Chaire de Physique Mésoscopique
Michel Devoret
Année 2011, 10 mai - 21 juin

AMPLIFICATION ET RETROACTION QUANTIQUES

QUANTUM AMPLIFICATION AND FEEDBACK

Cinquième Leçon / *Fifth Lecture*

LU:110624

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction to quantum-limited amplification and feedback

Lecture II: How do we model open, out-of-equilibrium, non-linear quantum systems?

Lecture III: Can we maintain the noise at the quantum limit while increasing gain, bandwidth and dyn^{amic} range?

Lecture IV: What are the minimal requirements for an active circuit to be fully directional and noiseless?

Lecture V: Can a continuous quantum measurement be viewed as a form of Brownian motion?

Lecture VI: How can we maintain a dynamic quantum state alive?

CALENDAR OF SEMINARS

May 10: Fabien Portier, SPEC-CEA Saclay
The Bright Side of Coulomb Blockade

May 17, 2011: Jan van Ruitenbeek (Leiden University, The Netherlands)
Quantum Transport in Single-molecule Systems

May 31, 2011: Irfan Siddiqi (UC Berkeley, USA)
Quantum Jumps of a Superconducting Artificial Atom

June 7, 2011: David DiVicenzo (IQI Aachen, Germany)
Quantum Error Correction and the Future of Solid State Qubits

June 14, 2011: Andrew Cleland (UC Santa Barbara, USA)
Images of Quantum Light

June 21, 2011: Benjamin Huard (LPA - ENS Paris)
Building a Quantum Limited Amplifier from Josephson Junctions and Resonators

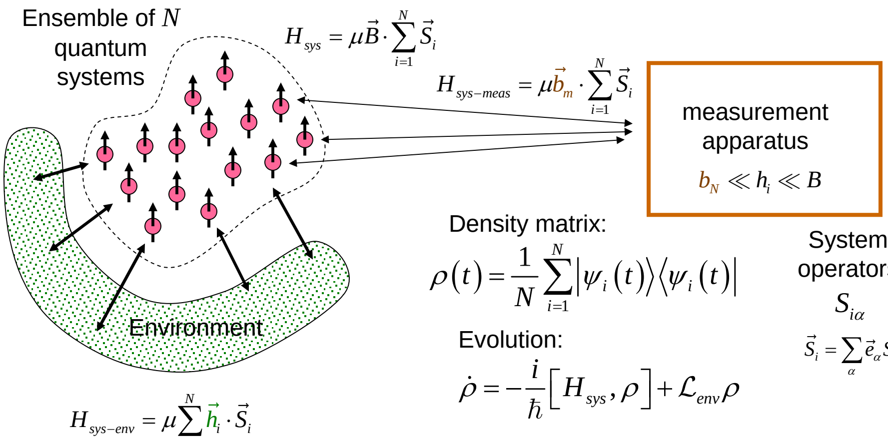
June 21, 2011 (3pm): Andrew Cleland (UC Santa Barbara, USA)
How to Be in Two Places at the Same Time ?

LECTURE V : AMPLIFIERS AND MEASUREMENTS

OUTLINE

1. Ensemble measurements versus continuous measurement of a single system
2. Monitoring of the Z component of a qubit by a quantum-limited amplifier
3. Monitoring of the charge of a LC oscillator by a quantum-limited amplifier
4. Quantum stochastic equation for density matrix of qubit under measurement

"TRADITIONAL" QUANTUM MEASUREMENT



Ensemble of N quantum systems

$$H_{sys} = \mu\vec{B}\cdot\sum_{i=1}^{N}\vec{S}_i$$

$$H_{sys-meas} = \mu\vec{b}_m\cdot\sum_{i=1}^{N}\vec{S}_i$$

measurement apparatus

$$b_N \ll h_i \ll B$$

Environment

$$H_{sys-env} = \mu\sum_{i=1}^{N}\vec{h}_i\cdot\vec{S}_i$$

Density matrix:

$$\rho(t) = \frac{1}{N}\sum_{i=1}^{N}\left|\psi_i(t)\right\rangle\left\langle\psi_i(t)\right|$$

System operators:

$$S_{i\alpha}$$

$$\vec{S}_i = \sum_{\alpha}\vec{e}_\alpha S_{i\alpha}$$

Evolution:

