COLLÈGE DE FRANCE 1530

Chaire de Physique Mésoscopique
Michel Devoret
Année 2011, 10 mai - 21 juin

AMPLIFICATION ET RETROACTION QUANTIQUES

QUANTUM AMPLIFICATION AND FEEDBACK

Quatrième Leçon / *Fourth Lecture*

Transparents des leçons disponibles à http://www.physinfo.fr/lectures.html

11-IV-1

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction to quantum-limited amplification and feedback

Lecture II: How do we model open, out-of-equilibrium, non-linear quantum systems?

Lecture III: Can we maintain the noise at the quantum limit while increasing gain, bandwidth and dyn^{amic} range?

Lecture IV: What are the minimal requirements for an active circuit to be fully directional and noiseless?

Lecture V: Can continuous quantum measurements be viewed as a form of Brownian motion?

Lecture VI: How can we maintain a dynamic quantum state alive?

11-IV-2

CALENDAR OF SEMINARS

May 10: Fabien Portier, SPEC-CEA Saclay
The Bright Side of Coulomb Blockade

May 17, 2011: Jan van Ruitenbeek (Leiden University, The Netherlands)
Quantum Transport in Single-molecule Systems

May 31, 2011: Irfan Siddiqi (UC Berkeley, USA)
Quantum Jumps of a Superconducting Artificial Atom

June 7, 2011: David DiVicenzo (IQI Aachen, Germany)
Quantum Error Correction and the Future of Solid State Qubits

June 14, 2011: Andrew Cleland (UC Santa Barbara, USA)
Images of Quantum Light

June 21, 2011: Benjamin Huard (LPA - ENS Paris)
Building a Quantum Limited Amplifier from Josephson Junctions and Resonators

June 21, 2011 (3pm): Andrew Cleland (UC Santa Barbara, USA)
How to Be in Two Places at the Same Time ?

11-IV-3

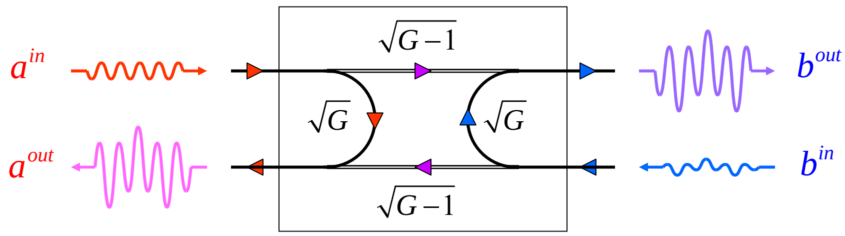
LECTURE IV : SYMMETRY PROPERTIES OF QUANTUM-LIMITED AMPLIFIERS

OUTLINE

1. Fundamental symmetries of scattering by active circuits
2. Passive non-reciprocity by Faraday rotation
3. Proof-of-principle of active, noiseless non-reciprocity
4. Amplifiers versus photomultipliers

11-IV-4

PHASE-PRESERVING MINIMAL AMPLIFIER



Reflection amplifier

Idler port brings in minimum added noise

$$a^{out} = \sqrt{G}a^{in} + e^{i\phi}\sqrt{G-1}\,b^{in\dagger}$$

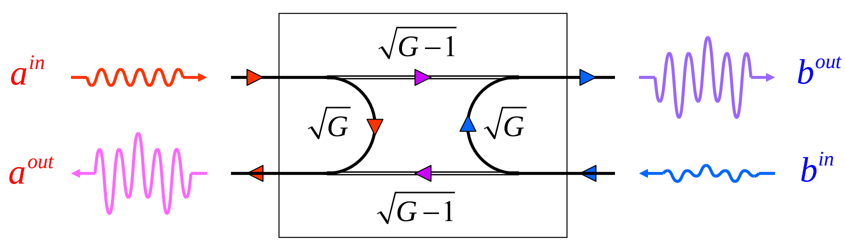
$$N_a^{out} = GN_a^{in} + |G-1|\,N_b^{in}$$
$$= GN_a^{in}(1+\mathcal{A})$$

$$N_a = \frac{1}{2}\langle a^\dagger a + aa^\dagger \rangle$$
$$N_b = \frac{1}{2}\langle b^\dagger b + bb^\dagger \rangle$$

Added noise in units of photon number

11-IV-5

PHASE-PRESERVING MINIMAL AMPLIFIER



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$$= GN_a^{in}(1+\mathcal{A})$$

$$N_a = \frac{1}{2}\langle a^\dagger a + aa^\dagger \rangle$$
$$N_b = \frac{1}{2}\langle b^\dagger b + bb^\dagger \rangle$$

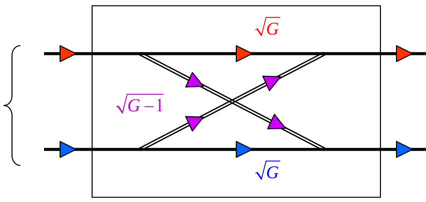
Reaches Caves' lower bound when idler port brings only vacuum noise $N_b = 1/2$ ⇨ $\mathcal{A}^{\min} = \frac{1}{2}\left|1-\frac{1}{G}\right|$

