Chaire de Physique Mésoscopique
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INTRODUCTION À LA PHYSIQUE MÉSOSCOPIQUE: ÉLECTRONS ET PHOTONS

INTRODUCTION TO MESOSCOPIC PHYSICS: ELECTRONS AND PHOTONS

Première Leçon / *First Lecture*

AIM OF THESE TWO LECTURES

Discuss a selection of basic concepts of mesoscopic physics and contrast their treatment with that of atomic physics

OUTLINE

1. General remarks on mesoscopic physics
   - a) fundamental constants
   - b) survival of quantum effects in macrosystems
2. What are "electrons"?
   - a) example of mesoscopic resistor
   - b) screening
   - c) finite lifetime
3. What are "photons"?
   - a) example of transmission line
   - b) longitudinal and transverse modes
   - c) 1-D Planck's law
4. Conclusion: quantum transport v.s. quantum optics

part 1 (items 1–2b)

part 2 (items 2c–4)

General remarks

Purpose: explain the novelty of mesoscopic phenomena by discussing status of variables and parameters

MACRO *versus* micro

MACRO SYSTEM

classical mechanics

many particles, strongly coupled to environment

$$H = \frac{p_\theta^2}{2M} + \frac{k\theta^2}{2} + \ldots\ldots$$

can have engineered structural parameters

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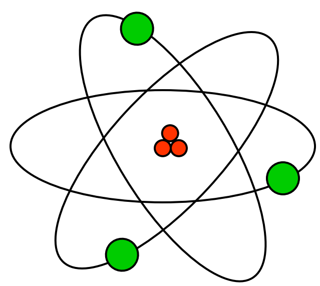
micro system

quantum mechanics



few particles, almost isolated from environment

$$\hat{H} = \frac{\hat{p}^2}{2m_e} - \frac{e^2}{4\pi\varepsilon_0 \hat{r}} + \ldots\ldots$$

always "God-given" structural parameters

MESOSCOPIC PHENOMENA:

Number of particles: macroscopic
Collective degrees of freedom: quantum

High-Q nanomechanical resonators @ low temperatures:
clearly mesoscopic
Superconducting magnets for MRI: clearly not mesoscopic

Characteristic energies in mesoscopic phenomena involve not only fundamental constants, but also geometric dimensions adjusted by design and fabrication

REVIEW OF UNIVERSAL CONSTANTS

S. I. Units : s, m, kg, A, K

Classical constants		Quantum constants	
speed of light	c	charge quantum	e
impedance of vacuum	$Z_{vac} \simeq 377\Omega$	action quantum	$\hbar$
electrical permittivity of vacuum	$\varepsilon_0 = \frac{1}{cZ_{vac}}$	flux quantum	$\Phi_0 = \frac{h}{2e}$
magnetic permeability of vacuum	$\mu_0 = \frac{Z_{vac}}{c}$	resistance quantum	$R_K = \frac{h}{e^2} \simeq 26\mathrm{k}\Omega$
Boltzmann's constant	k_B	fine structure cst.	$\alpha = \frac{Z_{vac}}{2R_K} \simeq \frac{1}{137}$
gravitational constant	G		

UNIVERSAL AND MICROSCOPIC QUANTUM CONSTANTS

Universal constants

charge of electron: e

Planck's constant: $\hbar$

flux quantum: $\Phi_0 = \frac{h}{2e}$

resistance quantum: $R_K = \frac{h}{e^2} \simeq 26\text{k}\Omega$

fine structure cst.: $\alpha = \frac{Z_{vac}}{2R_K} \simeq \frac{1}{137}$

Microscopic constants

mass of electron: m_e

magn. mt. of electron: μ_B

mass of proton: m_p

mass of neutron m_n

magn. mt. of proton μ_p

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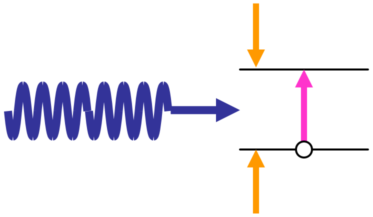
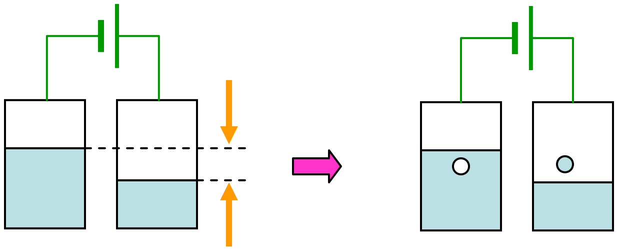
FORMS OF CHARACTERISTIC ENERGIES