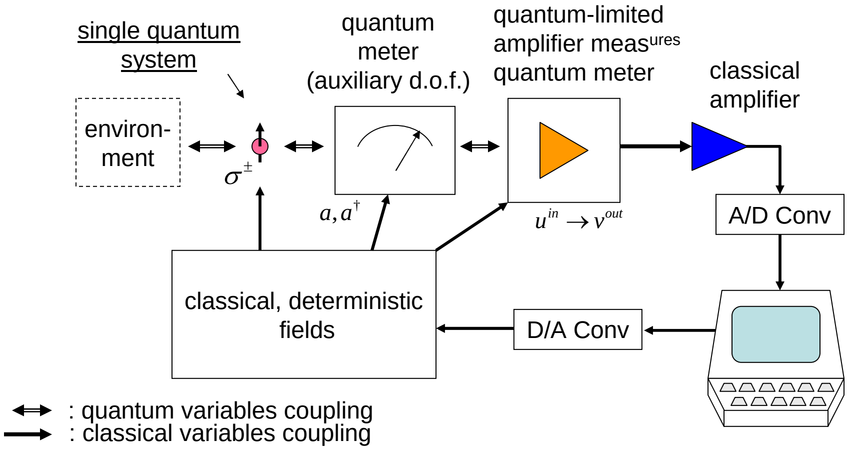
$$\dot{\rho} = -\frac{i}{\hbar}\left[H_{sys},\rho\right] + \mathcal{L}_{env}\rho$$

Measurement accesses:

$$\left\langle S_\alpha(t)\right\rangle = \mathrm{Tr}\left[S_\alpha\rho(t)\right]$$

Noise of measurement apparatus and its back-action on the quantum system can be neglected as long as the coupling between them is very weak and N is very large.

"MODERN" QUANTUM MEASUREMENT



single quantum system

environ-ment

$\sigma^{\pm}$

quantum meter (auxiliary d.o.f.)

$a, a^{\dagger}$

quantum-limited amplifier meas^{ures} quantum meter

$u^{in} \to v^{out}$

classical amplifier

A/D Conv

D/A Conv

classical, deterministic fields

⟷ : quantum variables coupling

⟶ : classical variables coupling

Notes:

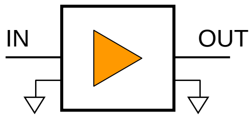
1) quantum meter is interface between quantum system and measurement

2) environment of quantum system can eventually be limited to meter and amplifier

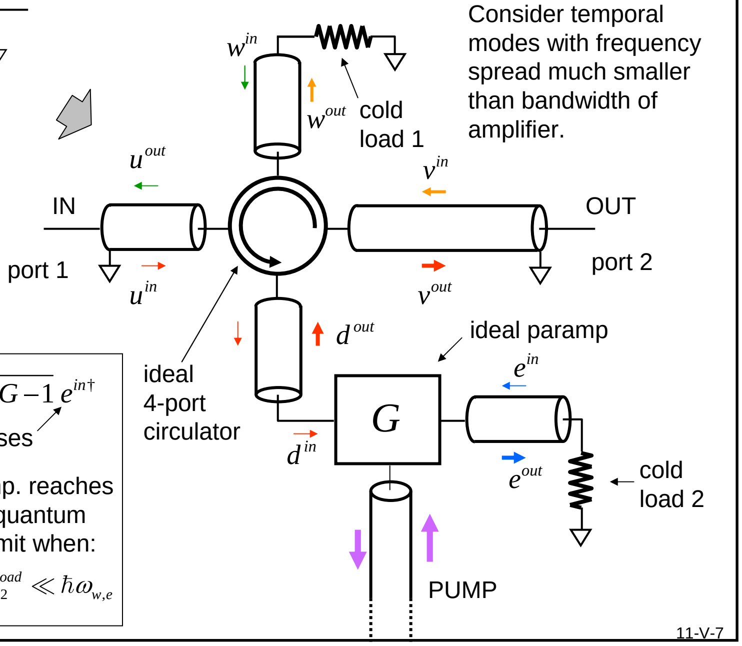
Operators can be treated class^{lly} when: $\left\langle[A,B]\right\rangle \ll \langle A\rangle\langle B\rangle$

Example: $\left\langle\left[a,a^{\dagger}a\right]\right\rangle = \langle a\rangle \ll \langle a\rangle\left\langle a^{\dagger}a\right\rangle$ when $\bar{n} = \left\langle a^{\dagger}a\right\rangle \gg 1$

DESCRIPTION OF AMPLIFIER



Courty et al. (1999)



Consider temporal modes with frequency spread much smaller than bandwidth of amplifier.

$$v^{out} = \sqrt{G}\, u^{in} + \sqrt{G-1}\, e^{in\dagger}$$
$$u^{out} = w^{in}$$

noises

$$S = \begin{bmatrix} 0 & 0 \\ \sqrt{G} & 0 \end{bmatrix}$$

Amp. reaches quantum limit when:

$$kT_{1,2}^{load} \ll \hbar\omega_{w,e}$$

IT IS POSSIBLE IN THIS SETUP TO CONTROL PRECISELY THE QUANTUM PROCESS THAT WE CALL MEASUREMENT.

