega') = \int dt dt' e^{i(\omega t+\omega' t')} \left\langle \hat{I}(0,t,t'?)\hat{I}(0,t,t'?)\hat{I}(0,t,t'?) \right\rangle$$

Two results:

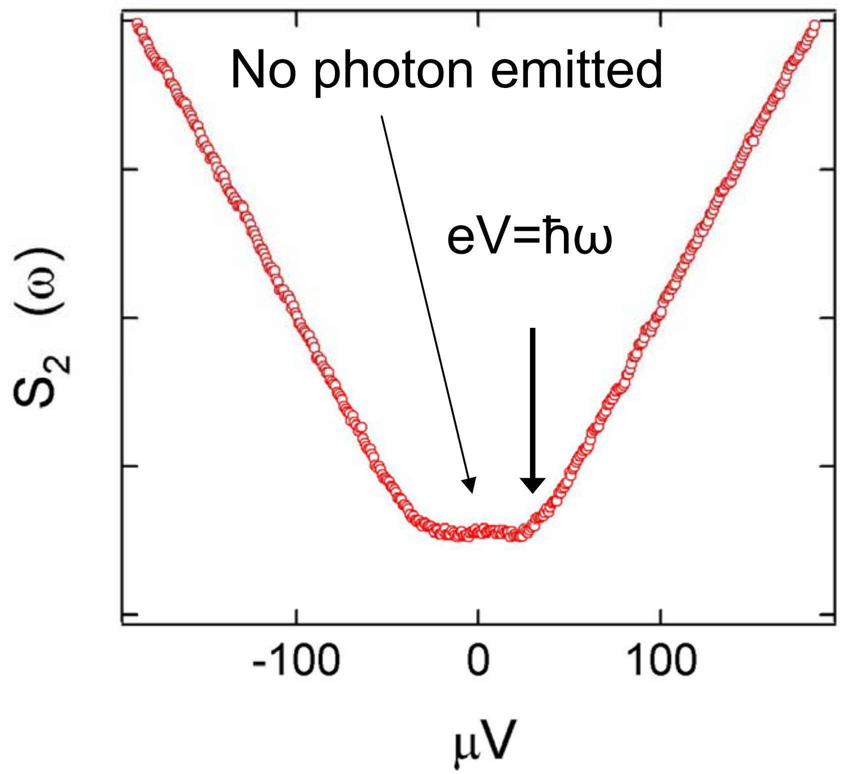
$$S_3(0,0) = \frac{e^2}{h} V \cdot \begin{cases} t(1-t)(1-2t) \\ t^2(1-t) \end{cases}$$

Keldysh ordering

Fully symmetrized

$t \approx 10^{-5}$ for a tunnel junction

The third cumulant in the quantum regime: $S_3(0,\omega)$ with $\hbar\omega > k_B T, eV$