11-IV-5

TRANSFORMATION OF A QUANTUM STATE OF μWAVE LIGHT BY THE AMPLIFIER

Examine scattering time-wise:

$$\left|\Psi_{ab}^{in}\right\rangle = O\left(a^{in}, a^{in\dagger}, b^{in}, b^{in\dagger}\right)\left|vac\right\rangle$$



$$\left|\Psi_{ab}^{out}\right\rangle = U\left|\Psi_{ab}^{in}\right\rangle = O'\left(a^{out}, a^{out\dagger}, b^{out}, b^{out\dagger}\right)\left|vac'\right\rangle$$

important

Examples:

$$\left|\Psi_{ab}^{in}\right\rangle = \left|1\right\rangle_a \left|2\right\rangle_b = a^{in\dagger}\left(b^{in\dagger}\right)^2\left|vac\right\rangle$$

$$\left|\Psi_{ab}^{in}\right\rangle = \left|\alpha\right\rangle_a \left|0\right\rangle_b = e^{\alpha a^{in\dagger} - \alpha^* a a^{in}}\left|vac\right\rangle$$

$$a^{out} = e^{+i\delta}e^{+i\alpha}\sqrt{G}a^{in} + e^{+i\delta}e^{+i\beta}\sqrt{G-1}\,b^{in\dagger}$$

$$a^{out\dagger} = e^{-i\delta}e^{-i\alpha}\sqrt{G}a^{in\dagger} + e^{-i\delta}e^{-i\beta}\sqrt{G-1}\,b^{in}$$

$$b^{out} = e^{-i\delta}e^{+i\alpha}\sqrt{G}b^{in} + e^{-i\delta}e^{+i\beta}\sqrt{G-1}\,a^{in\dagger}$$

$$b^{out\dagger} = e^{+i\delta}e^{-i\alpha}\sqrt{G}b^{in\dagger} + e^{+i\delta}e^{-i\beta}\sqrt{G-1}\,a^{in}$$

$\left|vac'\right\rangle \neq \left|vac\right\rangle$ for an amplifier with photon gain

$\left|vac'\right\rangle = \left|vac\right\rangle$ for a conservative circuit

11-IV-6

AMPLIFIER PRODUCES NOISE AT OUPUT EVEN IF BOTH INPUT SIGNALS ARE IN THEIR GROUND STATE!

11-IV-7

FULL SCATTERING MATRIX OF QUANTUM-LIMITED AMPLIFIER

convention:
$\hat{o}[-\omega] = \hat{o}[\omega]^\dagger$

$$\begin{bmatrix} a^{out}[+\omega_S] \\ a^{out}[-\omega_S] \\ b^{out}[+\omega_I] \\ b^{out}[-\omega_I] \end{bmatrix} = \begin{bmatrix} e^{i\delta}e^{i\alpha}\sqrt{G} & 0 & 0 & e^{i\delta}e^{i\beta}\sqrt{G-1} \\ 0 & e^{-i\delta}e^{-i\alpha}\sqrt{G} & e^{-i\delta}e^{-i\beta}\sqrt{G-1} & 0 \\ 0 & e^{-i\delta}e^{i\beta}\sqrt{G-1} & e^{-i\delta}e^{i\alpha}\sqrt{G} & 0 \\ e^{i\delta}e^{-i\beta}\sqrt{G-1} & 0 & 0 & e^{i\delta}e^{-i\alpha}\sqrt{G} \end{bmatrix} \begin{bmatrix} a^{in}[+\omega_S] \\ a^{in}[-\omega_S] \\ b^{in}[+\omega_I] \\ b^{in}[-\omega_I] \end{bmatrix}$$

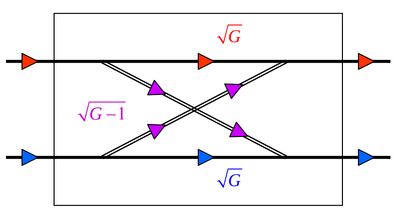
$$e^{i\delta}e^{i\alpha}\sqrt{G} = \frac{\eta_a^*\eta_b^* + |\rho|^2}{\eta_a\eta_b^* - |\rho|^2} \qquad e^{i\delta}e^{-i\alpha} = \frac{\eta_a\eta_b + |\rho|^2}{\eta_a\eta_b^* - |\rho|^2} \qquad e^{i\delta}e^{i\beta}\sqrt{G-1} = \frac{-2i\rho}{\eta_a\eta_b^* - |\rho|^2}$$

$$\eta_a = 1 - i\frac{\omega_S - \omega_a}{\Gamma_a} \qquad \eta_b = 1 - i\frac{\omega_I - \omega_b}{\Gamma_b} \qquad \rho = \frac{g^{(3)}\sqrt{\bar{n}_c}\,e^{-i\phi}}{\sqrt{\Gamma_a\Gamma_b}} < 1$$

3 symmetry properties:
- Conservation of commutators (volume and structure of phase space)
- Phase of the pump enters in scattering matrix in non-reciprocal manner
- Causality is enforced in frequency dependence of matrix

AMPLIFIER TRANSFORMS VACUUM INTO SQUEEZED VACUUM

$$|\Psi_{ab}^{in}\rangle = |vac\rangle$$



$$|\Psi_{ab}^{out}\rangle = U(G)|vac\rangle$$

$$U(G)|vac\rangle = \frac{1}{\sqrt{G}}\sum_{n=0}^{\infty}\left(e^{i\phi}\sqrt{\frac{G-1}{G}}\right)^n |n\rangle_a |n\rangle_b$$

← A pure, entangled state!

Energy has increased, entropy is still zero!

However, each port separately displays a thermal photon distribution:

$$\rho_a = \frac{1}{1+\bar{n}_a}\sum_{n_a}\left(\frac{\bar{n}_a}{1+\bar{n}_a}\right)^{n_a}|n_a\rangle\langle n_a| \qquad \bar{n}_a = (G-1) = \frac{kT_{eff}}{\hbar\omega_a}$$

