Chaire de Physique Mésoscopique
Michel Devoret
Année 2008, 13 mai - 24 juin

CIRCUITS ET SIGNAUX QUANTIQUES

QUANTUM SIGNALS AND CIRCUITS

Deuxième Leçon / *Second Lecture*

VISIT THE WEBSITE OF THE CHAIR OF MESOSCOPIC PHYSICS

http://www.college-de-france.fr

and follow links to:

http://www.physinfo.fr/lectures.html

PDF FILES OF ALL LECTURES WILL BE POSTED ON THIS WEBSITE

Questions, comments and corrections are welcome!

CALENDAR OF SEMINARS

May 13: Denis Vion, (Quantronics group, SPEC-CEA Saclay)
Continuous dispersive quantum measurement of an electrical circuit

May 20: Bertrand Reulet (LPS Orsay)
Current fluctuations : beyond noise

June 3: Gilles Montambaux (LPS Orsay)
Quantum interferences in disordered systems

June 10: Patrice Roche (SPEC-CEA Saclay)
Determination of the coherence length in the Integer Quantum Hall Regime

June 17: Olivier Buisson, (CRTBT-Grenoble)
A quantum circuit with several energy levels

June 24: Jérôme Lesueur (ESPCI)
High Tc Josephson Nanojunctions: Physics and Applications

NOTE THAT THERE IS NO LECTURE AND NO SEMINAR ON MAY 27 !

08-II-3

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction and overview

Lecture II: Modes of a circuit and propagation of signals

Lecture III: The "atoms" of signals

Lecture IV: Non-linear circuit elements: length and energy scales of superconductivity

Lecture V: Hamiltonian vs scattering description of circuits

Lecture VI: Amplifying quantum signals with dispersive circuits

08-II-4

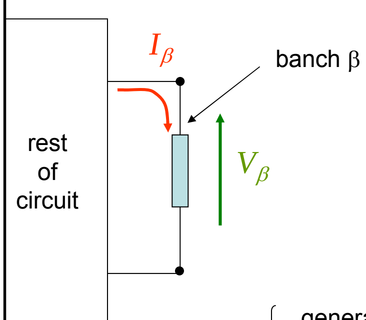
LECTURE II : MODES OF A CIRCUIT, PROPAGATION OF SIGNALS

OUTLINE

1. Introduction, purpose of this lecture
2. Finding the Hamiltonian of an arbitrary circuit
3. Comparison with cavity QED
4. Transmission lines and waveguides
5. Coupled LC oscillators: model of transmission line

08-II-5

REVIEW OF LAST LECTURE



Introduce branch flux and charge

$$\phi_\beta(t) = \int_{-\infty}^{t} V_\beta(t')\,dt'$$

$$Q_\beta(t) = \int_{-\infty}^{t} I_\beta(t')\,dt'$$

generalized mass : $C \longleftrightarrow M$
generalized spring constant: $1/L \longleftrightarrow k$

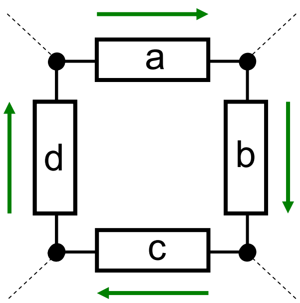
constitutive relations:

$\phi_L = LI_L \longleftrightarrow X_S = f_S/k$ for inductances / springs

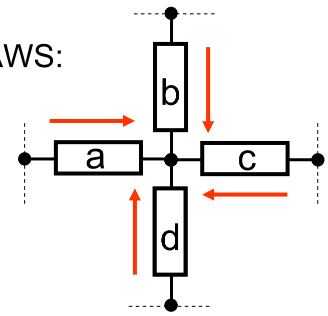
$Q_C = CV_C \longleftrightarrow P_M = MV_M$ for capacitances / masses

08-II-6b

PROBLEM: NOT ALL BRANCH VARIABLES ARE INDEPENDENT



KIRCHHOFF'S LAWS:



$$\sum_{\text{branches } \lambda \text{ around loop}} V_\lambda = 0$$

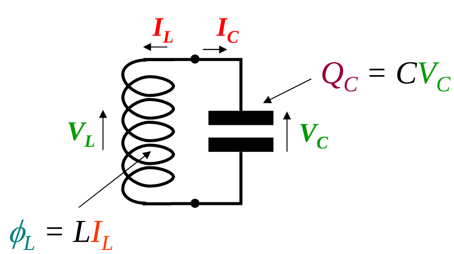
$$\sum_{\text{branches } \nu \text{ tied to node}} I_\nu = 0$$

IMPOSE CONSTRAINTS ON BRANCH VARIABLES

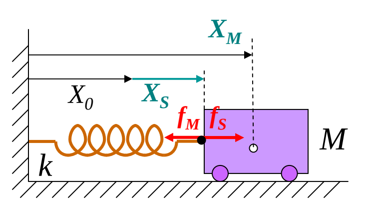
08-II-7

BRANCH CONSTRAINTS IN LC OSCILLATOR

electrical world



mechanical world



$X_S = f_S/k$ $\quad P_M = MV_M$

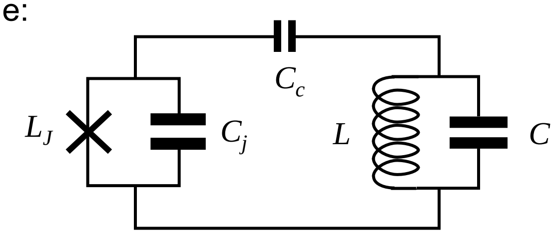
X_0 = equilibrium length of spring

Kirchhoff's law (loop) :	$V_L = V_C$	⟷	$V_S = V_M$	mass-spring rigid link
Kirchhoff's law (node):	$I_L = -I_C$	⟷	$f_S = -f_M$	action - reaction principle
Constitutive relation (inductive) :	$\phi_L = LI_L$	⟷	$X_S = f_S/k$	Hookes's Law
Constitutive relation (capacitive):	$Q_C = CV_C$	⟷	$P_M = MV_M$	Linear momentum
Faraday's Law:	$V_L = \dot{\phi}_L$	⟷	$V_S = \dot{X}_S$	Def. of velocity
Charge conservation:	$I_C = \dot{Q}_C$	⟷	$f_M = \dot{P}_M$	Newton's Law

08-II-8b

HOW DO WE FIND THE HAMILTONIAN OF AN ARBITRARY CIRCUIT?