Situation similar, but not identical, to that in Rydberg atom experiments.
(see Haroche and Raimond, "Exploring the Quantum")
Here atoms are fixed, and microwave photons are detected.

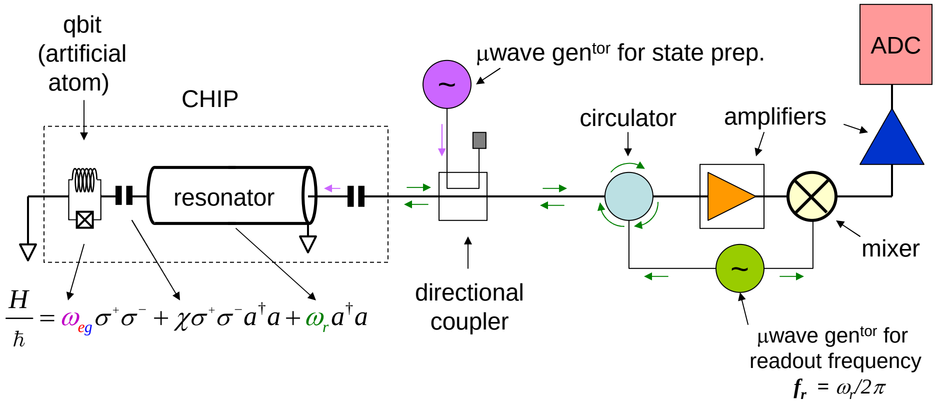
LECTURE IV : AMPLIFIERS AND MEASUREMENTS

OUTLINE

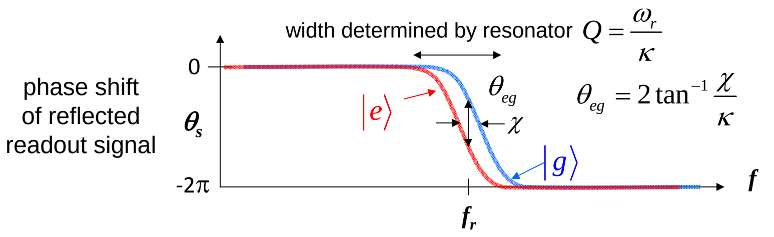
1. Ensemble measurements versus continuous measurement of a single system
2. Monitoring of the Z component of a qubit by a quantum-limited amplifier
3. Monitoring of the charge of a LC oscillator by a quantum-limited amplifier
4. Quantum stochastic equation for density matrix of qubit under measurement

11-V-4

DISPERSIVE CQED QUBIT MEASUREMENT



$$\frac{H}{\hbar} = \omega_{eg}\sigma^+\sigma^- + \chi\sigma^+\sigma^- a^\dagger a + \omega_r a^\dagger a$$



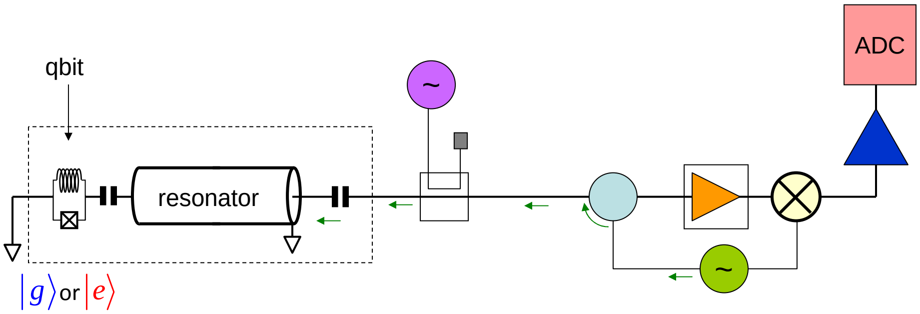
$$\theta_{eg} = 2\tan^{-1}\frac{\chi}{\kappa}$$