DEGENERATE PARAMETRIC AMPLIFIER

$$\begin{bmatrix} a^{out}[+\omega_S] \\ a^{out}[-\omega_S] \\ a^{out}[+\omega_I] \\ a^{out}[-\omega_I] \end{bmatrix} = \begin{bmatrix} e^{i\delta}\sqrt{G} & 0 & 0 & e^{i\delta}e^{i\beta}\sqrt{G-1} \\ 0 & e^{-i\delta}\sqrt{G} & e^{-i\delta}e^{-i\beta}\sqrt{G-1} & 0 \\ 0 & e^{-i\delta}e^{i\beta}\sqrt{G-1} & e^{-i\delta}\sqrt{G} & 0 \\ e^{i\delta}e^{-i\beta}\sqrt{G-1} & 0 & 0 & e^{i\delta}\sqrt{G} \end{bmatrix} \begin{bmatrix} a^{in}[+\omega_S] \\ a^{in}[-\omega_S] \\ a^{in}[+\omega_I] \\ a^{in}[-\omega_I] \end{bmatrix}$$

$$e^{i\delta}\sqrt{G} = \frac{1+\vartheta^2+\rho^2}{(1-i\vartheta)^2-\rho^2} \qquad e^{i\delta}e^{i\beta}\sqrt{G-1} = \frac{-2i\rho}{(1-i\vartheta)^2-\rho^2} \qquad \rho = \frac{g^{(3)}\sqrt{\bar{n}_c}e^{-i\phi}}{\Gamma_a} < 1$$

$$\vartheta = \frac{\omega_S-\omega_a}{\Gamma_a}$$

For mode whose spectral width is that of single pole of circuit:

$$U(G)|vac\rangle = \frac{1}{\sqrt{G}}\sum_n \frac{\sqrt{(2n)!}}{2^n n!}\left(e^{i\phi}\sqrt{\frac{G-1}{G}}\right)^n |2n\rangle_a$$

Contains only an even number of photons!

$$X_\parallel^{out} = e^{\lambda}X_\parallel^{in} \qquad \cosh\lambda = \sqrt{G}$$

$$X_\perp^{out} = e^{-\lambda}X_\perp^{in} \qquad X_\parallel + iX_\perp \sim \int d\omega \frac{ia[\omega]}{1-i\vartheta}$$

11-IV-10

OUTLINE

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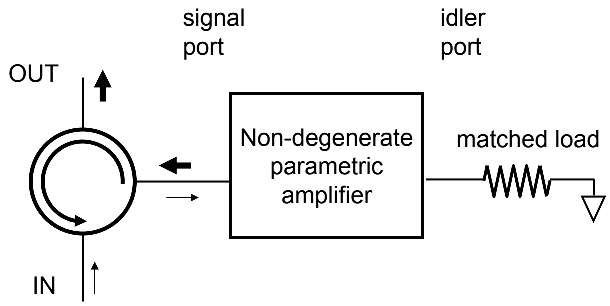
11-IV-4

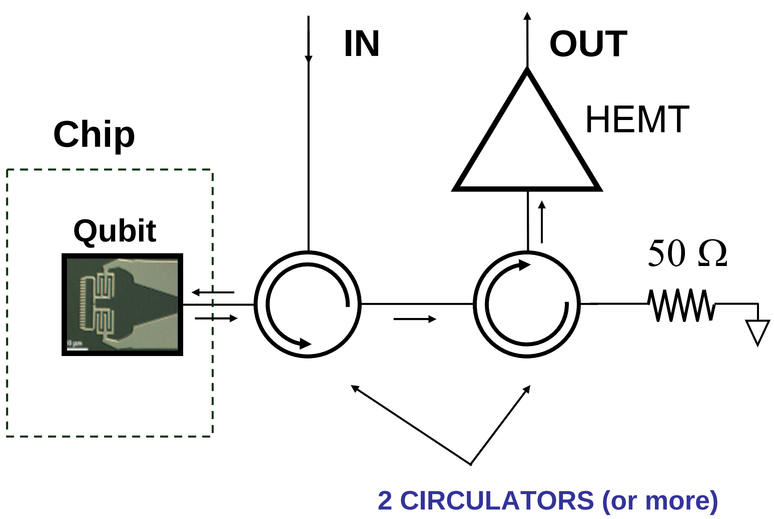
FOR DIRECTIONALITY, CIRCULATOR NEEDS TO BE ADDED TO PARAMETRIC AMPLIFIER



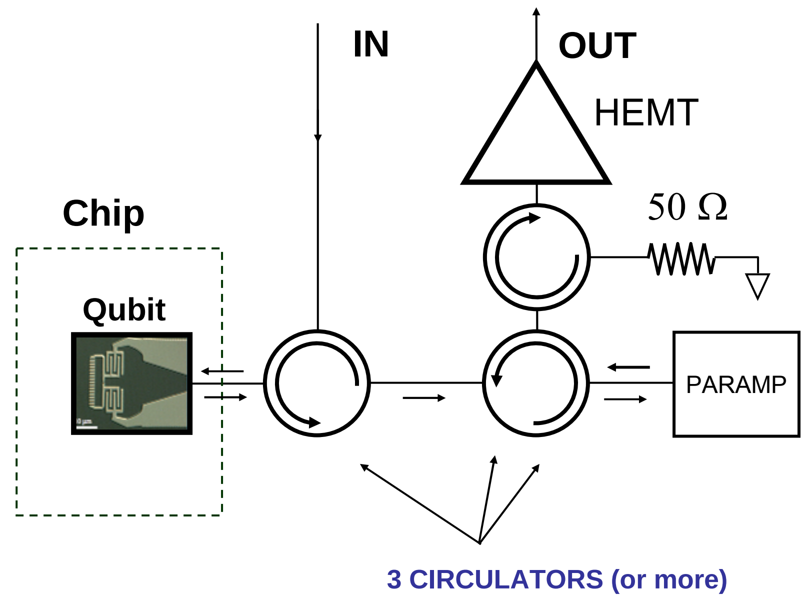
2+1 port device!

N.B. Phase-sensitive paramp operation can be recovered with non-degenerate paramp if information in idler port is recombined with that of signal port.