Example:



Josephson junction coupled to 1 resonator mode

Wish also to extend the notions of circuits to distributed elements

Progagation of signals on transmission lines : standing modes to propagating modes
Dissipative dynamical evolution: treating resistors and measuring instruments

08-II-9

OUTLINE

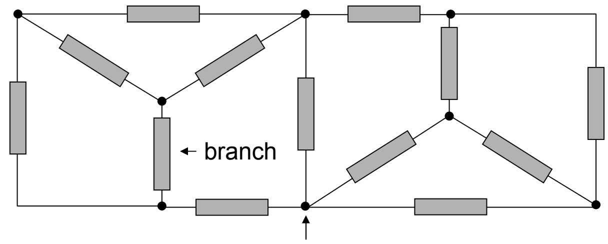
1. Introduction, purpose of this lecture
2. Finding the Hamiltonian of an arbitrary circuit
3. Comparison with cavity QED
4. Transmission lines and waveguides
5. Coupled LC oscillators: model of transmission line

08-II-5b

FIRST, HOW DO WE FIND A COMPLETE SET OF INDEPENDENT VARIABLES?

Method of nodes

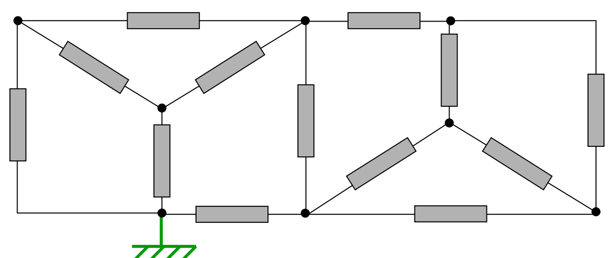
Example of circuit:



08-II-10

FINDING A COMPLETE SET OF INDEPENDENT VARIABLES

Method of nodes

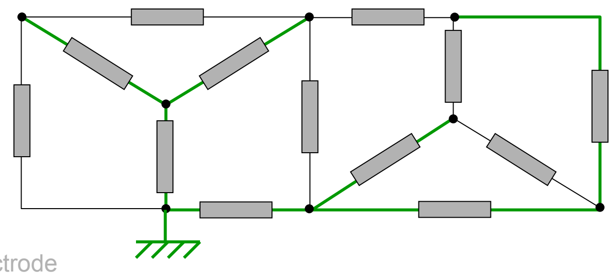


1) Choose a reference electrode (ground)

08-II-10a

FINDING A COMPLETE SET OF INDEPENDENT VARIABLES

<u>Method of nodes</u>



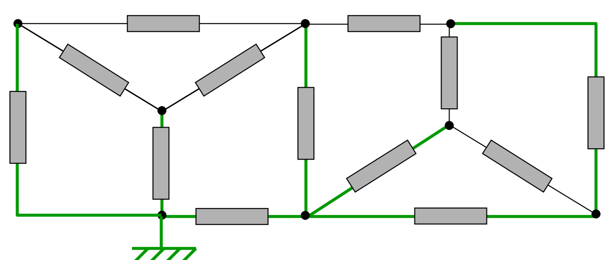
1) Choose a reference electrode (ground)

2) Choose a spanning tree (accesses every node, no loop)

08-II-10b

FINDING A COMPLETE SET OF INDEPENDENT VARIABLES

<u>Method of nodes</u>



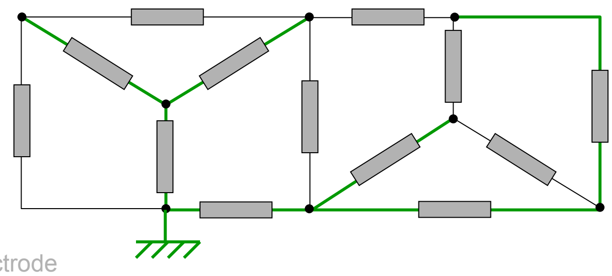
1) Choose a reference electrode (ground)

2) Choose a spanning tree (accesses every node, no loop)

08-II-10c

FINDING A COMPLETE SET OF INDEPENDENT VARIABLES

Method of nodes



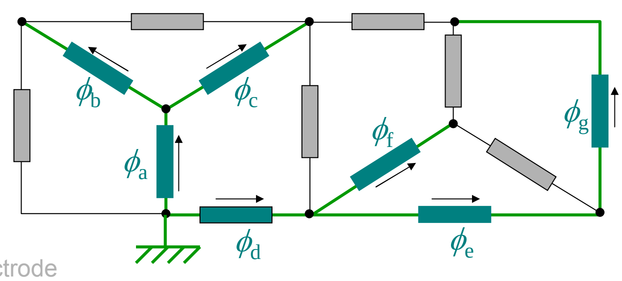
1) Choose a reference electrode (ground)

2) Choose a spanning tree (accesses every node, no loop)

08-II-10d

FINDING A COMPLETE SET OF INDEPENDENT VARIABLES

Method of nodes



1) Choose a reference electrode (ground)

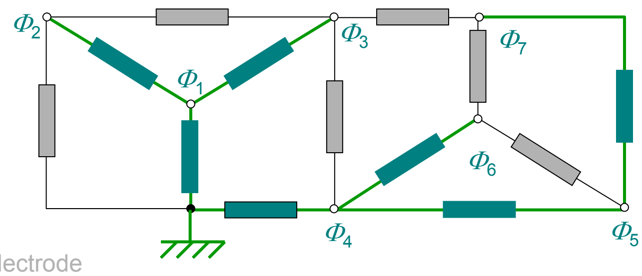
2) Choose a spanning tree (accesses every node, no loop)

3) Select tree branch fluxes (closure branches left out)

08-II-10e

FINDING A COMPLETE SET OF INDEPENDENT VARIABLES

Method of nodes



1) Choose a reference electrode (ground)

2) Choose a spanning tree (accesses every node, no loop)

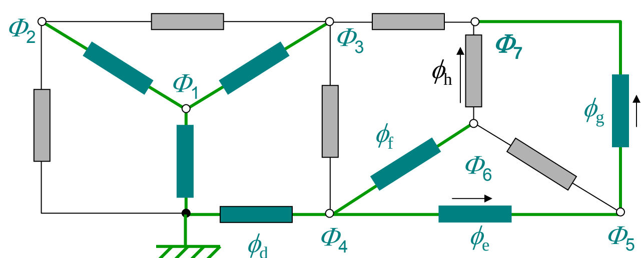
3) Select tree branch fluxes (closure branches left out)

4) Node flux is sum of branch fluxes to ground (closure branch fluxes are expressed as differences between node fluxes)

08-II-10f

FINDING A COMPLETE SET OF INDEPENDENT VARIABLES

Method of nodes



1) Choose a reference electrode (ground)

2) Choose a spanning tree (accesses every node, no loop)