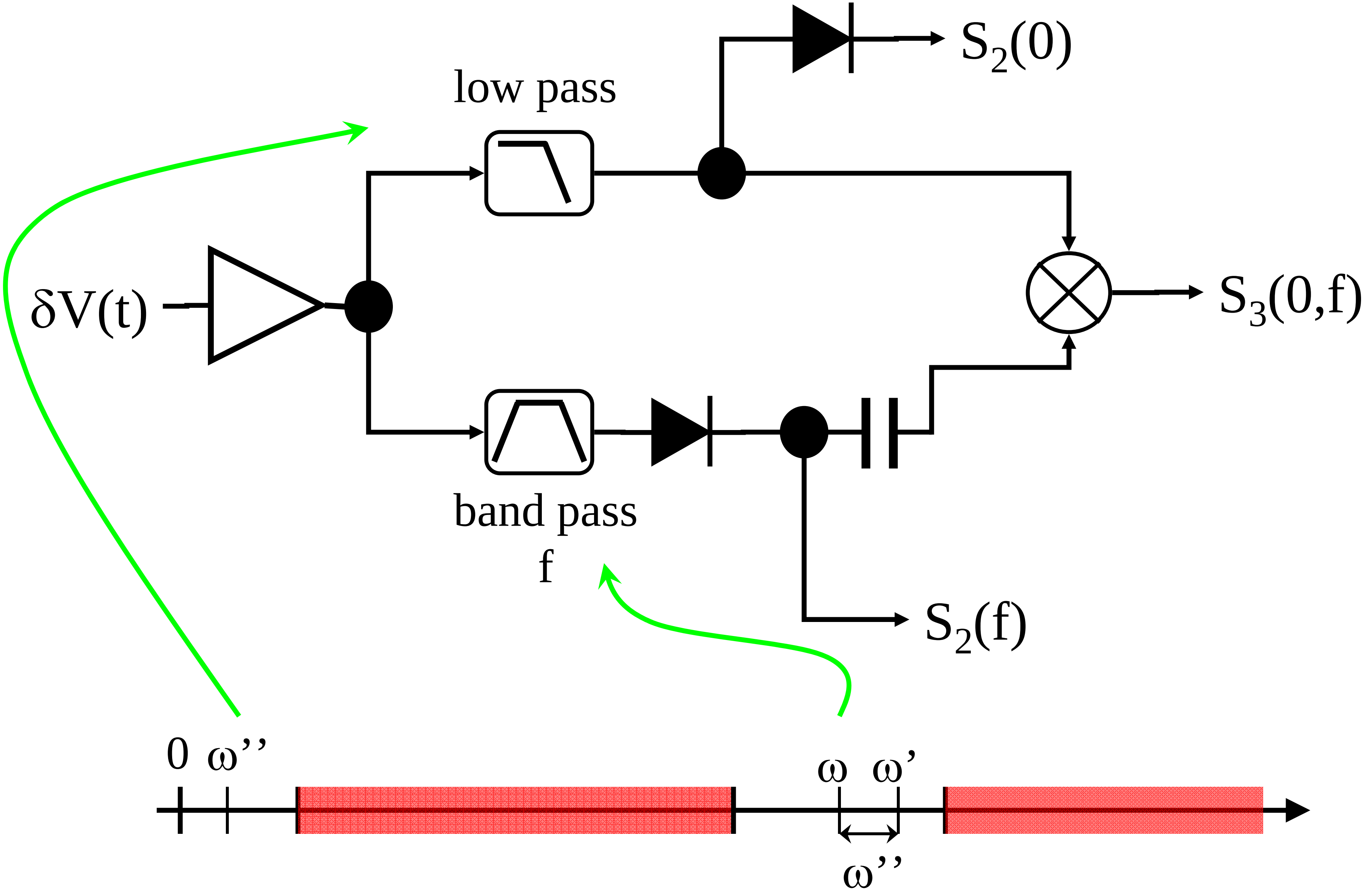


Prediction:

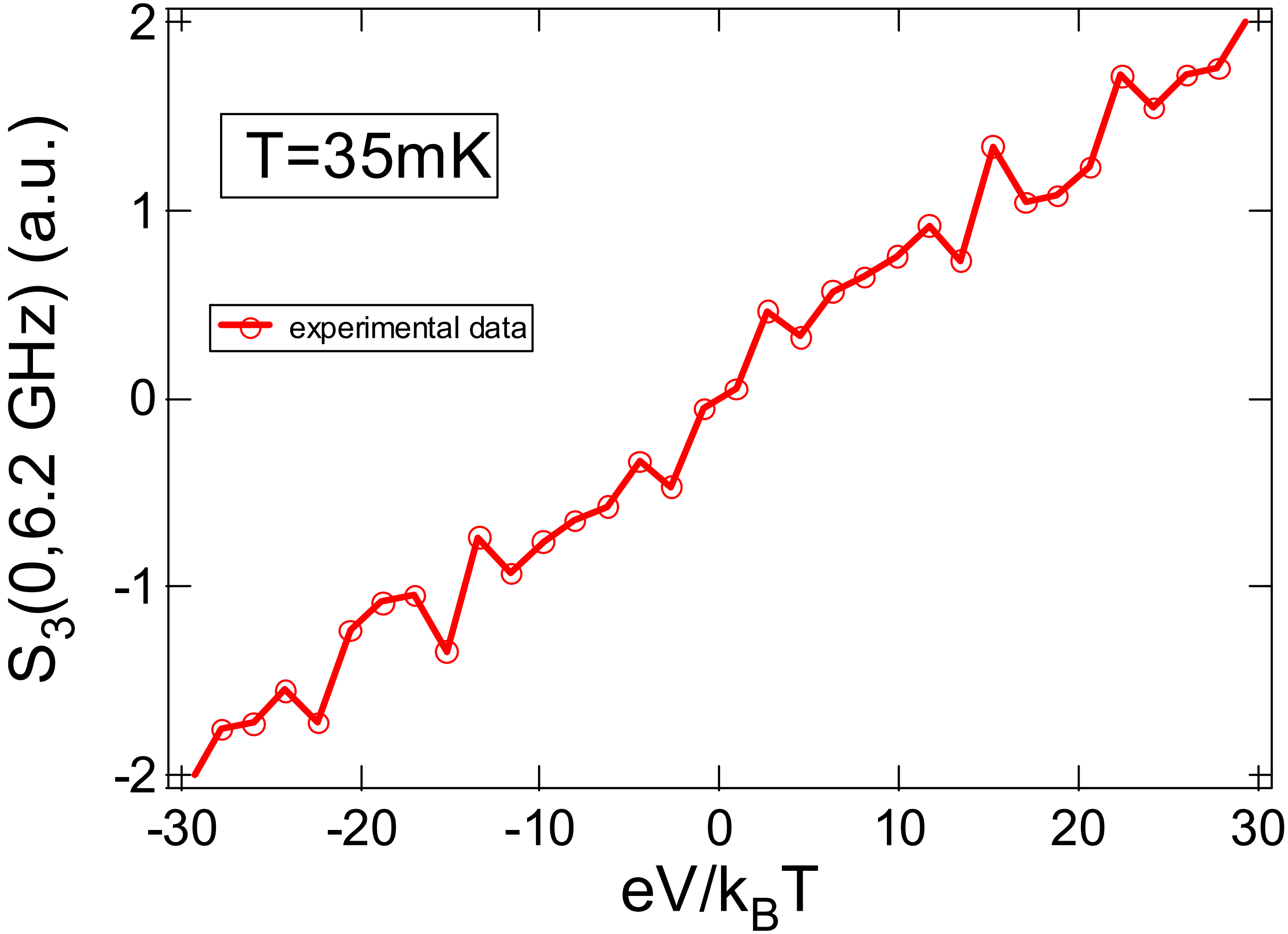
$S_3(\omega,\omega') = e^2 I$

indep of frequency !!

How to measure $S_3(0,f)$?



$S_3(0,\omega)$: experiment - minus environment in the Q regime



The third cumulant in the quantum regime: $S_3(0,\omega)$ with $\hbar\omega > k_B T, eV$

$$S_2(\omega) = \langle \delta I(\omega)\delta I(-\omega) \rangle$$

~ power of light emitted at frequency ω
=0 for $\hbar\omega > eV$

$$S_3(0,\omega) = \langle \delta I(0)\delta I(\omega)\delta I(-\omega) \rangle$$

~ correlations between low frequency current fluctuations and power of light at frequency ω.

Prediction:

$S_3(\omega,\omega')=e^2 I$ independant of frequency !?

WARNING: zero point motion of electrons !!

Another way to measure $S_3(0,f)$?