TYPICAL QBIT MEASUREMENT SCHEMATIC

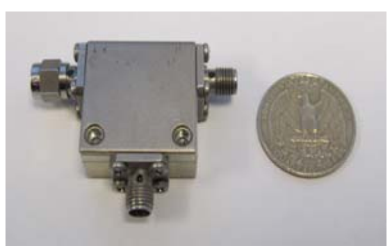


QBIT MEASUREMENT WITH PARAMP



11-IV-13

COMMERCIAL CIRCULATORS



Characteristics

Bandwidth 4 GHz
@ 6 & 10 GHz
Loss <1 dB
Isolation 15-20 dB

Drawbacks

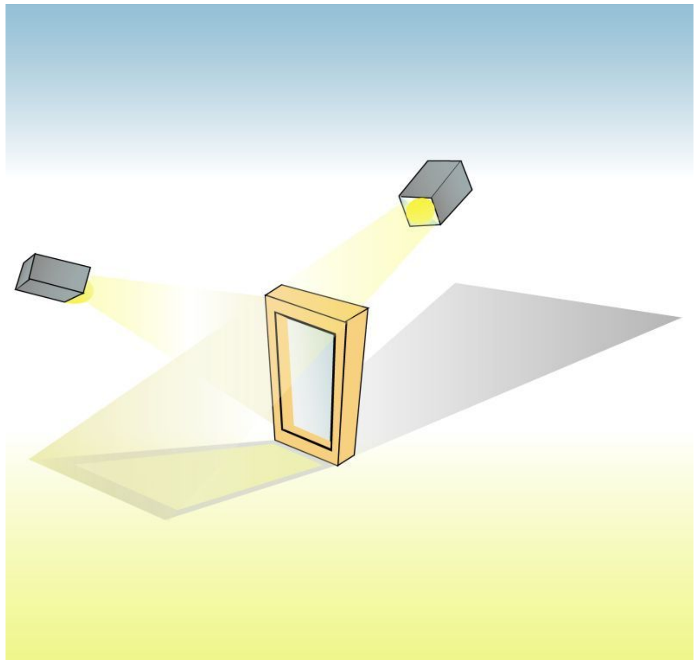
Large size
Permanent magnet

⇩

No on-chip integration

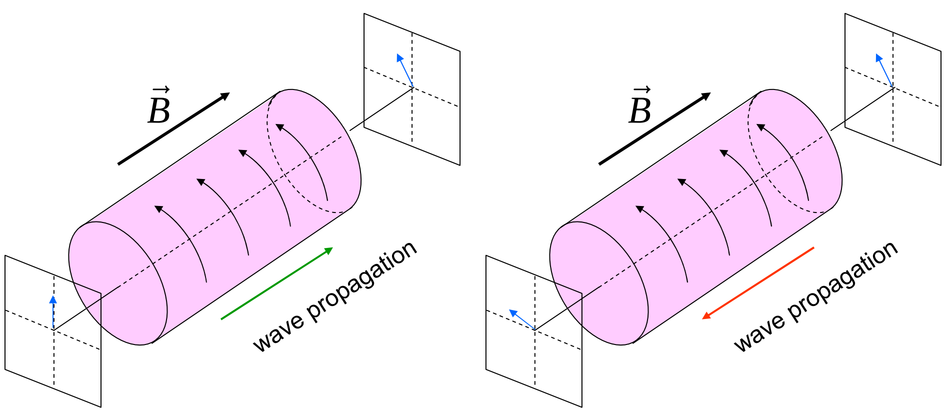
11-IV-14

CIRCULATOR ACTS AS VALVE FOR MICROWAVE LIGHT



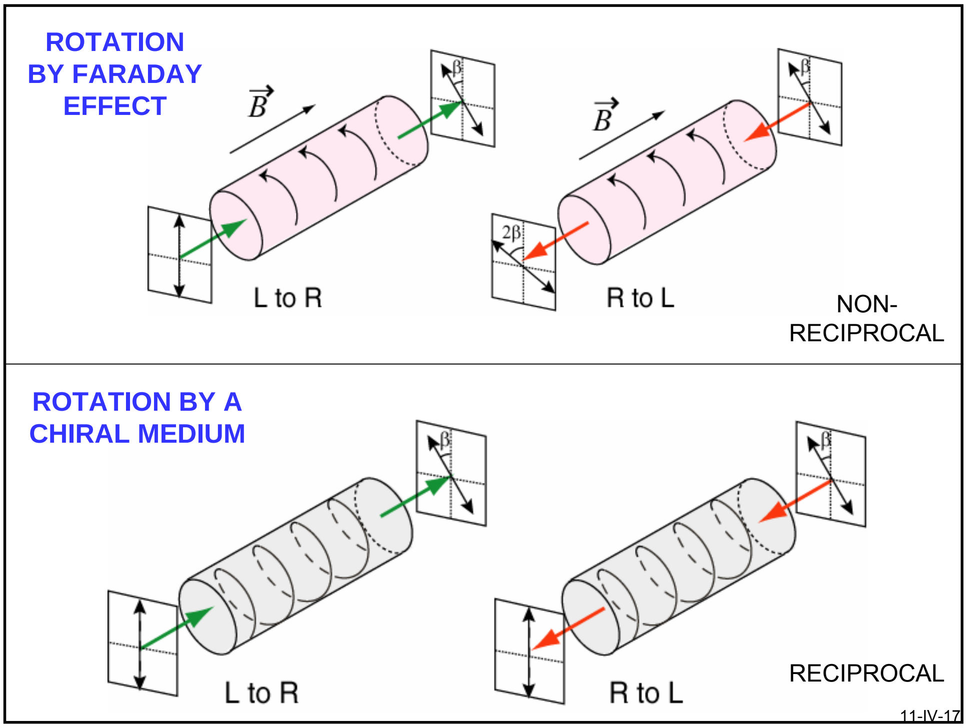
11-IV-15

FARADAY ROTATION

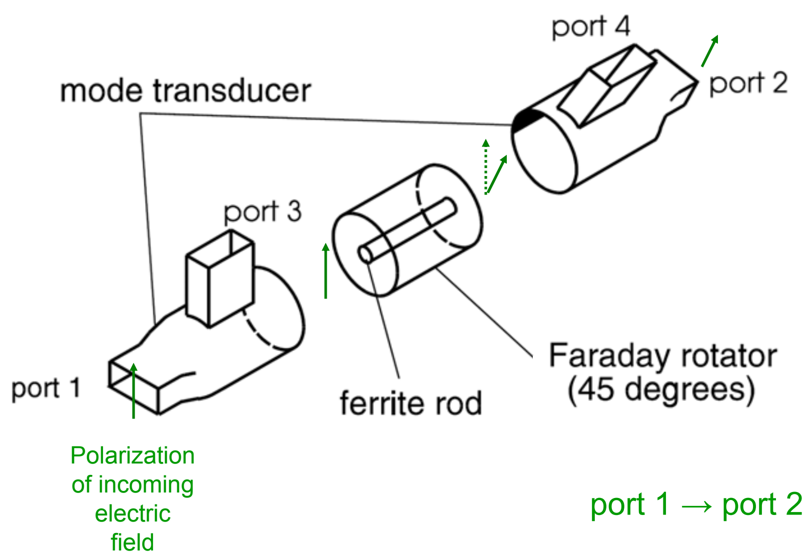


Breaks reciprocity (Principle of Inverse Return of Light)

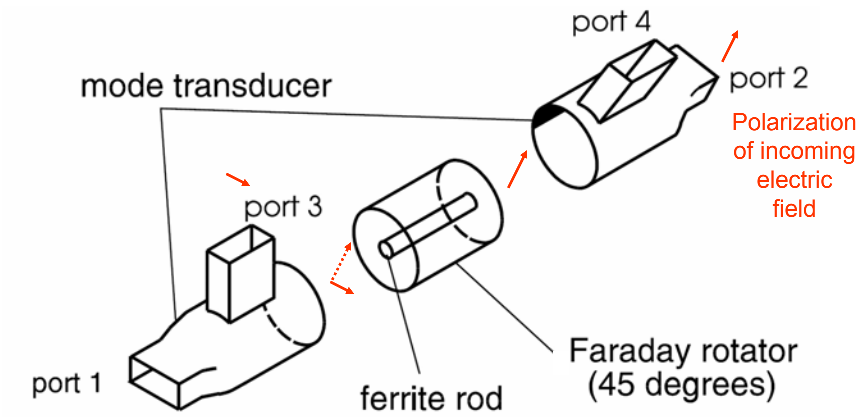
11-IV-16

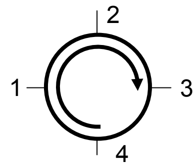


FERRITE ROTATION CIRCULATOR