3) Select tree branch fluxes (closure branches left out)

4) Node flux is sum of branch fluxes to ground (closure branch fluxes are expressed as differences between node fluxes)

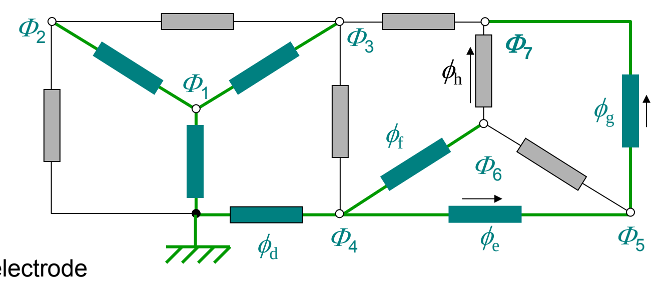
example: $\Phi_7 = \phi_d + \phi_e + \phi_g$

$\phi_h = \Phi_7 - \Phi_6 + \text{cst}$

08-II-10g

FINDING A COMPLETE SET OF INDEPENDENT VARIABLES

Method of nodes



1) Choose a reference electrode (ground)

2) Choose a spanning tree (accesses every node, no loop)

3) Select tree branch fluxes (closure branches left out)

4) Node flux is sum of branch fluxes to ground (closure branch fluxes are expressed as differences between node fluxes)

example: $\Phi_7 = \phi_d + \phi_e + \phi_g$

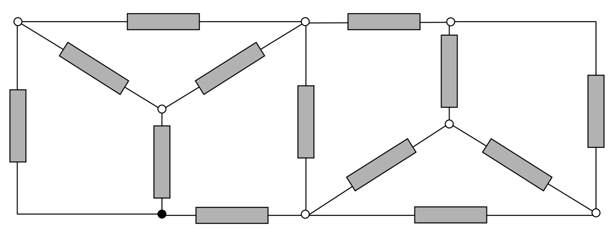
$\phi_h = \Phi_7 - \Phi_6 + \text{cst}$

$$\Phi_n = \sum_{\substack{\text{tree branches } \beta \\ \text{leading to } n}} \phi_\beta$$

$$\phi_\gamma = \Phi_{n_+(\gamma)} - \Phi_{n_-(\gamma)} + \text{cst}$$

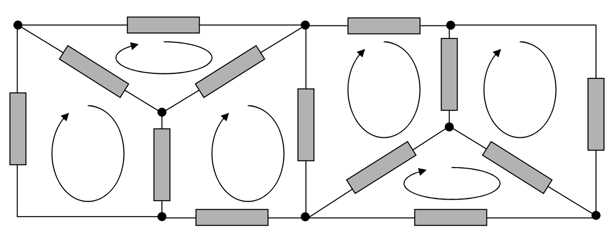
TWO METHODS FOR DEFINING A COMPLETE SET OF INDEPENDENT VARIABLES

Method of nodes



Method of loops

Defines loop charges



SHORTCUT

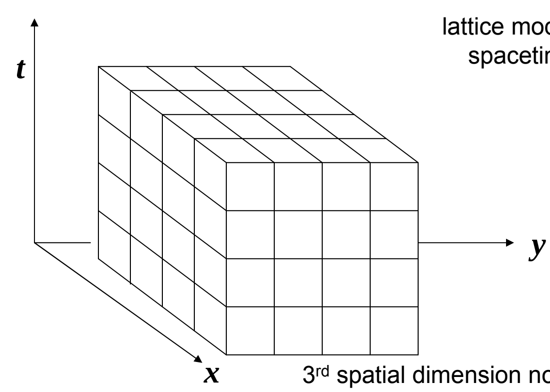
It is advantageous in the method of nodes to choose a spanning tree that passes only through inductors

Not necessary, just avoid tedious details in most cases

08-II-12

GAUGE INVARIANCE

The choice of spanning tree or set of independent loops is analogous to the choice of gauge in continuous media electromagnetism



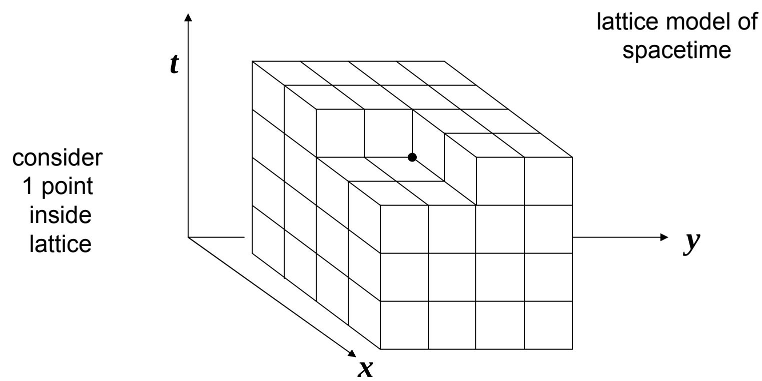
lattice model of spacetime

3rd spatial dimension not represented

08-II-13

GAUGE INVARIANCE

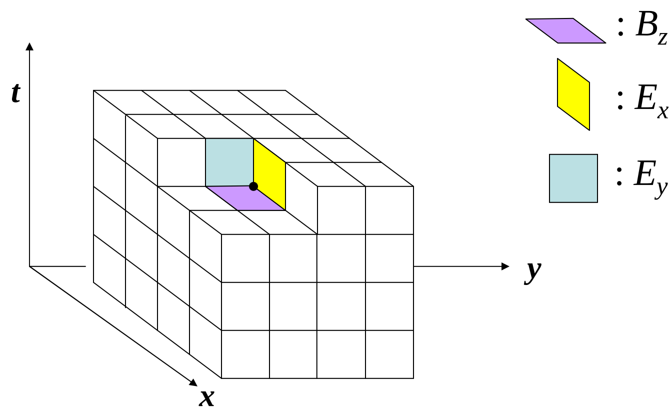
The choice of spanning tree or set independent loops is analogous to the choice of gauge in continuous media electromagnetism



08-II-13-bis

GAUGE INVARIANCE

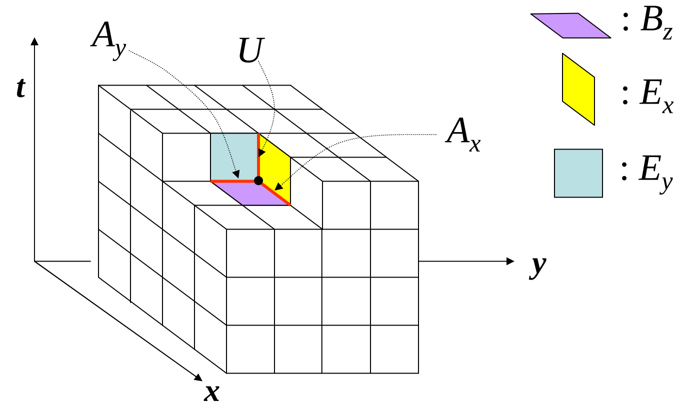
The choice of spanning tree or set independent loops is analogous to the choice of gauge in continuous media electromagnetism