transmon cavity pull:

$$\chi \cong \frac{2E_c g^2}{\Delta^2}$$

11-I-28bis, 11-V-9

POWERING THE RESONATOR

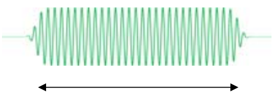


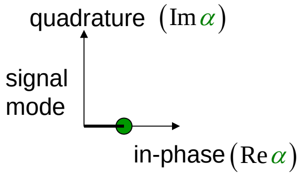
$|g\rangle$ or $|e\rangle$

or $\frac{|g\rangle+|e\rangle}{\sqrt{2}}$

THE METER →

= a bunch
of flying photons





~400ns → $T_m \gg \kappa^{-1}$ ← ~20ns

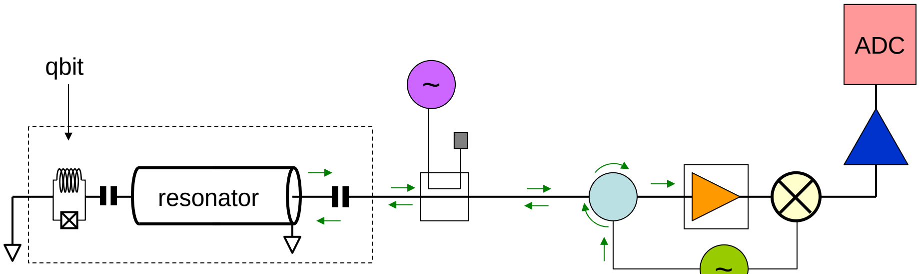
$T_m \ll T_1$ ← ~10µs

f_r ~ 8GHz > f_{eg}

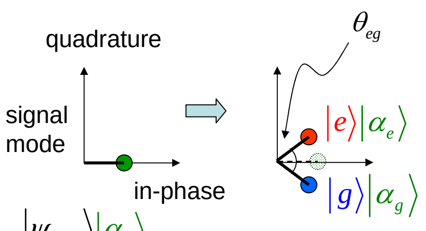
$|\psi_{qbit}\rangle|\alpha_0\rangle$

Average number of
photons used in readout → $\bar{n}_m = |\alpha|^2 \sim 20$

RECEIVING SIGNAL FROM RESONATOR



$|g\rangle$ or $|e\rangle$ or $\frac{|g\rangle+|e\rangle}{\sqrt{2}}$

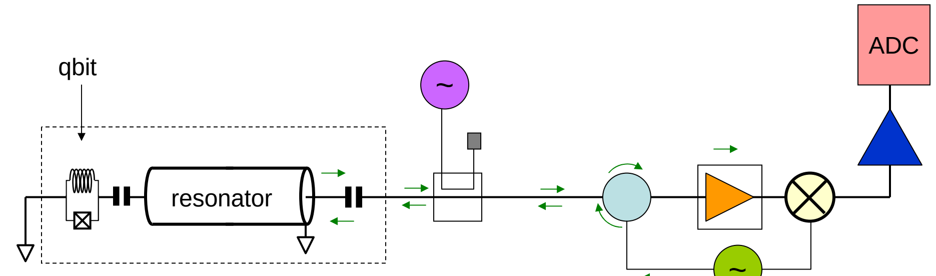


$|\psi_{qbit}\rangle|\alpha_0\rangle$

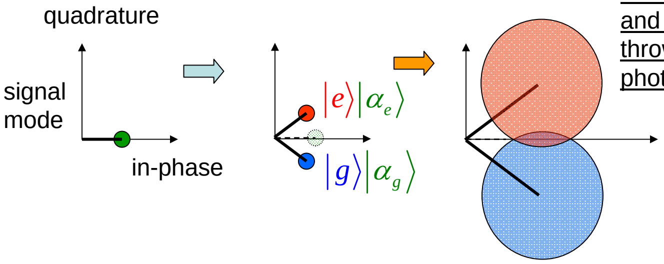
or $\frac{|g\rangle|\alpha_g\rangle+|e\rangle|\alpha_e\rangle}{\sqrt{2}}$

Entanglement between qubit and
resonator preserved as long as
qubit or resonator is not dephased
or relaxed.