FERRITE ROTATION CIRCULATOR





port 2 → port 3

11-IV-19

OUTLINE

1. Fundamental symmetries of scattering by active circuits
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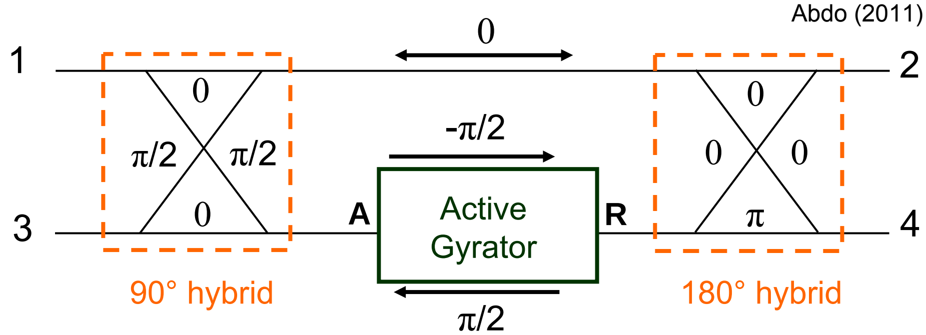
11-IV-4

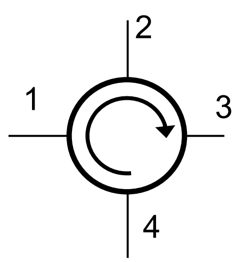
CAN AN ACTIVE MEDIUM REPLACE
A FARADAY MEDIUM IN A MAGNETIC FIELD?

11-IV-20

ON-CHIP CIRCULATOR SCHEME

Abdo (2011)





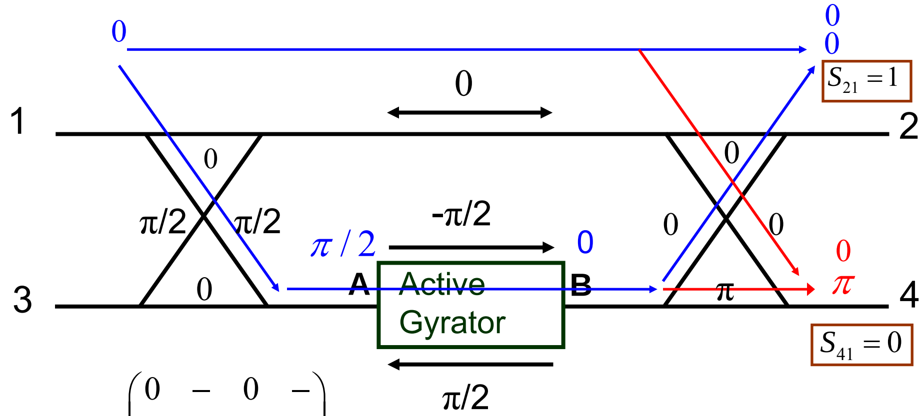
CONVERTS PHASE NON-RECIPROCITY
INTO AMPLITUDE NON-RECIPROCITY

Other proposal by A. Kamal, J. Clarke and MHD
Nature Physics 7, 311-315 (2011)

On chip hybrid already realized (Boulder)
Ku et al., *IEEE Trans. Appl. Superconductivity*, online, http://dx.doi.org/10.1109 (2010)