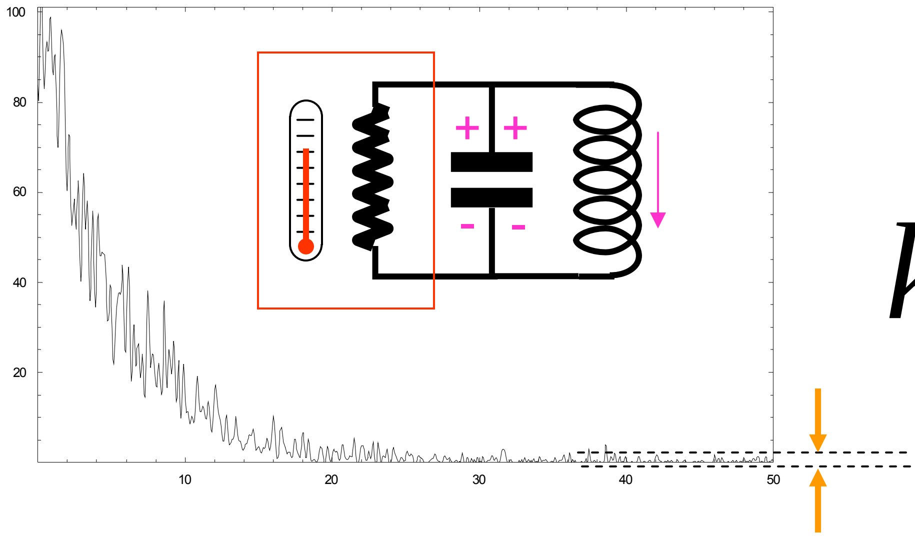
$\hbar\omega$

20 GHz



eV

80 μV

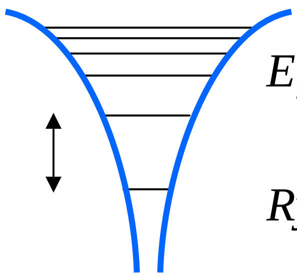


$k_B T$

1K

USUAL ENERGY SCALES
OF MICROSCOPIC QUANTUM EFFECTS

Atomic matter

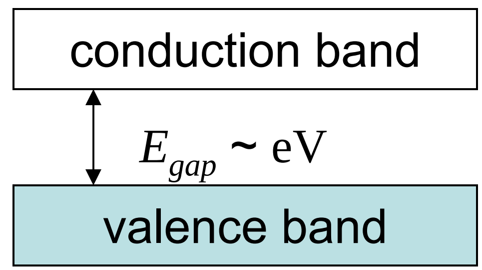


$$E_n = \frac{-Ry}{n^2}$$

$$Ry = \frac{m_e c^2}{2}\alpha^2 = 13.6\text{eV}$$

$\Delta E \sim \text{eV}$

Bulk condensed matter



$E_{gap} \sim \text{eV}$

$E_{Debye} \sim 10\text{meV}$

$$\Delta_{sc} = E_{Debye} e^{-\frac{1}{N(0)V}} \sim \text{meV}$$

IN MESOSCOPIC QUANTUM EFFECTS, NEW ENERGY SCALES EMERGE!

MESOSCOPIC PHYSICS CHALLENGES COMMON WISDOM ABOUT THE SUPPRESSION OF COLLECTIVE QUANTUM EFFECTS IN LARGE AND DIRTY SYSTEMS

- THERMAL ENERGY CAN BE BIGGER THAN SINGLE PARTICLE ENERGY LEVEL SPACINGS
- DISORDER DOES NOT FULLY SUPPRESS INTERFERENCES, IT IS DECOHERENCE WHICH IS IMPORTANT
- A SINGLE ELECTRON CAN COUNT
- LOCAL CIRCUIT THEORY WILL UNEXPECTEDLY FAIL