AMPLIFYING SIGNAL FROM RESONATOR



$|g\rangle$ or $|e\rangle$ or $\frac{|g\rangle+|e\rangle}{\sqrt{2}}$

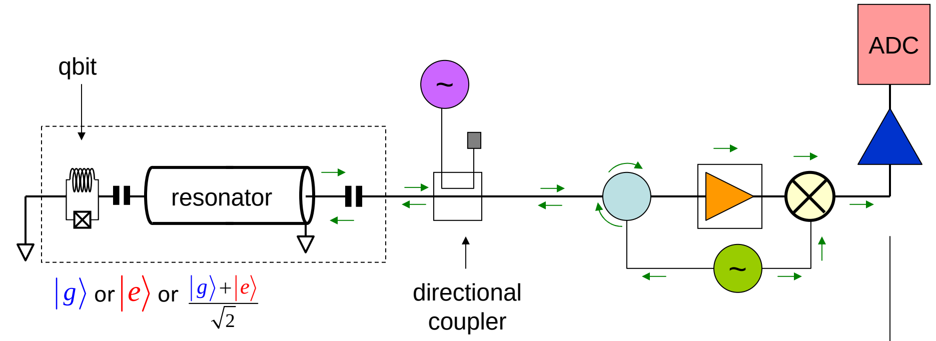


Entanglement between qubit and resonator is now killed by throwing out the idler port photons !

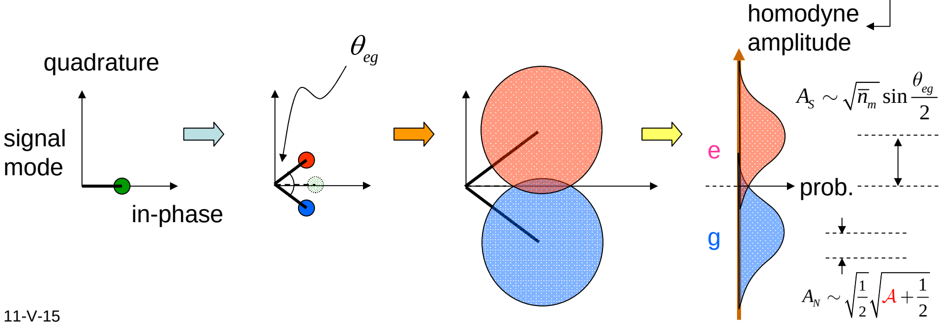
Boundary between quantum and classical part of apparatus is amplifier (when gain and signal are large)