08-II-13-ter

GAUGE INVARIANCE

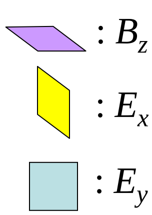
The choice of spanning tree or set independent loops is analogous to the choice of gauge in continuous media electromagnetism



08-II-13-4

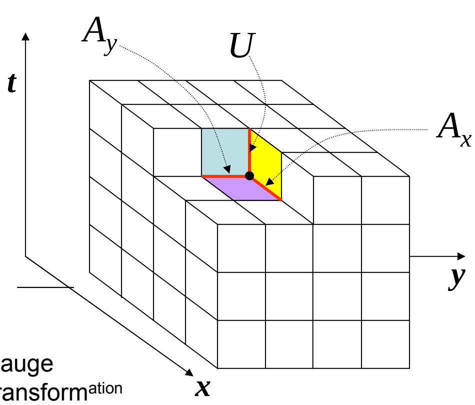
GAUGE INVARIANCE

The choice of spanning tree or set independent loops is analogous to the choice of gauge in continuous media electromagnetism



$\{\vec{A}, U\}$ = gauge

$$\left.\begin{aligned} \vec{A} &\rightarrow \vec{A} + \vec{\nabla}\Lambda \\ U &\rightarrow U - \frac{\partial}{\partial t}\Lambda \end{aligned}\right\}$$ gauge transformation

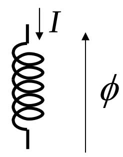


Charge transported by edges is conserved. Gauge field components, which give these currents, are not independent. In 4D, only 3 components are needed to specify field at one point.

08-II-13-5

INDUCTIVE vs CAPACITIVE ELEMENTS

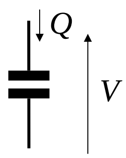
Inductance : current I function only of flux ϕ



$$E = \int_{-\infty}^{t} I \cdot V dt' = \int_{0}^{\phi} I(\phi') d\phi'$$

Electrical equivalent of spring: $\phi \leftrightarrow X$; $I \leftrightarrow f$

Capacitance : voltage V function only of charge Q



$$E = \int_{-\infty}^{t} V \cdot I dt' = \int_{0}^{Q} V(Q') dQ'$$

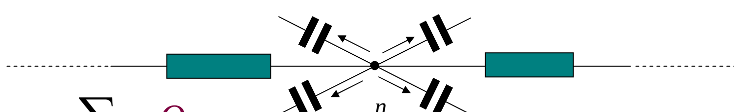
Electrical equivalent of mass: $Q \leftrightarrow P$; $V \leftrightarrow V$

08-II-14a

08-II-15a

NODE CHARGES

The conjugate coordinates of node fluxes
are node charges: they are the sum of all the charges
going into capacitances linked to this node.



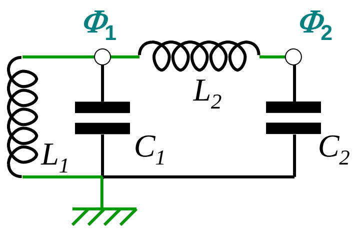
$$Q_n = \sum_{\substack{\text{capacitive} \\ \text{branches} \\ \text{exiting from } n}} Q_\beta$$

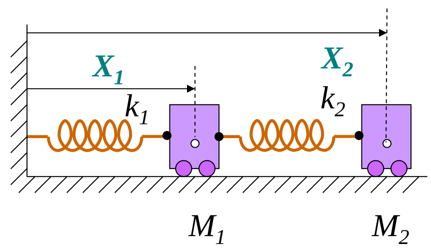
for each node: $[\Phi_n, Q_n] = i\hbar$

This can be demonstrated by writing the Lagrangian
of the circuit from its dynamical equations
and performing a Legendre transform to obtain the Hamiltonian

Dynamical equations ⇨ Lagrangian $L(x, \dot{x}, t)$ ⇨ conjugate coordinates $p = \partial L / \partial \dot{x}$ ⇨ Hamiltonian $H(x, p, t)$

A SIMPLE EXAMPLE





$$\hat{H}\left(\hat{\Phi}_1,\hat{Q}_1;\hat{\Phi}_2,\hat{Q}_2\right)=$$

$$\frac{\hat{Q}_1^2}{2C_1}+\frac{\hat{\Phi}_1^2}{2L_1}+\frac{\hat{Q}_2^2}{2C_2}+\frac{\left(\hat{\Phi}_2-\hat{\Phi}_1\right)^2}{2L_2}$$

$$\hat{H}\left(\hat{X}_1,\hat{P}_1;\hat{X}_2,\hat{P}_2\right)=$$

$$\frac{\hat{P}_1^2}{2M_1}+\frac{k_1\hat{X}_1^2}{2}+\frac{\hat{P}_2^2}{2M_2}+\frac{k_2\left(\hat{X}_2-\hat{X}_1\right)^2}{2}$$

Each electrical branch (mechanical element) contributes to the Hamiltonian by its energy expressed in terms of the variables of the chosen set of canonical coordinates

Method is equally valid for non-linear elements!

08-II-16b

REVIEWS ON QUANTUM CIRCUIT THEORY

Yurke B. and Denker J.S., Phys. Rev. A 29, 1419 (1984)

Devoret M. H. in "Quantum Fluctuations", S. Reynaud, E. Giacobino, J. Zinn-Justin, Eds. (Elsevier, Amsterdam, 1997) p. 351-385

G. Burkard, R. H. Koch, and D. P. DiVincenzo, Phys. Rev. B 69, 064503 (2004)

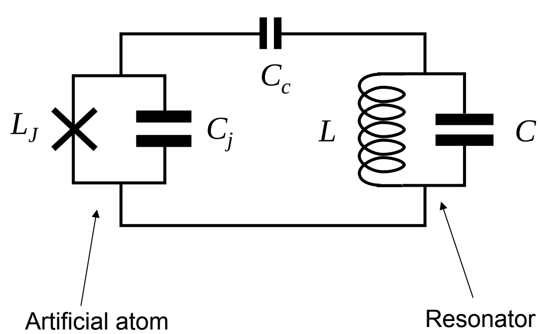
08-II-17

OUTLINE

1. Introduction, purpose of this lecture
2. Finding the Hamiltonian of an arbitrary circuit
3. Comparison with cavity QED
4. Transmission lines and waveguides
5. Coupled LC oscillators: model of transmission line