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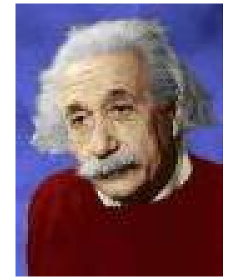
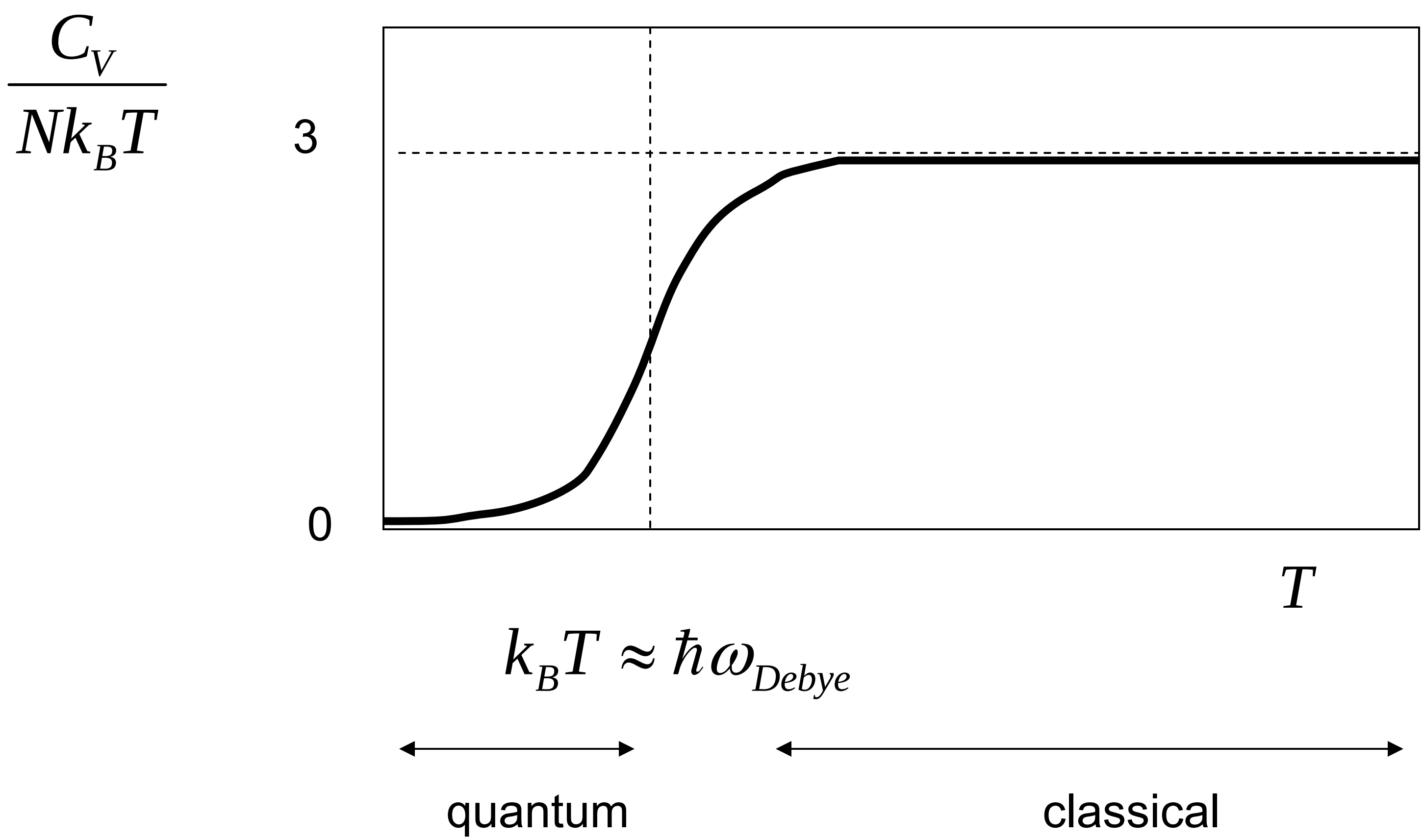