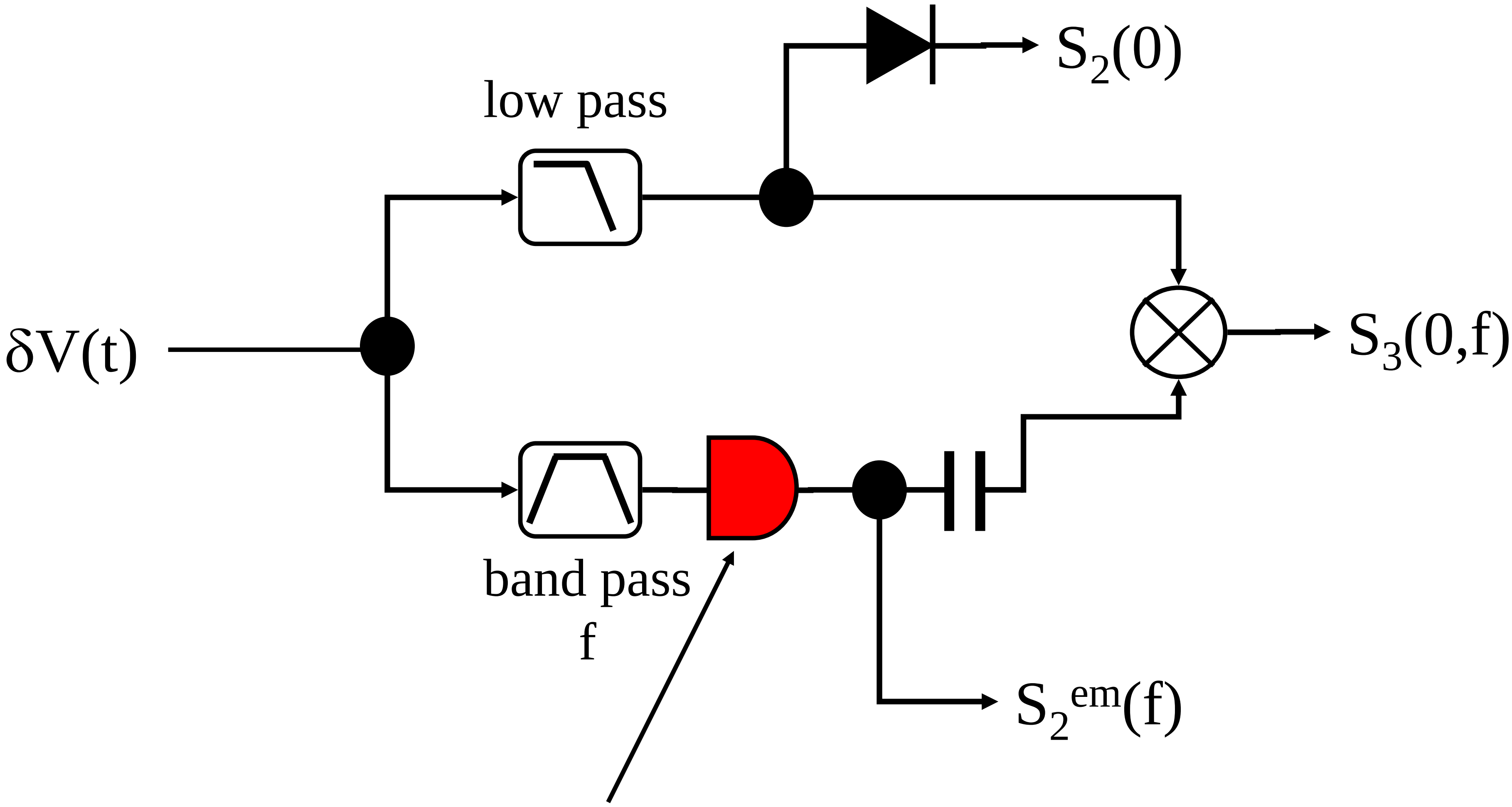