11-V-13

PROCESSING SIGNAL FROM RESONATOR

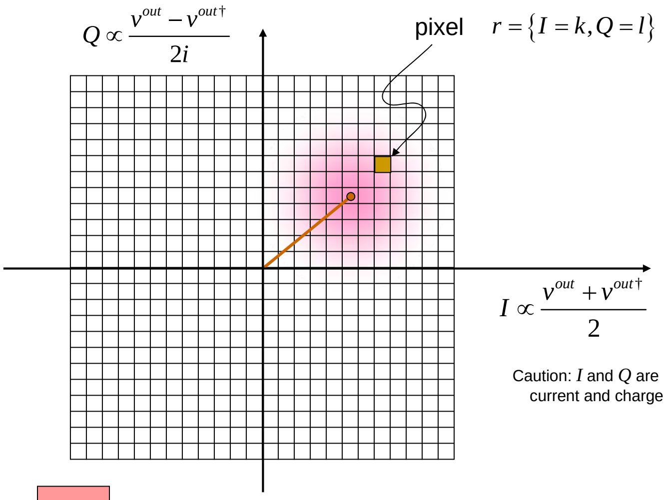


$|g\rangle$ or $|e\rangle$ or $\frac{|g\rangle+|e\rangle}{\sqrt{2}}$

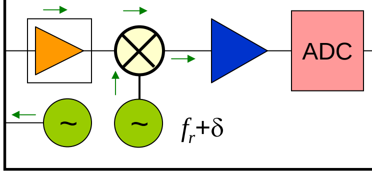


11-V-15

OUTPUT SIGNAL MODE IN I-Q PLANE



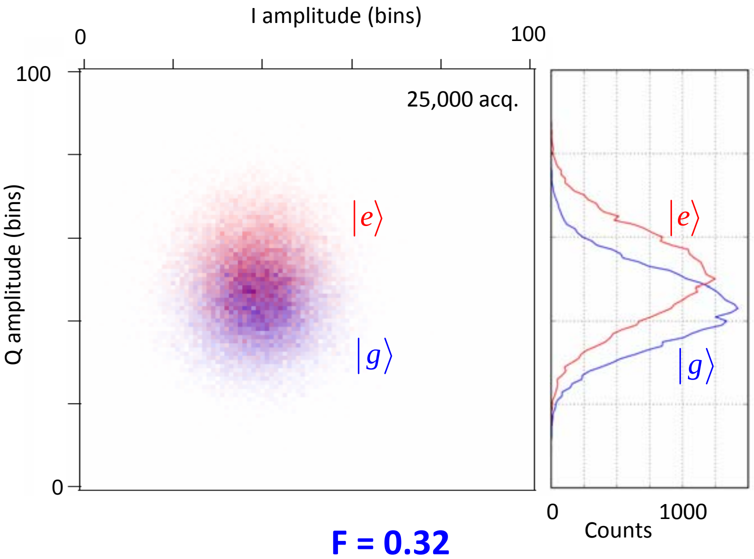
Caution: I and Q are not current and charge!



$$I = \int dt S \cos \delta t$$
$$Q = \int dt S \sin \delta t$$

Obtained in practice by heterodyning, rather than homodyning, output signal, and integrating the signal components.

EXAMPLE OF DATA TAKEN WITH JOSEPHSON PARAMP

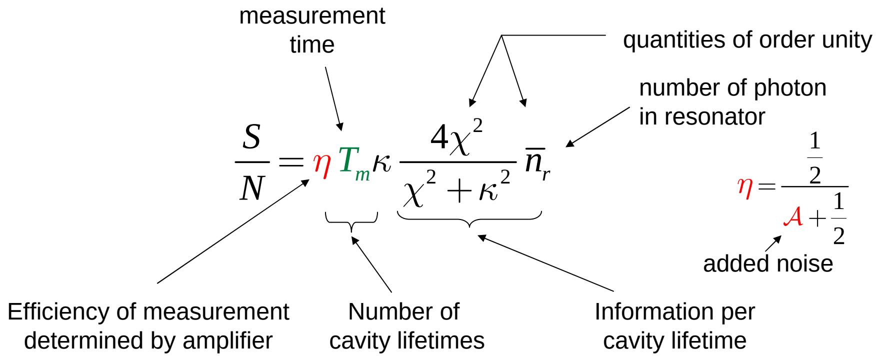


F = 0.32

M. Hatridge (2011)

SIGNAL TO NOISE RATIO OF MEASUREMENT

Gambetta et al. (2008)



measurement time

quantities of order unity

number of photon in resonator

$$\frac{S}{N} = \eta \, T_m \kappa \, \frac{4\chi^2}{\chi^2+\kappa^2} \, \overline{n}_r$$

$$\eta = \frac{\frac{1}{2}}{\mathcal{A}+\frac{1}{2}}$$

added noise

Efficiency of measurement determined by amplifier

Number of cavity lifetimes

Information per cavity lifetime

The full discrete measurement with duration T_m can be thought of as the unfolding of a continuous measurement acquiring information at the effective rate:

$$\gamma_m = \kappa \, \overline{n}_r \frac{4\chi^2}{\chi^2+\kappa^2}$$

"measurement rate"

GENERALIZED MEASUREMENT OPERATOR

Two different Hilbert spaces: system and meter. Meter space is larger than system's.

Initial state of system and meter $\quad |\Psi(t)\rangle = |\alpha(t)\rangle |\psi(t)\rangle$

Co-evolution of system and meter $\quad |\Psi(t+\Delta t)\rangle = U(\Delta t) |\alpha(t)\rangle |\psi(t)\rangle$

Entanglement !

GENERALIZED MEASUREMENT OPERATOR

Two different Hilbert spaces: system and meter. Meter space is larger than system's.

Initial state of system and meter $\left|\Psi(t)\right\rangle = \left|\alpha(t)\right\rangle \left|\psi(t)\right\rangle$

Co-evolution of system and meter $\left|\Psi(t+\Delta t)\right\rangle = U(\Delta t)\left|\alpha(t)\right\rangle \left|\psi(t)\right\rangle$

Entanglement !