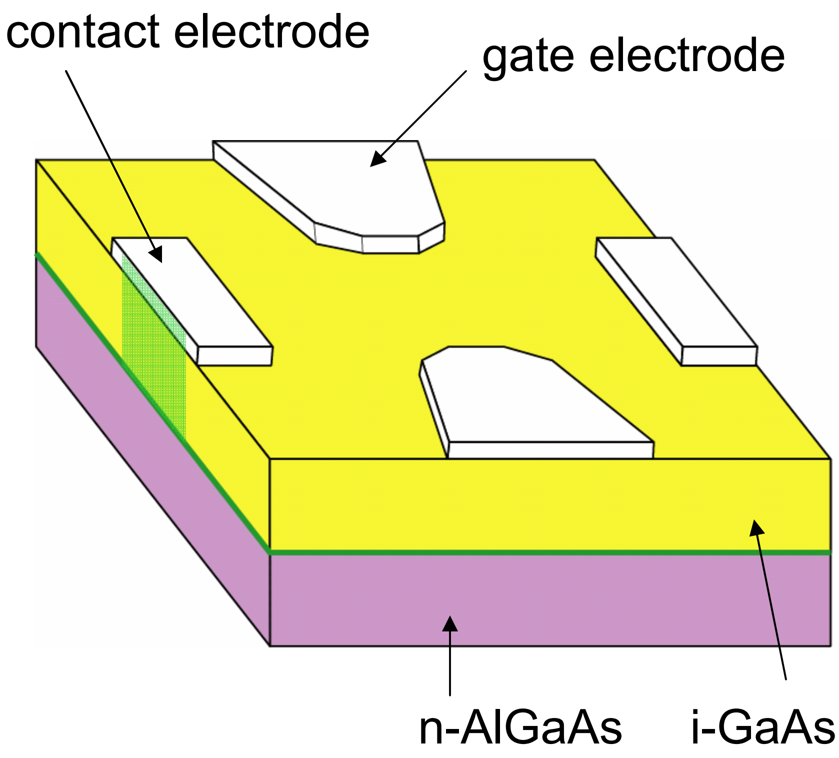
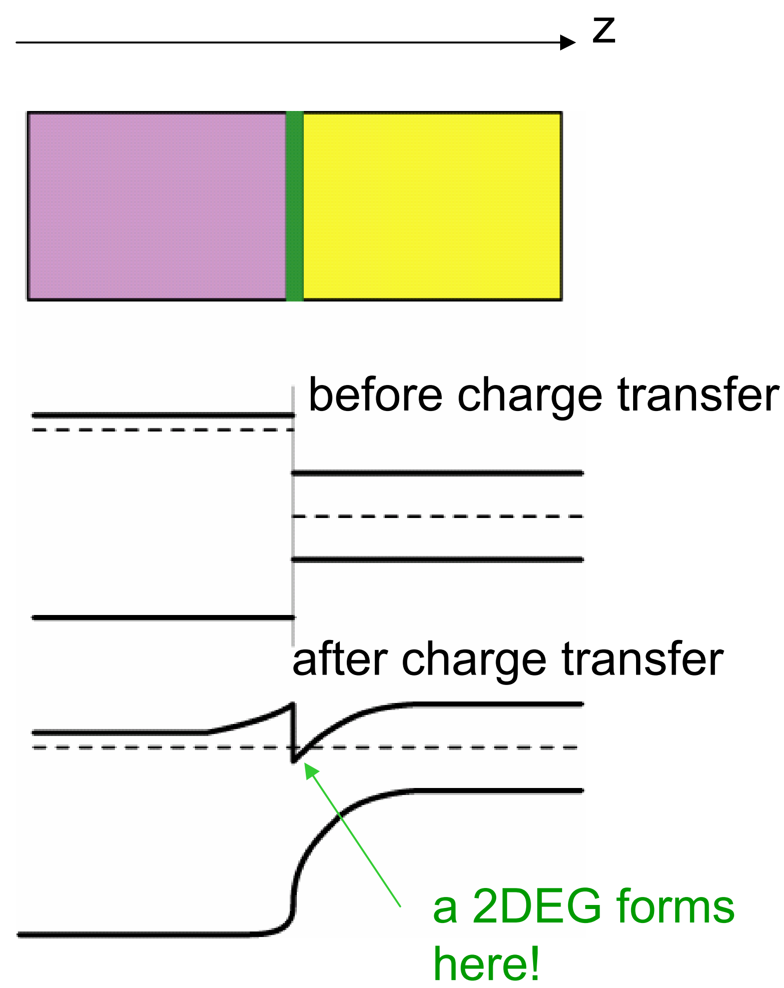
EXAMPLE OF COMMON WISDOM AT WORK: THE CROSSOVER TO DULONG-PETIT LAW