08-II-5c

HAMILTONIAN OF COOPER PAIR BOX COUPLED TO 1 RESONATOR MODE



BASIC CIRCUIT ANALOG OF CAVITY QED EXPERIMENTS

08-II-18

EARLIER EXPERIMENT

VOLUME 62, NUMBER 15 PHYSICAL REVIEW LETTERS 10 APRIL 1989

Escape Oscillations of a Josephson Junction Switching Out of the Zero-Voltage State

Emmanuel Turlot, Daniel Esteve, Cristian Urbina, John M. Martinis,[a] and Michel H. Devoret
Service de Physique du Solide et de Résonance Magnétique, Centre d'Etudes Nucleaires de Saclay, 91191 Gif-sur-Yvette CEDEX, France

Sebastian Linkwitz and Hermann Grabert
Fachbereich Physik, Universität Essen, D-4300 Essen, Federal Republic of Germany
(Received 25 October 1988)

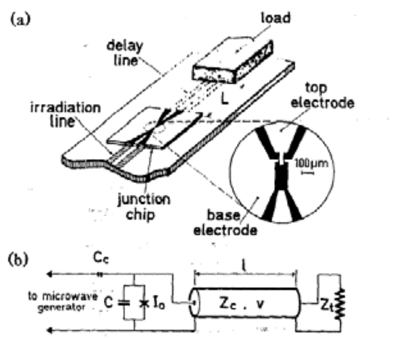


FIG. 2. (a) Josephson junction (see inset) connected to a delay line and capacitively coupled to an irradiation line. A movable microwave-absorbing block (load) terminates the delay line. (b) Equivalent circuit at microwave frequencies. The capacitor C_c represent the high-impedance capacitive coupling between the junction and irradiation line.

08-II-19

QUANTUM JOSEPHSON JUNCTION COUPLED TO A TRANSMISSION LINE

Physica Scripta. Vol. T25, 118–121, 1989.

Effect of an Adjustable Admittance on the Macroscopic Energy Levels of a Current Biased Josephson Junction

Michel H. Devoret, Daniel Esteve, John M. Martinis* and Cristian Urbina
Service de Physique de Solide et de Résonance Magnétique, C.E.N. Saclay, 91191 Gif-sur-Yvette Cedex, France
Received April 6, 1988; accepted June 22, 1988

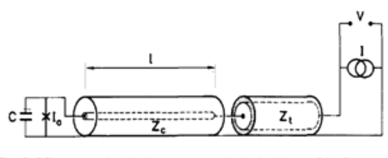
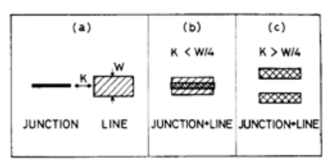


Fig. 2. Microwave circuit whose quantum behavior is tested in the experiment. It consists of a Josephson junction (cross) of critical current I_0 shunted by its self-capacitance C and by a transmission line of characteristic impedance Z_c followed by an attenuating transmission line of characteristic impedance Z_t. The length of the first line is l and its wave propagation velocity v. The junction is d.c. biased by a current source I using the inner and outer conductors of the transmission lines. The d.c. voltage V across the junction is measured using the same conductors.



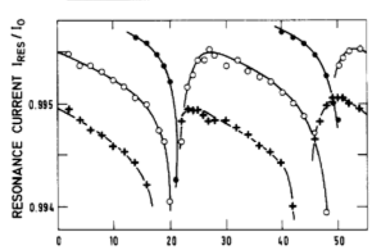
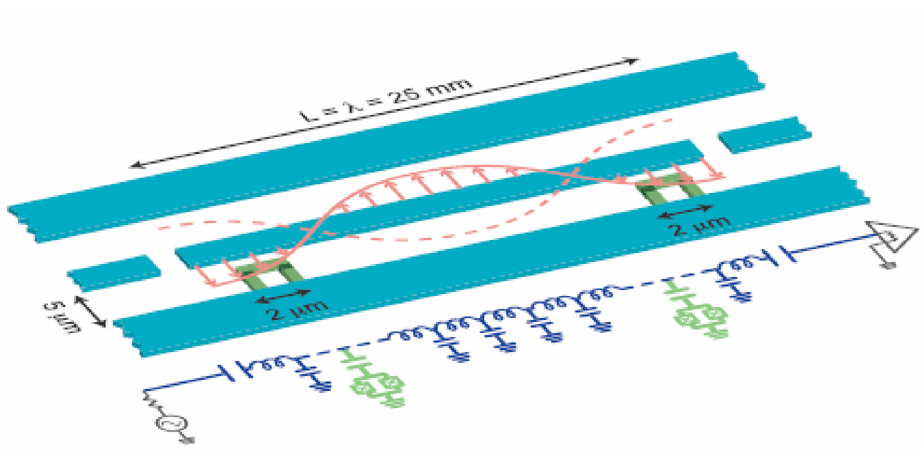


Fig. 4. Resonant current as a function of the position of the load for three irradiation frequencies. Crosses: $\Omega = 3.8$ GHz; open circles: $\Omega = 3.6$ GHz; full circles: $\Omega = 3.4$ GHz. Solid lines are guide for the eye. The half-wavelengths at the three frequencies are respectively 30, 32 and 34 mm.