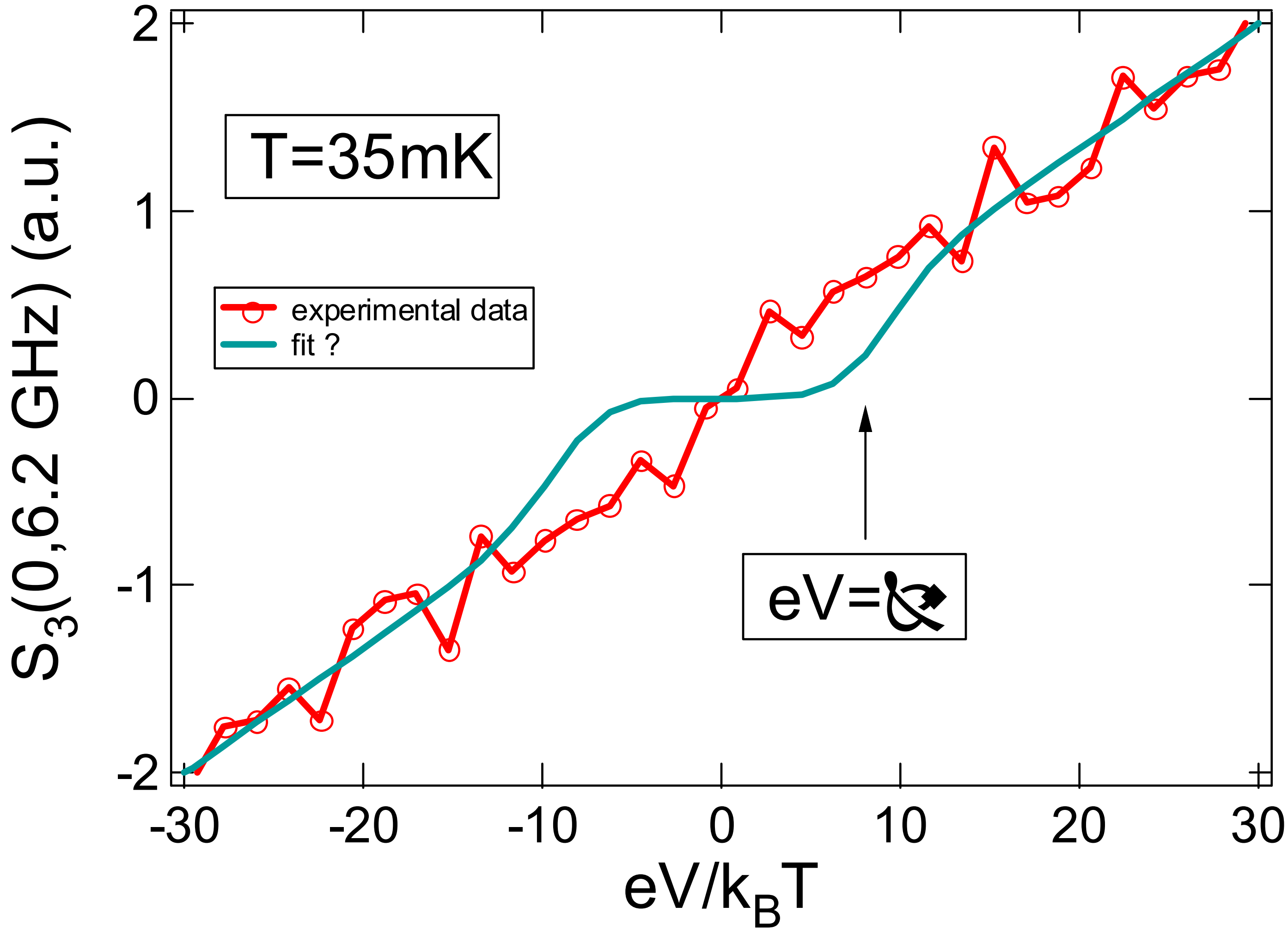