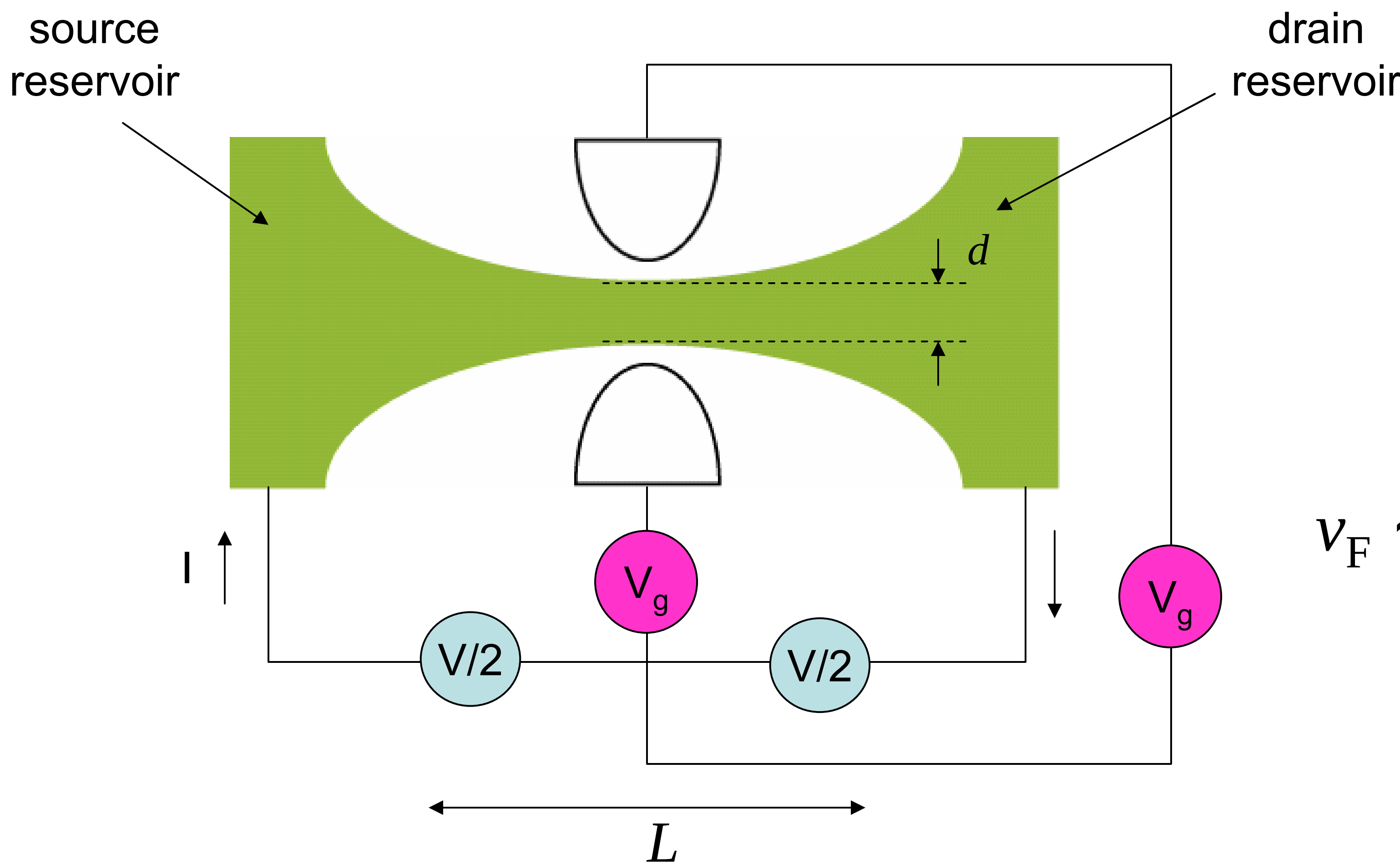


2-DIMENSIONAL ELECTRON GAS SYSTEM





PINCHING THE 2DEG INTO A SMALL WIRE