08-II-19b

SCHEMATIC OF COOPER PAIR BOXES IN A MICROWAVE CAVITY

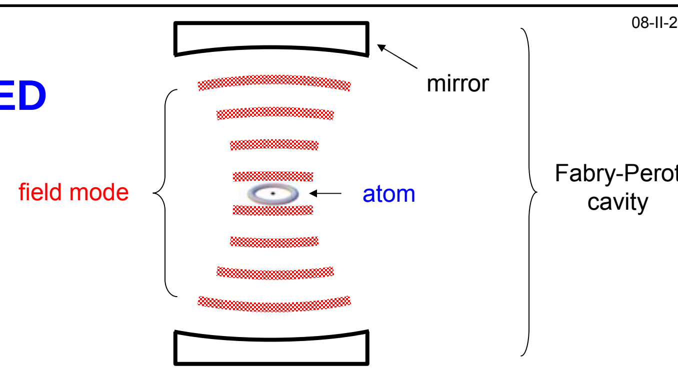


Blais A. et al., Phys. Rev. A **75**, 032329 2007

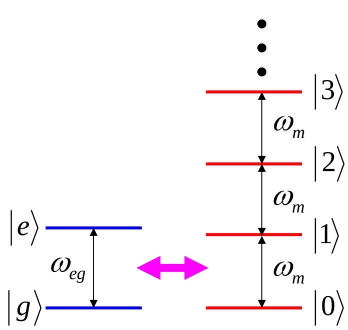
08-II-20

08-II-21b

CAVITY QED



See S. Haroche
College lectures



JAYNES-CUMMINGS HAMILTONIAN

$$\hat{H} = \hat{H}_{field} + \hat{H}_{atom} + \hat{H}_{coupling}$$

$$\hat{H}_{field} = \hbar\omega_m \hat{a}^\dagger \hat{a}$$

$$\hat{H}_{atom} = \hbar\omega_{eg}\hat{\sigma}^+\hat{\sigma}^-$$

$$\hat{H}_{coupling} = -\frac{i\hbar}{2}\Omega_0\left(\hat{a}\hat{\sigma}^+ - \hat{a}^\dagger\hat{\sigma}^-\right)$$

vacuum Rabi frequency

MICRO/MACRO CHARACTER OF COUPLING CONSTANTS IN CAVITY QED

atomic dipole moment — zero-point field amplitude

$$\Omega_0 = \frac{2d\mathcal{E}_0}{\hbar} = \left(\frac{2\alpha}{\pi}\right)^{3/2} \frac{R_\infty}{\hbar}\frac{1}{n}\frac{1}{\sqrt{k}}$$

n: atom principal quantum number

k: e.m. field mode order

$$k = \frac{d_c}{\lambda_n / 2}$$

d_c: distance between cavity mirrors

See book by S. Haroche and J.M. Raimond

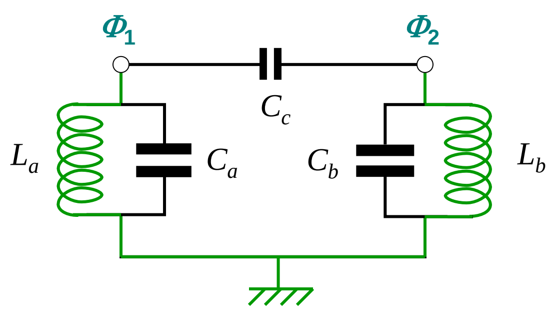
$\alpha = \frac{1}{2}\frac{e^2}{h} Z_{vac}$ Fine structure constant

$Z_{vac} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ Impedance of vacuum

$R_\infty = \alpha^2 \frac{m_e c^2}{2}$ Rydberg constant (infinite core mass)

08-II-22c

HAMILTONIAN OF TWO CAPACITIVELY COUPLED RESONATOR MODES



Reduced capacitance matrix:

$$\mathbf{C} = \begin{bmatrix} C_a + C_c & -C_c \\ -C_c & C_b + C_c \end{bmatrix}$$

Inverse of reduced capacitance matrix:

$$\mathbf{C}^{-1} = \frac{1}{C_a C_b + C_a C_c + C_b C_c} \begin{bmatrix} C_b + C_c & C_c \\ C_c & C_a + C_c \end{bmatrix}$$

$$\hat{H}\left(\hat{\Phi}_1, \hat{Q}_1; \hat{\Phi}_2, \hat{Q}_2\right) = \frac{\hat{\Phi}_1^2}{2L_a} + \frac{\hat{\Phi}_2^2}{2L_b} + \frac{\hat{Q}_1^2}{2C_1} + \frac{\hat{Q}_2^2}{2C_2} + \frac{\hat{Q}_1 \hat{Q}_2}{C_3}$$

08-II-23d

FORM OF COUPLING TERM

$$\frac{\hat{Q}_1\hat{Q}_2}{C_3} = \frac{C_c}{C_aC_b + C_aC_c + C_bC_c}\hat{Q}_1\hat{Q}_2$$

$$\simeq \hat{Q}_1\hat{Q}_2 = \frac{C_c}{C_a}\hat{Q}_1\hat{V}_2 = \frac{C_c}{2\sqrt{C_aC_b}}\hbar\sqrt{\omega_1\omega_2}\left[\hat{a}_1\hat{a}_2^{\dagger} + h.c.\right]$$

just capacitances

have kept here only RWA terms

$\Omega_0/2 =$ "g" in cQED

$$H\left(\hat{\phi},\hat{Q}\right) = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

These expressions are useful for obtaining value of g

$$\hat{H} = \hbar\omega_r\left(\hat{a}^{\dagger}\hat{a} + 1/2\right)$$

$$\hat{a} = \frac{\hat{\phi}}{\phi_r} + i\frac{\hat{Q}}{Q_r}; \quad \hat{a}^{\dagger} = \frac{\hat{\phi}}{\phi_r} - i\frac{\hat{Q}}{Q_r}$$

$$\phi_r = \sqrt{2\hbar\omega_r L}$$

$$Q_r = \sqrt{2\hbar\omega_r C}$$

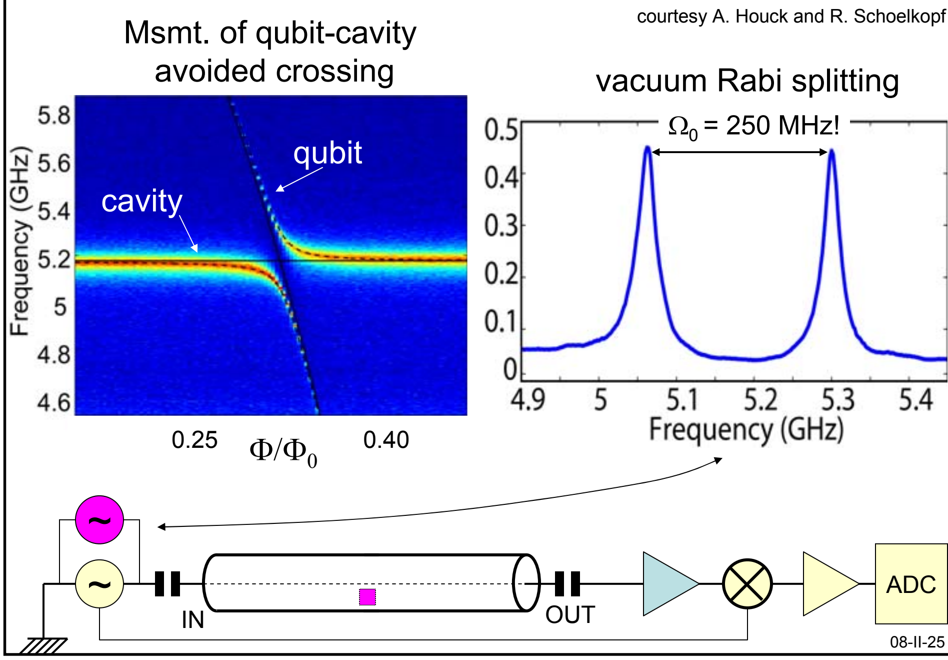
$$\hat{\phi} = \sqrt{\frac{\hbar\omega_r L}{2}}\left(\hat{a} + \hat{a}^{\dagger}\right)$$

$$\hat{Q} = \sqrt{\frac{\hbar\omega_r C}{2}}\left(\frac{\hat{a} - \hat{a}^{\dagger}}{i}\right)$$