Operator R of meter $R\left|r\right\rangle = r\left|r\right\rangle$ *(Pixel in I-Q space)*

Projective measurement of meter only, using R , and yielding result r

$$\left|\Psi_r(t+\Delta t)\right\rangle = \frac{\left|r\right\rangle\left\langle r\right|U(\Delta t)\left|\alpha(t)\right\rangle\left|\psi(t)\right\rangle}{\sqrt{\Pr(R=r)}}$$

Define M_r, operator in system Hilbert space $M_r = \left\langle r\right|U(\Delta T)\left|\alpha(t)\right\rangle$

Result probability (probability pixel r lights up) $\Pr(R=r) = \left\langle \psi(t)\right|M_r^\dagger M_r\left|\psi(t)\right\rangle$

Probability operator $E_r = M_r^\dagger M_r$ is a generalization of projector (Set E_r is named POVM)

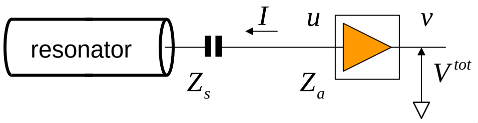
LECTURE IV : AMPLIFIERS AND MEASUREMENTS

OUTLINE

1. Ensemble measurements versus continuous measurement of a single system
2. Monitoring of the Z component of a qubit by a quantum-limited amplifier
3. Monitoring of the charge of a LC oscillator by a quantum-limited amplifier
4. Quantum stochastic equation for density matrix of qubit under measurement

MONITORING AN OSCILLATOR INSTEAD OF A QUBIT

For the qubit measurement, the content of the resonator is "eaten" by the amplifier. What if our system was the resonator? Could we monitor it directly with the amplifier?



One too many sources of noise...!

$$V_N = \sqrt{\frac{\hbar\omega Z_a}{2G}}\left(v^{out} + v^{in}\right)$$

$$I_N = \sqrt{\frac{\hbar\omega}{2Z_a}}\left(u^{out} - u^{in}\right)$$

when input shortened

$$v^{out} = \sqrt{G}\, u^{in} + \sqrt{G-1}\, e^{in\dagger}$$

$$u^{out} = w^{in}$$

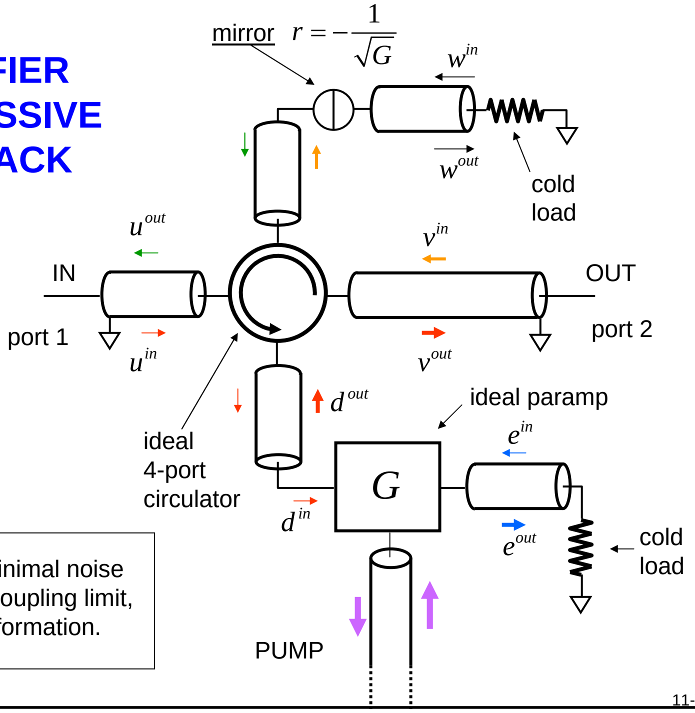
$$V_N^{tot} = \sqrt{\frac{\hbar\omega Z_a}{2}}\left[\left(\frac{Z_s - Z_a}{Z_s + Z_a}\right) w^{in} - e^{in\dagger}\right]$$

The only way for the measurement to be minimally noisy appears to imply $Z_s = Z_a$, which corresponds to matching the resonator to the amplifier and thus critical damping!

Clerk et al., RMP (2010)

A solution exists to repair this problem.....

AMPLIFIER WITH PASSIVE FEEDBACK



Can reach minimal noise in the weak coupling limit, no wasted information.