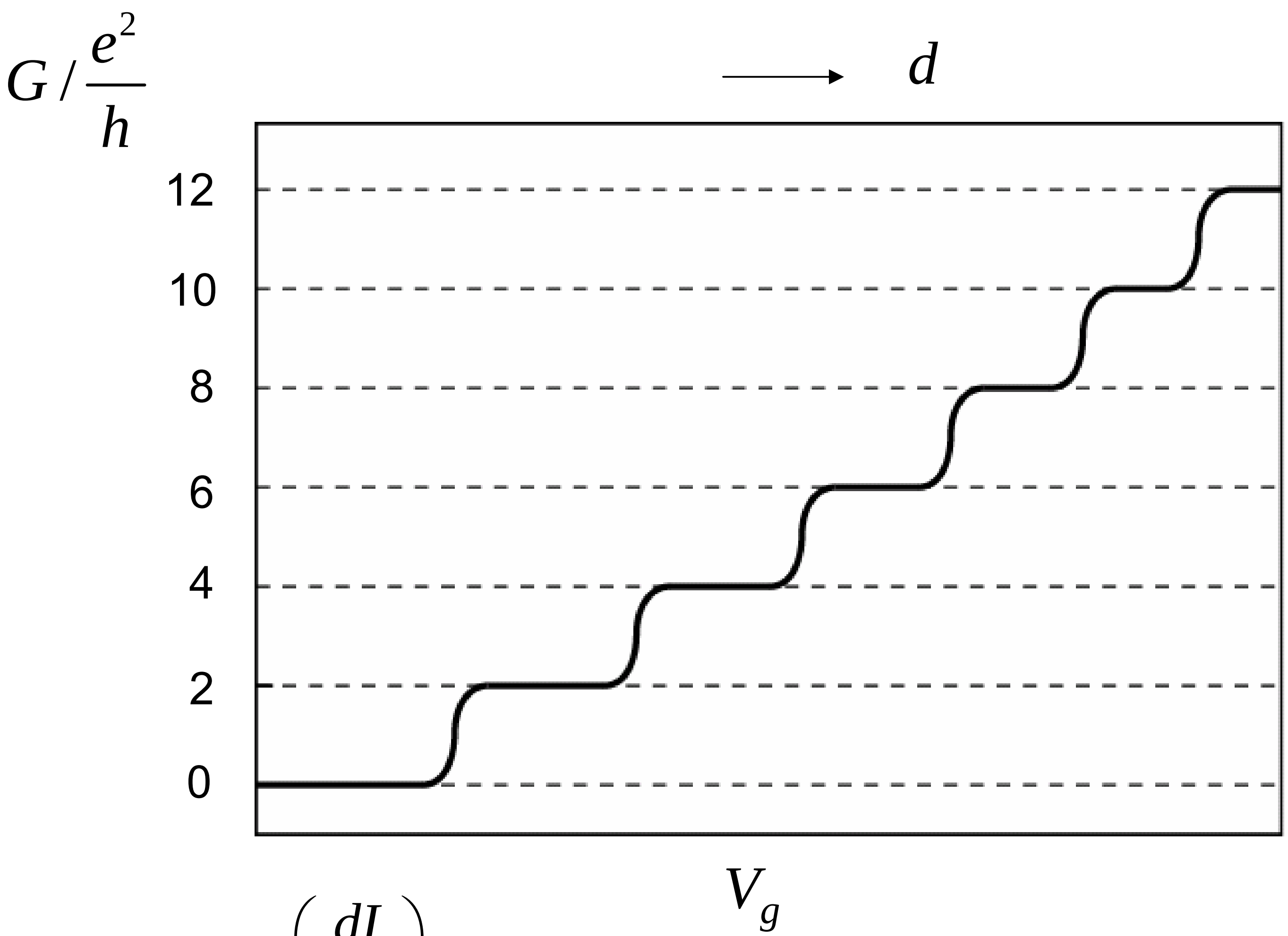
small
effective
mass

$v_F \sim$ 1nm/ps

Ballistic transport: $L \ll \ell_e$

what if $k_B T \ll \frac{\hbar v_F}{d}$?