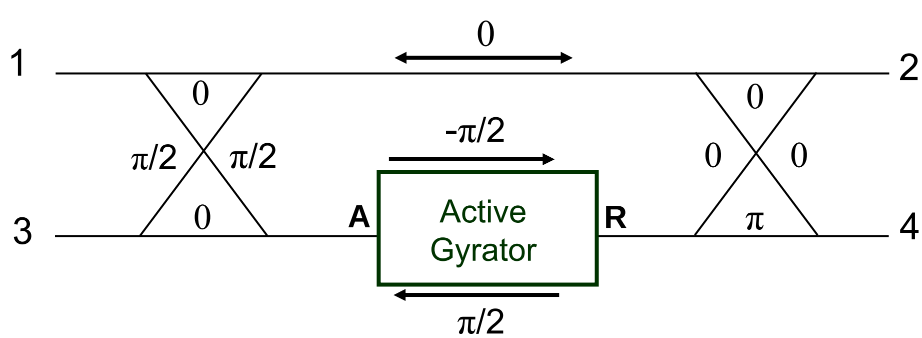
11-IV-21

EXAMINE INTERFERENCE WITH A SOURCE AT PORT 1



$$S_{4x4}^{circ} = \begin{pmatrix} 0 & - & 0 & - \\ 1 & 0 & - & 0 \\ 0 & - & 0 & - \\ 0 & 0 & - & 0 \end{pmatrix}$$

CIRCULATOR SCATTERING MATRIX

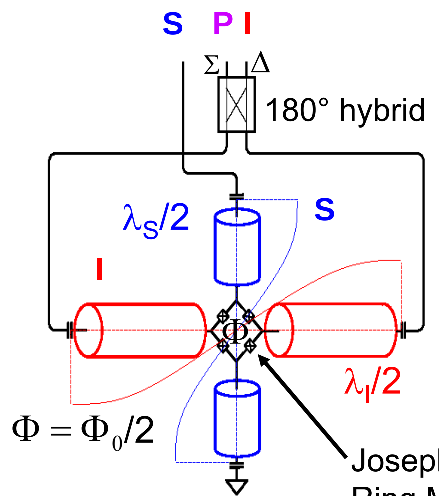


$$S_{4x4}^{circ} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \end{pmatrix}$$

$$S \neq {}^{t}S$$

BASIC GYRATOR BUILDING BLOCK: JOSEPHSON PARAMETRIC CONVERTER (JPC)

S = Signal
I = Idler
P = Pump



Parameters:

$\omega_S \cong \omega_a$, $\omega_I \cong \omega_b$

$Q_a = \frac{\omega_a}{\gamma_a}$, $Q_b = \frac{\omega_b}{\gamma_b}$

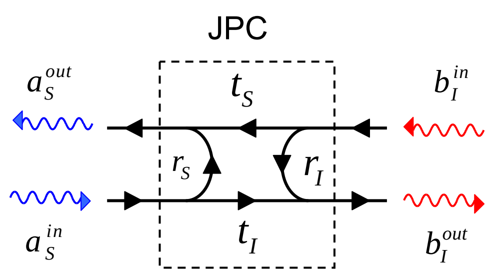
p_a , p_b

ω_P , I_P , I_0

B. Abdo et al., arXiv:1103.1405

11-IV-22

PURE CONVERSION MODE



$$\omega_P = |\omega_I - \omega_S|$$

No photon gain:

$$|r|^2 + |t|^2 = 1$$

Reflection

$$r_s = \frac{\eta_a \eta_b^* - |\rho|^2}{\eta_a^* \eta_b^* + |\rho|^2}$$

$$r_I = \frac{\eta_a^* \eta_b - |\rho|^2}{\eta_a^* \eta_b^* + |\rho|^2}$$

Transmission

$$t_S = \frac{2i\rho}{\eta_a^* \eta_b^* + |\rho|^2}$$

$$t_I = \frac{2i\rho^*}{\eta_a^* \eta_b^* + |\rho|^2}$$

where

$$\eta_a = 1 - iQ_a\left(\frac{\omega_S^2 - \omega_a^2}{\omega_S \omega_a}\right)$$

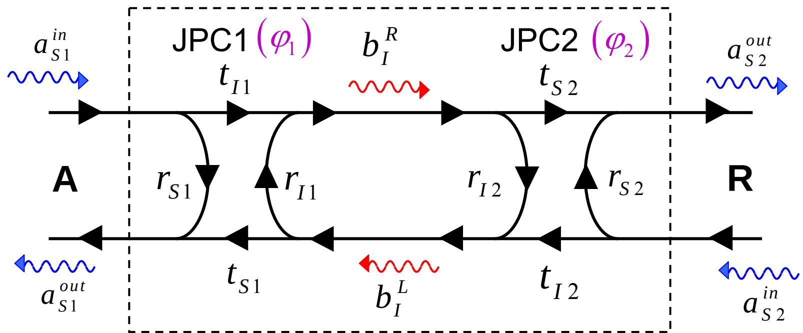
$$\eta_b = 1 - iQ_b\left(\frac{\omega_I^2 - \omega_b^2}{\omega_I \omega_b}\right)$$

$$\rho = \frac{1}{4}\sqrt{Q_a Q_b p_a p_b}\left|\frac{I_P}{I_0}\right| e^{i\varphi}$$

Full conversion: $r = 0, \ |t| = 1$ NO NOISE!