08-II-24b

STRONG QUBIT-CAVITY COUPLING

courtesy A. Houck and R. Schoelkopf



08-II-25

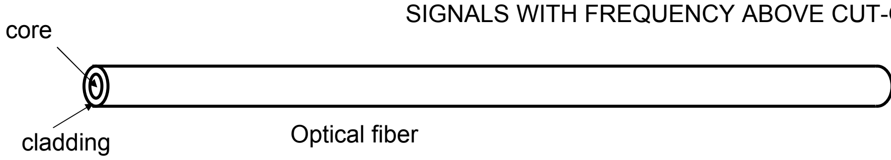
OUTLINE

1. Introduction, purpose of this lecture
2. Finding the Hamiltonian of an arbitrary circuit
3. Comparison with cavity QED
4. Transmission lines and waveguides
5. Coupled LC oscillators: model of transmission line

08-II-5d

SIMPLEST DISTRIBUTED ELEMENT SITUATION: GUIDED ELECTROMAGNETIC PROPAGATION

SIGNALS WITH FREQUENCY ABOVE CUT-OFF



Optical fiber

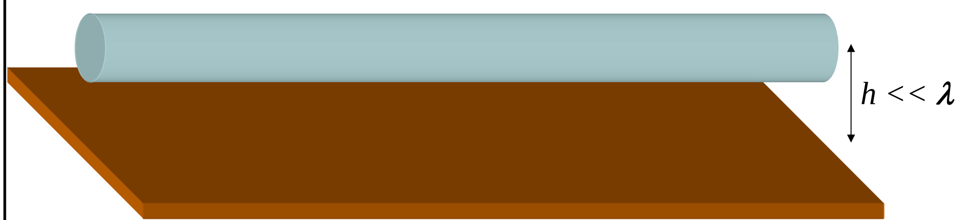
$$L \gg \lambda$$

Metallic wire

Metallic hollow waveguide

08-II-26

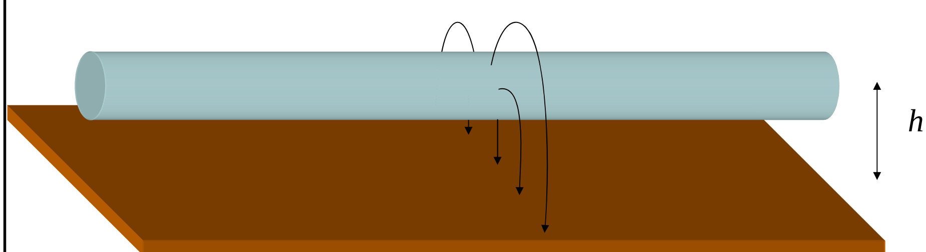
BETTER BOUNDARY CONDITIONS: WIRE ABOVE A GROUND PLANE



In coaxial line, ground plane is replaced by a cylindrical shield around wire.
In microwave microtechnology: wire of rectangular cross-section (stripline).
Also, ground plane can be split and extend on either side of wire (coplanar)

08-II-27

BOUNDARY CONDITIONS: WIRE ABOVE A GROUND PLANE



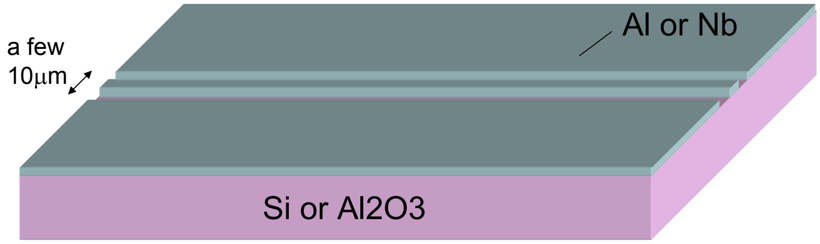
Electric field lines from wire end on ground plane: T.E.M.

PREVENTS RADIATION AROUND TURNS

08-II-27a

SUPERCONDUCTING TRANSMISSION LINES

COPLANAR WAVEGUIDE: 2D VERSION OF COAXIAL CABLE



1/e propagation length ~ 10km!

attenuation comparable to optical fibers

(1mm dia. copper wire: 700m)

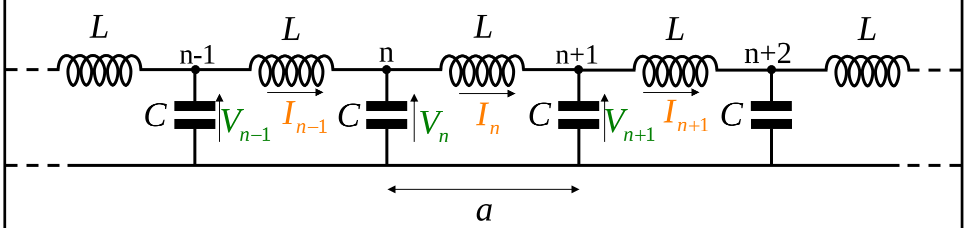
08-II-28

OUTLINE

1. Introduction, purpose of this lecture
2. Finding the Hamiltonian of an arbitrary circuit
3. Comparison with cavity QED
4. Transmission lines and waveguides
5. Coupled LC oscillators: model of transmission line

08-II-5e

LC LADDER MODEL OF TRANSMISSION LINE



Dynamical equations:

$$V_n - V_{n+1} = L\frac{d}{dt}I_n$$

$$I_{n-1} - I_n = C\frac{d}{dt}V_n$$

Continuum limit:

$$\frac{V_{n+1}-V_n}{a} \to \frac{\partial V}{\partial x}$$

$$\frac{I_{n+1}-I_n}{a} \to \frac{\partial I}{\partial x}$$

$$\frac{C}{a} \to C_\ell \; ; \; \frac{L}{a} \to L_\ell$$

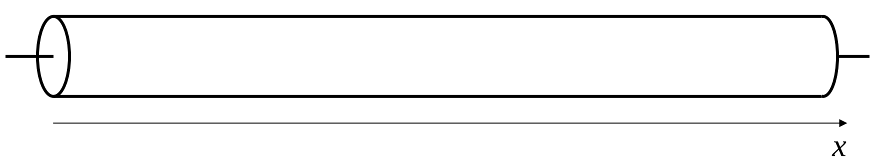
Field equations:

$$\frac{\partial V}{\partial x} = -L_\ell \frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial x} = -C_\ell \frac{\partial V}{\partial t}$$