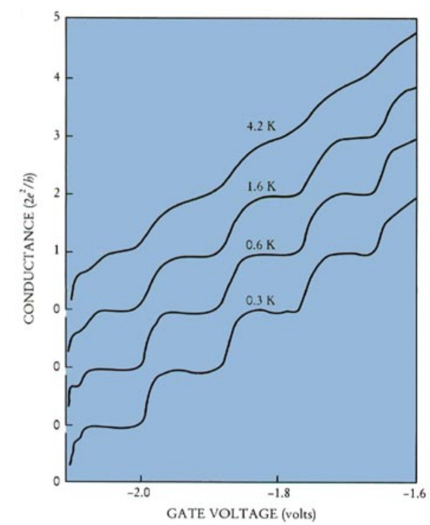
CONDUCTANCE QUANTIZATION



$$G = \left(\frac{dI}{dV} \right)_{eV/k_B T \to 0}$$

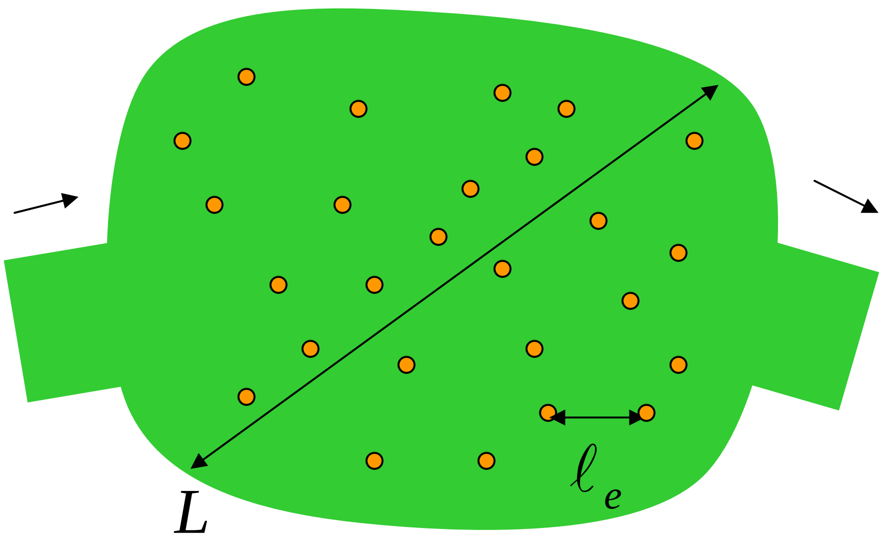
$$k_B T \ll \frac{\hbar v_F}{d}$$

van Wees *et al*. 1988:



waveguide effect!

UP TO WHAT SIZE CAN QUANTUM EFFECTS PERSIST IN PRESENCE OF DISORDER?



weak disorder:

$$\ell_e \gg 1/k_F$$

NON-TRIVIAL RESULT

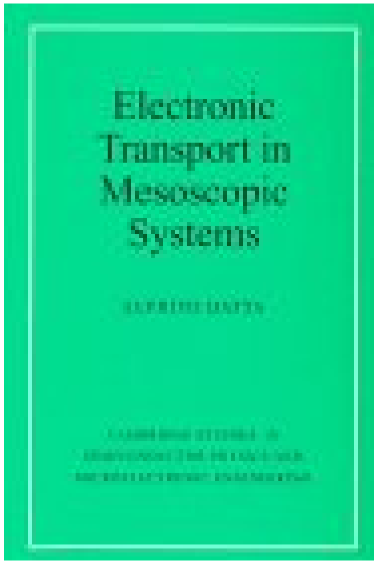
$$k_B T, \frac{\hbar}{\tau_\varphi} \ll \frac{\hbar v_F}{L}\frac{\ell_e}{L}$$

← Thouless energy

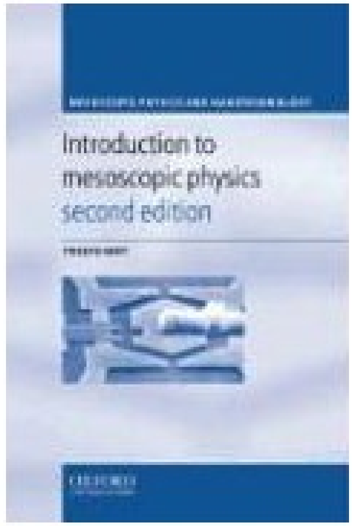
Can be much larger than level spacing in box of size L!