11-IV-23

2 CONVERTERS BACK-TO-BACK



Transmission

$$t_{RA} = \frac{t_{I1}t_{S2}}{1 - r_{I2}r_{I1}}$$

$$t_{AR} = \frac{t_{I2}t_{S1}}{1 - r_{I2}r_{I1}}$$

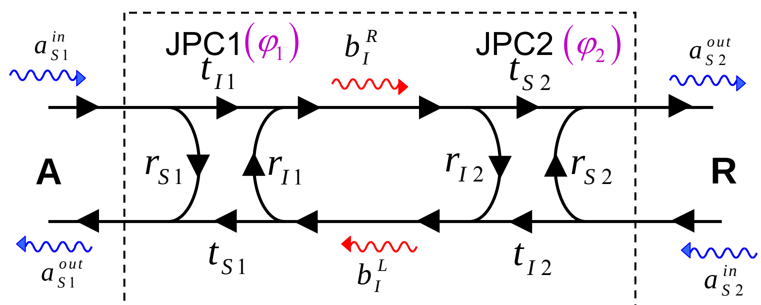
Reflection

$$r_{AA} = r_{S1} + \frac{r_{I2}t_{S1}t_{I1}}{1 - r_{I2}r_{I1}}$$

$$r_{RR} = r_{S2} + \frac{r_{I1}t_{S2}t_{I2}}{1 - r_{I2}r_{I1}}$$

11-IV-24

2 CONVERTERS BACK-TO-BACK = GYRATOR



$|\rho| = 1$

$\omega_S \cong \omega_a$

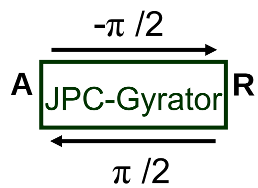
$\omega_I \cong \omega_b$

$$t_{RA} \cong t_{I1}t_{S2} \cong e^{i(\varphi_2 - \varphi_1 + \pi)}$$

$$t_{AR} \cong t_{I2}t_{S1} \cong e^{-i(\varphi_2 - \varphi_1 - \pi)}$$

$$r_{AA} = r_{RR} \cong 0$$

$$\varphi_2 - \varphi_1 = \pi / 2$$



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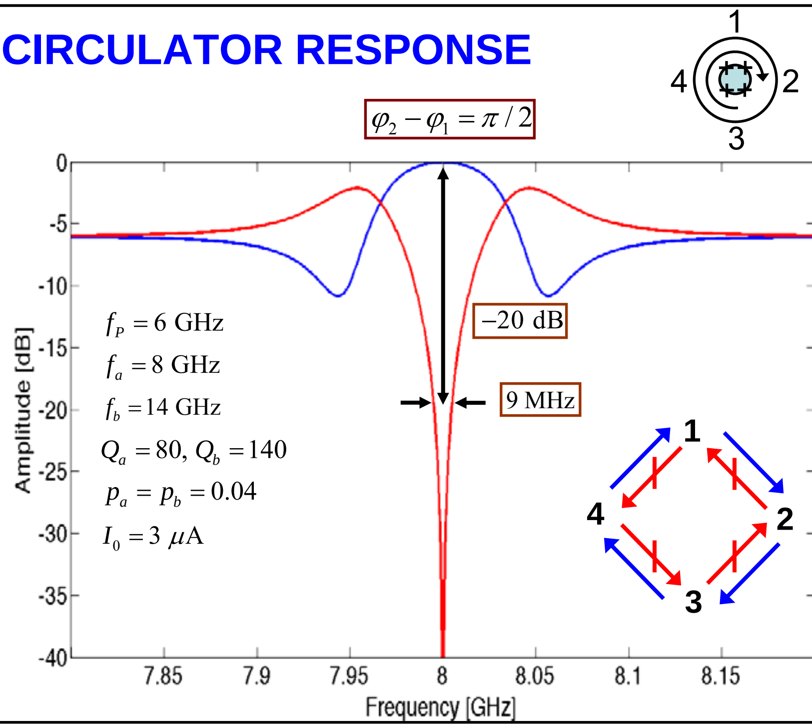
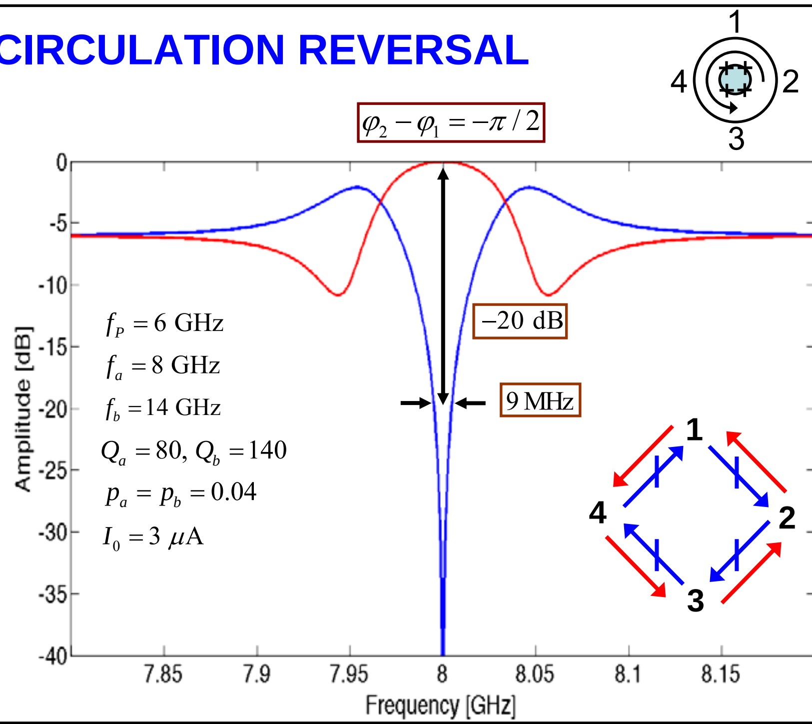
CIRCULATOR RESPONSE



11-IV-26

CIRCULATION REVERSAL



11-IV-26bis

OUTLINE

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11-IV-4

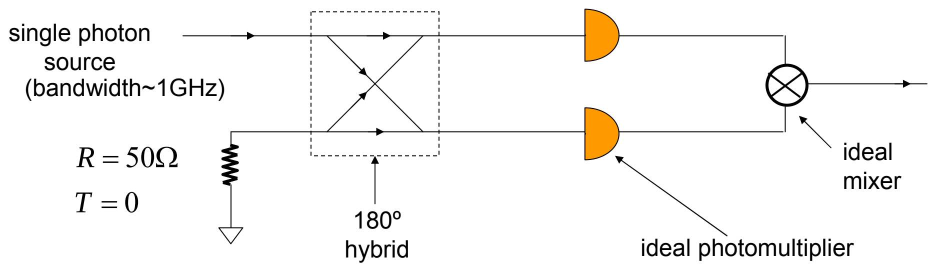
INFRARED vs μWAVE LIGHT

Infrared	μWave
λ = 1.5 μm	λ = 3 cm
ν = 200 THz	ν = 10 GHz
T = 10,000 K	T = 0.5 K
E= 0.8 eV	E= 40 μeV

IS A QUANTUM-LIMITED AMPLIFIER
EQUIVALENT TO A PHOTOMULTIPLIER?