08-II-29e

PROPAGATING WAVE AMPLITUDES



Introduce right- and left-moving fields:

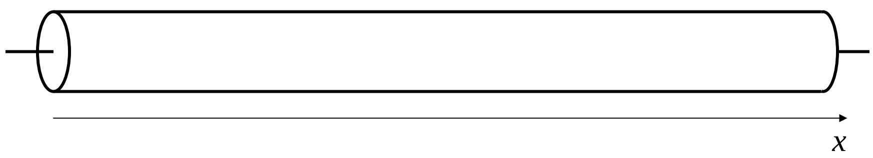
$$A^{\rightarrow}(x,t) = \frac{1}{\sqrt{Z_c}}V + \sqrt{Z_c}\,I$$

$$A^{\leftarrow}(x,t) = \frac{1}{\sqrt{Z_c}}V - \sqrt{Z_c}\,I$$

$$Z_c = \sqrt{\frac{L_\ell}{C_\ell}}$$ ← characteristic impedance

08-II-30

PROPAGATING WAVE AMPLITUDES



1st order equations

$$A^{\rightarrow}(x,t) = \frac{1}{\sqrt{Z_c}} V + \sqrt{Z_c}\, I \qquad \frac{\partial}{\partial x} A^{\rightarrow} = -\frac{1}{v_p} \frac{\partial}{\partial t} A^{\rightarrow}$$

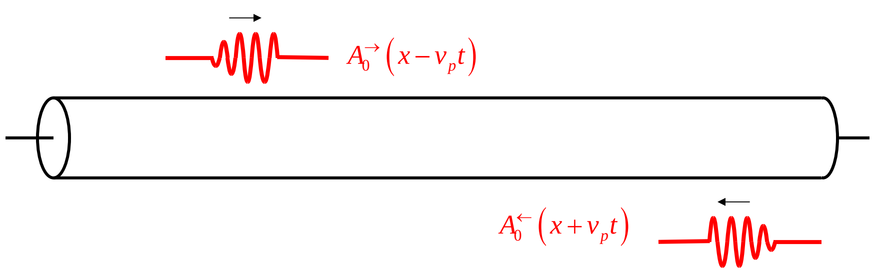
$$A^{\leftarrow}(x,t) = \frac{1}{\sqrt{Z_c}} V - \sqrt{Z_c}\, I \qquad \frac{\partial}{\partial x} A^{\leftarrow} = +\frac{1}{v_p} \frac{\partial}{\partial t} A^{\leftarrow}$$

$Z_c = \sqrt{\frac{L_\ell}{C_\ell}}$ ← characteristic impedance

$v_p = \sqrt{\frac{1}{L_\ell C_\ell}}$ ← propagation velocity

08-II-30a

PROPAGATING WAVE AMPLITUDES



$$A^{\rightarrow}(x,t) = \frac{1}{\sqrt{Z_c}} V + \sqrt{Z_c}\, I \qquad \frac{\partial}{\partial x} A^{\rightarrow} = -\frac{1}{v_p} \frac{\partial}{\partial t} A^{\rightarrow}$$

$$A^{\leftarrow}(x,t) = \frac{1}{\sqrt{Z_c}} V - \sqrt{Z_c}\, I \qquad \frac{\partial}{\partial x} A^{\leftarrow} = +\frac{1}{v_p} \frac{\partial}{\partial t} A^{\leftarrow}$$

$$Z_c = \sqrt{\frac{L_\ell}{C_\ell}} \qquad v_p = \sqrt{\frac{1}{L_\ell C_\ell}}$$

solution:

$$A^{\rightleftarrows}(x,t) = A_0^{\rightleftarrows}(x \mp v_p t)$$

08-II-30c

REFLEXION ON A RESISTIVE LOAD

Reflection coefficient: $r = \dfrac{R - Z_c}{R + Z_c}$

$R = Z_c$ ⇨ NO REFLECTION

INVERSELY, VIEWED FROM ITS TERMINAL,
A SEMI-INFINITE TRANSMISSION LINE CANNOT
BE DISTINGUISHED FROM A RESISTANCE!

08-II-31

NEXT LECTURE:

How do we quantize the propagating modes on the transmission line?

The answer is crucial for understanding friction in quantum regime

08-II-32

SELECTED BIBLIOGRAPHY

Books

Braginsky, V. B., and F. Y. Khalili, "Quantum Measurements" (Cambridge University Press, Cambridge, 1992)
Cohen-Tannoudji, C., Dupont-Roc, J. and Grynberg, G., "Atom-Photon Interactions" (Wiley, New York, 1992)
Haroche, S. and Raimond, J-M., "Exploring the Quantum" (Oxford University Press, 2006)
Mallat, S. M., "A Wavelet Tour of Signal Processing" (Academic Press, San Diego, 1999)
Nielsen, M. and Chuang, I., "Quantum Information and Quantum Computation" (Cambridge, 2001)
Pozar, D. M., "Microwave Engineering" (Wiley, Hoboken, 2005)
Tinkham, M., "Introduction to Superconductivity" (2nd edition, Dover, New York, 2004)

Review articles

Blais A., Gambetta J., Wallraff A., Schuster D. I., Girvin S., Devoret M.H., Schoelkopf R.J. Phys. Rev. (2007) A **75**, 032329
Courty J. and Reynaud S., Phys. Rev. A **46**, 2766-2777 (1992)
Devoret M. H. in "Quantum Fluctuations", S. Reynaud, E. Giacobino, J. Zinn-Justin, Eds. (Elsevier, Amsterdam, 1997) p. 351-385
Devoret M. H., Wallraff A., and Martinis J. M., e-print cond-mat/0411174
Makhlin Y., Schön G., and Shnirman A., Nature (London) **398**, 305 (1999).

Articles

Koch J, et al. Phys. Rev. A **76**, 042319 (2007)
Houck A. A., et al. arXiv:0803.4490, A. A. Houck, et al., Phys. Rev. Lett. **101**, 080502 (2008)
Schreier J., et al, .arXiv:0712.3581, J. A. Schreier, et al., Phys. Rev. B **77**, 180502(R) (2008)

08-II-33

END OF LECTURE