LECTURE IV : AMPLIFIERS AND MEASUREMENTS

OUTLINE

1. Ensemble measurements versus continuous measurement of a single system
2. Monitoring of the Z component of a qubit by a quantum-limited amplifier
3. Monitoring of the charge of a LC oscillator by a quantum-limited amplifier
4. Quantum stochastic equation for density matrix of qubit under measurement

---

MASTER EQUATION OF QUBIT UNDER MEASUREMENT

Hamiltonian of qubit coupled to cavity resonator

$$\frac{H}{\hbar} = \omega_{eg} \sigma^{+} \sigma^{-} + \chi \sigma^{+} \sigma^{-} a^{\dagger} a + \omega_{r} a^{\dagger} a$$

Coupling of resonator to amplifier :

$$\sqrt{\kappa} a = u^{out} - u^{in}$$

Master equation in the Markov approximation for a general system

$$\longrightarrow \quad \dot{\rho}_{tot} = -\frac{i}{\hbar}\left[H, \rho_{tot}\right] + \sum_{i} \mathcal{D}\left(A_i\right) \rho_{tot}$$

$$\mathcal{D}\left(A\right) \rho = A \rho A^{\dagger} - A^{\dagger} A \rho / 2 - \rho A^{\dagger} A / 2$$

i =decay channel

Master equation in the Markov approximation for qubit alone

Analogous to the Bloch equations

$$\longrightarrow \quad \dot{\rho} = \mathcal{L} \rho = -i \omega_{\acute{e}g} \left[\sigma^{+} \sigma^{-}, \rho \right] + \gamma_1 \mathcal{D}\left(\sigma^{-}\right) \rho$$

$$+ \frac{1}{2}\left[\gamma_{\phi} + \frac{\gamma_m}{2}\right] \mathcal{D}\left(\sigma_z\right) \rho$$

secular

relaxation

dephasing

additional dephasing from meas^{ment}

MEASUREMENT RECORD

quadrature signal increment

z-component of qubit, given previous meas. result

Wiener increment

$$dQ_r = Z_r dt + \frac{dW}{\sqrt{\gamma_m \eta}}$$

Wiener increment defining relations

$$\begin{cases} E\left[dW\right] = 0 \\ dW^2 = dt \end{cases}$$

Idealized white noise

We can thus infer Z_r from a stochastic differential equation giving the evolution of the density matrix. The random "force" is position-dependent, unlike in the usual Langevin equation.

ITO AND STRATONOVITCH STOCHASTIC EQUATION FORMALISMS

Stratonovitch

$$\int_{T_0}^{T} g(t)\,dB(t) = \lim_{N\to\infty}\sum_{i=1}^{N}\frac{\left[g(t_{i+1})+g(t_i)\right]}{2}\left[B(t_{i+1})-B(t_i)\right]$$

Acausal

Ito

$$\int_{T_0}^{T} g(t)\,dB(t) = \lim_{N\to\infty}\sum_{i=1}^{N} g(t_i)\left[B(t_{i+1})-B(t_i)\right]$$

Causal

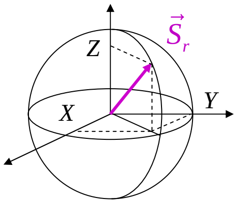
Ito rules of differentiation:

$$d\left(A\cdot B\right) = A\cdot dB + dA\cdot B + dA\cdot dB$$

extra term which can contain parts of same order as first 2

QUANTUM STOCHASTIC EQUATION FOR THE DENSITY MATRIX CONDITIONED BY MEASUREMENT RESULT

Neglecting relaxation and dephasing processes, the 3 components of the Bloch vector obey:



$$\begin{cases} dX_r = -\dfrac{\gamma_m}{2} X_r dt - \sqrt{\eta\gamma_m} X_r Z_r dW \\ dY_r = -\dfrac{\gamma_m}{2} Y_r dt - \sqrt{\eta\gamma_m} Y_r Z_r dW \\ dZ_r = \sqrt{\eta\gamma_m} \left(1 - \left|Z_r\right|^2\right) dW \end{cases}$$

Non-linear Brownian motion equations for the information on qubit!

(Ito formalism)

They are equivalent to the Schrödinger equ$^{\text{tion}}$ + projection postulate.

The evolution is of the form:

dissipation - fluctuation

$$d\vec{S}_r = -\vec{H} \times \vec{S}_r + \frac{1}{2} d\vec{M} \times \left(d\vec{M} \times \vec{S}_r \right) + \eta \left[d\vec{M} - \vec{S}_r \cdot \left(\vec{S}_r \cdot d\vec{M} \right) \right]$$

If η=1, $E\left[d\left(\vec{S}_r\right)^2 \right] = 0$: length of Bloch vector is conserved!

with: $d\vec{M} = \sqrt{\gamma_m} d\vec{W}$

END OF LECTURE