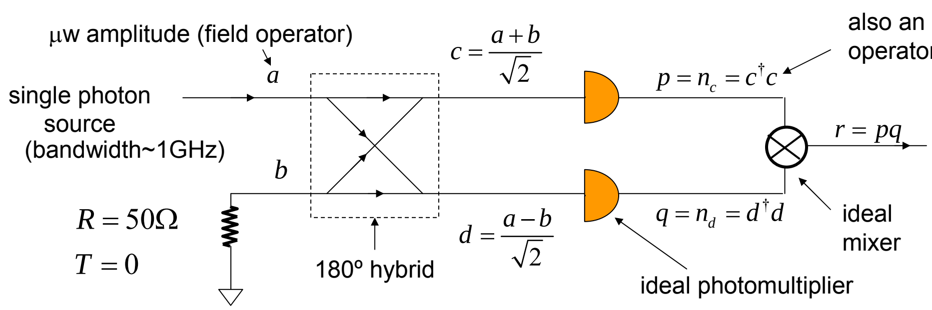
11-IV-27

MICROWAVE HANBURY BROWN & TWISS EXPT



Original demonstration of use of intensity-intensity correlations, optical domain:
R. Hanbury Brown and R.Q. Twiss, *Nature* **178** , 1046 (1956)

MICROWAVE HANBURY BROWN & TWISS EXPT



$\left[a, a^\dagger\right] = \left[b, b^\dagger\right] = 1$

Original demonstration of use of intensity-intensity correlations, optical domain:
R. Hanbury Brown and R.Q. Twiss, *Nature* **178** , 1046 (1956)

$$p = \frac{1}{2}\left(n_a + n_b + b^\dagger a + a^\dagger b\right) \qquad q = \frac{1}{2}\left(n_a + n_b - b^\dagger a - a^\dagger b\right)$$

$$r = \frac{1}{2}\left[\left(n_a + n_b\right)^2 - \left(b^\dagger a + a^\dagger b\right)^2\right] = \frac{1}{2}\left[\left(n_a + n_b\right)^2 - b^\dagger b^\dagger aa - a^\dagger a^\dagger bb - n_a\left(n_b + 1\right) - n_b\left(n_a + 1\right)\right]$$

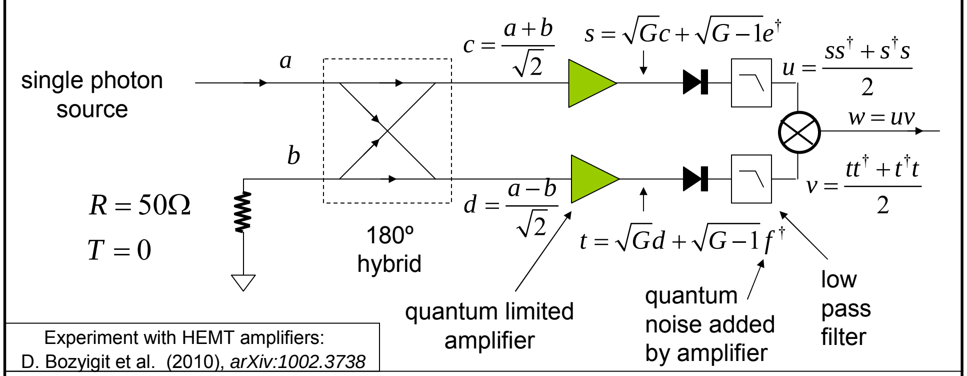
$$\langle pq \rangle = \mathrm{tr}\left[r \left|\Psi_{in}\right\rangle\left\langle\Psi_{in}\right|\right] \text{ with } \left|\Psi_{in}\right\rangle = \left|1_a\right\rangle\left|0_b\right\rangle \;\Rightarrow\; \langle pq \rangle = \frac{1}{4}\left(n_a - n_a\right) = 0$$

photon numbers at input

contrast function: $C = \frac{\langle pq \rangle - \langle p \rangle \langle q \rangle}{\langle p \rangle \langle q \rangle} = -1$

Note that if source is Glauber then one finds $C = 0$.

MICROWAVE HANBURY BROWN & TWISS VARIANT



Experiment with HEMT amplifiers:
D. Bozyigit et al. (2010), *arXiv:1002.3738*

$$u = \frac{G}{2}\left(n_a + n_b + 1 + \left[ab^{\dagger} + h.c.\right]\right) + (G-1)\left(n_e + \frac{1}{2}\right) + \sqrt{\frac{G(G-1)}{2}}\left[(a+b)e^{\dagger} + h.c.\right]$$

$$v = \frac{G}{2}\left(n_a + n_b + 1 - \left[ab^{\dagger} + h.c.\right]\right) + (G-1)\left(n_f + \frac{1}{2}\right) + \sqrt{\frac{G(G-1)}{2}}\left[(a-b)f^{\dagger} + h.c.\right]$$

In the large gain limit: $C = \frac{\langle uv\rangle - \langle u\rangle\langle v\rangle}{\langle u\rangle\langle v\rangle} = \frac{G\frac{8}{4}}{G\frac{9}{4}} - 1 = -\frac{1}{9}$

If source is Glauber then again $C = 0$.

11-IV-29

END OF LECTURE