Photo-multiplier: absorbs photons

$S_3(0,\omega)$: experiment - minus environment in the Q regime



Another way to measure $S_3(0,f)$?

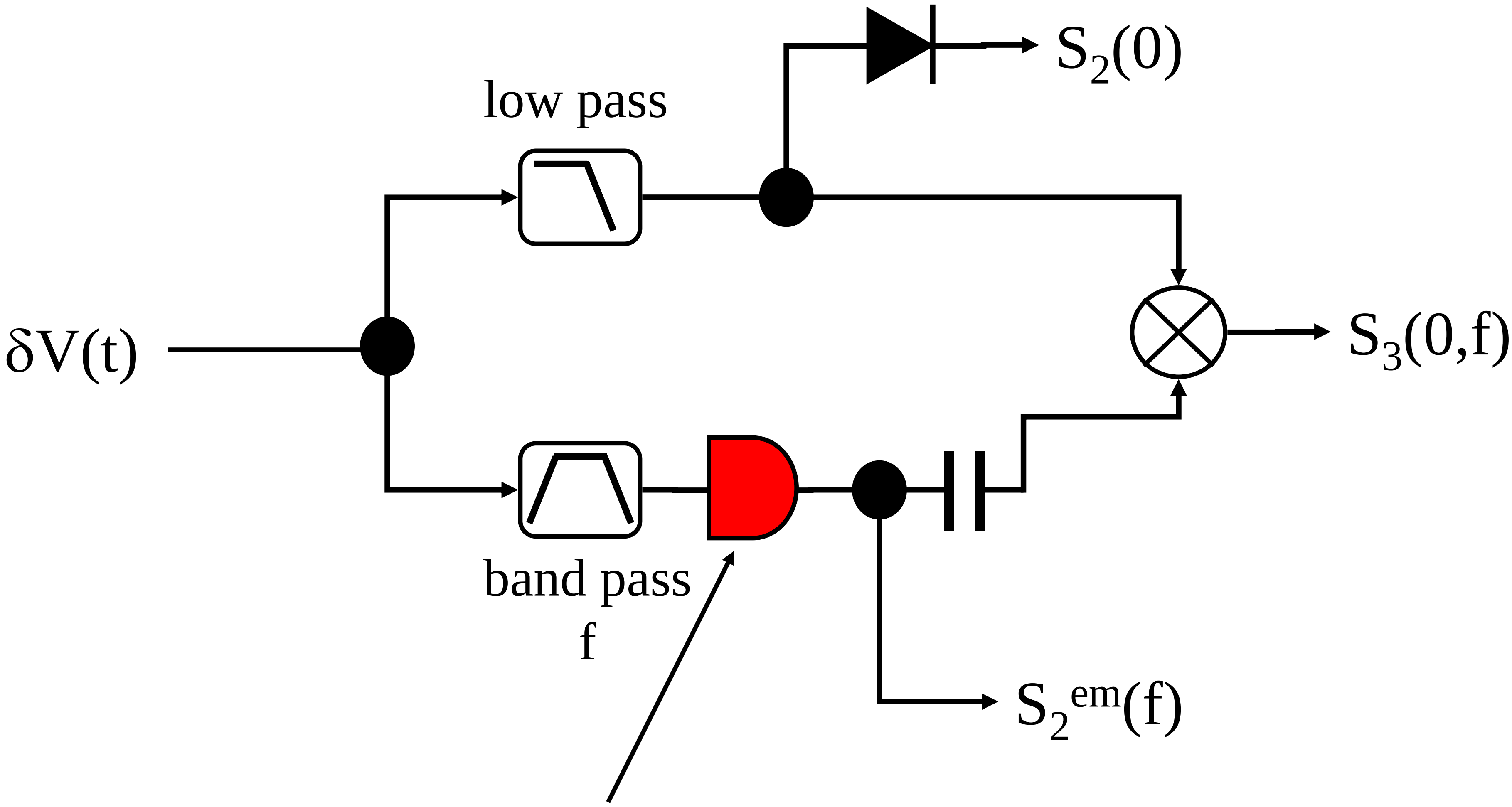
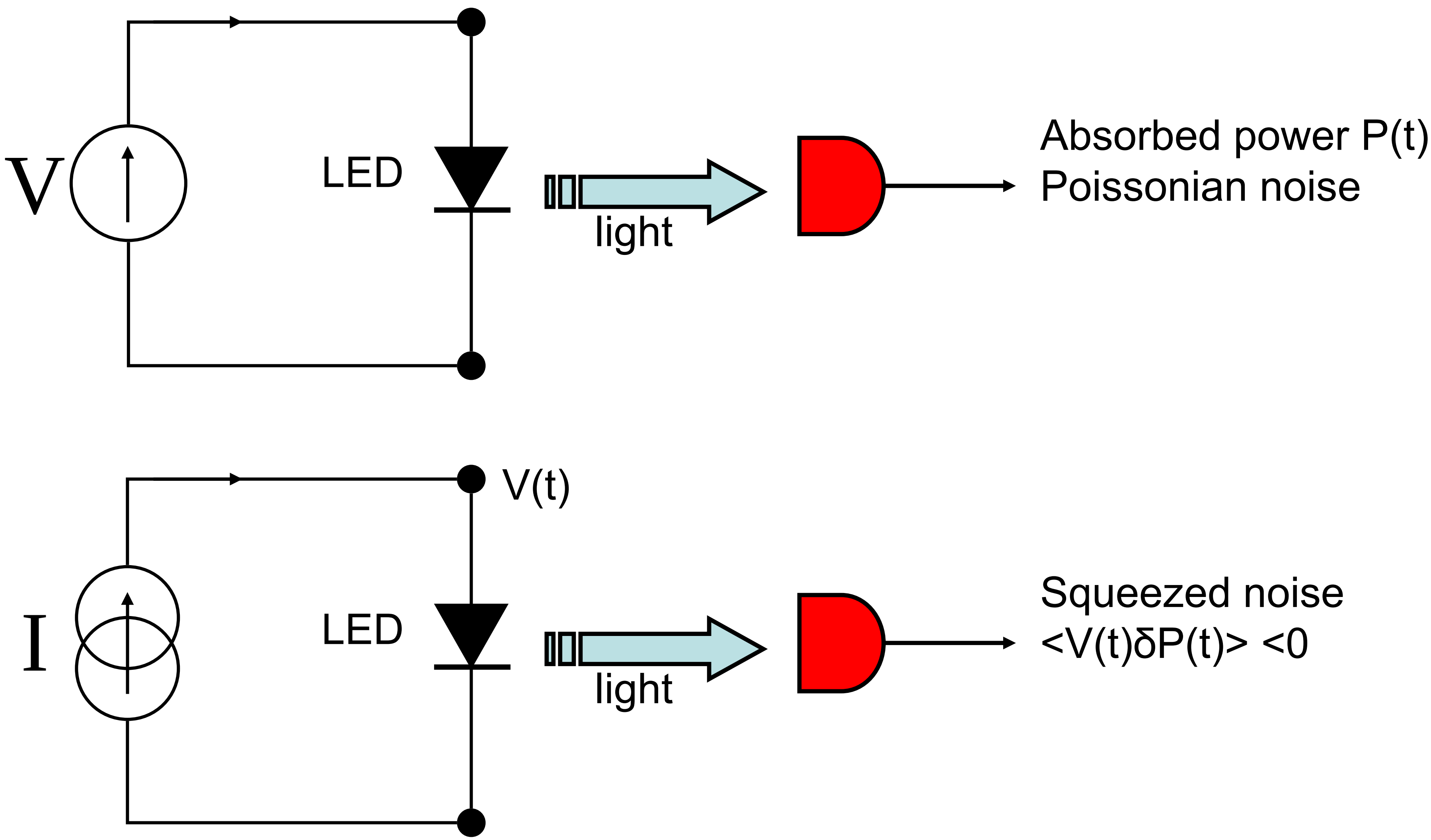


Photo-multiplier: absorbs photons

Similar Q optics experiments…



Conclusion

NOISE SUSCEPTIBILITY

Probe for inelastic mechanisms:
* electron-phonon time vs T in normal metals at very low T
* electron-electron time
* in more subtle systems (carbon nanotubes, quasi-crystals, etc.)

Link with dynamical Coulomb blockade:
* Van der Waals interaction between noise sources ?
* Coulomb blockade at room T ?

THIRD CUMULANT OF NOISE

Test for deep understanding of transport mechanisms ?
Measurement theory: how to order three current operators in quantum mechanics ?

Which system exhibits gaussian noise ?

* Not a radiator, not a light bulb run through by a current (more current, more heat or light)…
* Not the cosmologic background (?)
* Not a chemical reaction (cf tunnel junction)
* Not a finite system, even at equilibrium (there is S_4…)

ONLY a perfectly transmitting channel…