$$\frac{\hbar v_F k_F}{(L k_F)^2} \lll \frac{\hbar v_F \ell_e}{L^2}$$

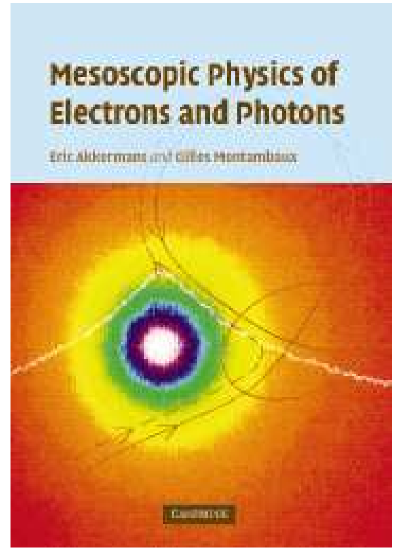
BOOKS ON MESOSCOPIC PHYSICS



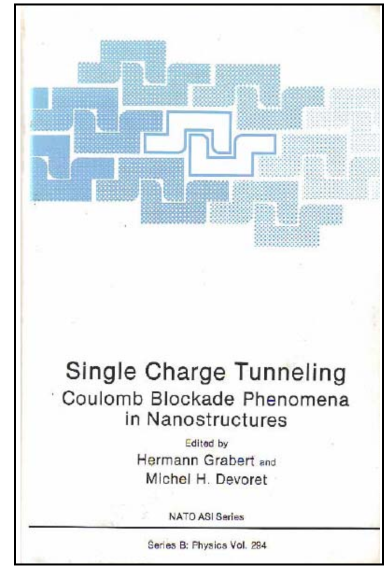
S. Datta



Y. Imry



E. Akkermans and G. Montambaux



H. Grabert and M. Devoret

REQUISITES:

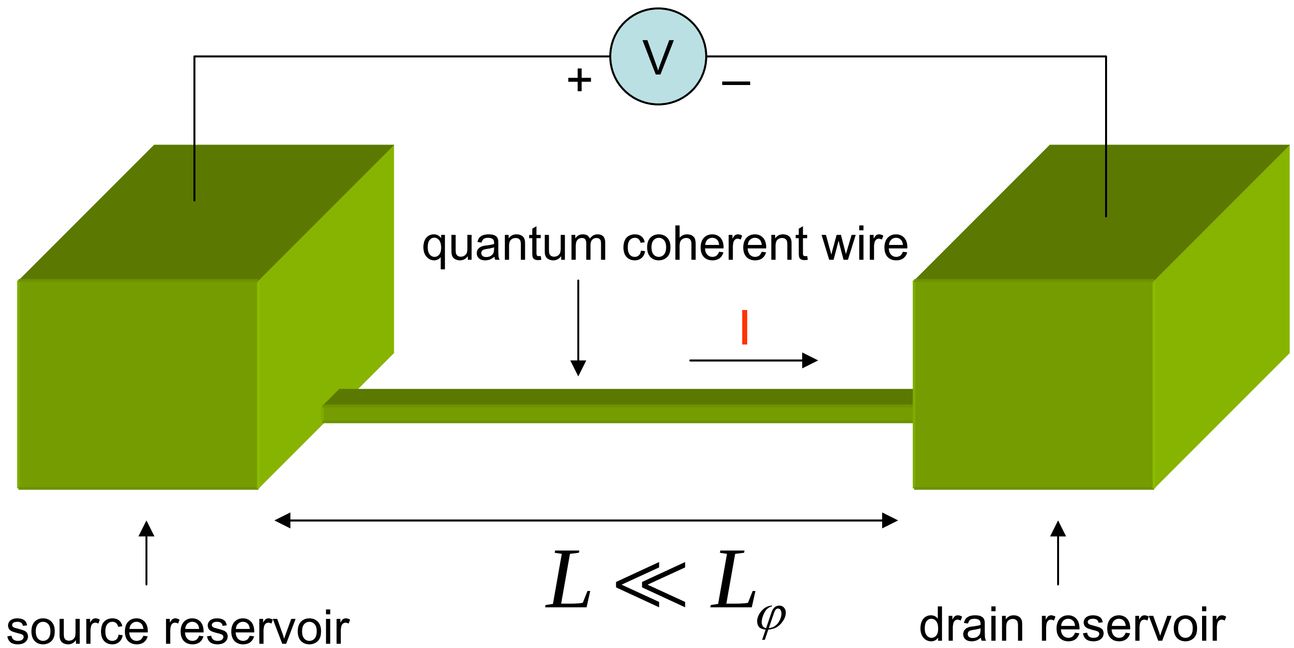
Quantum mechanics
Solid state physics
Statistical mechanics

Useful reference books on quantum statistical physics: Feynman; Schrieffer; Pines and Nozières

2. How mesoscopic physics models the "electron"

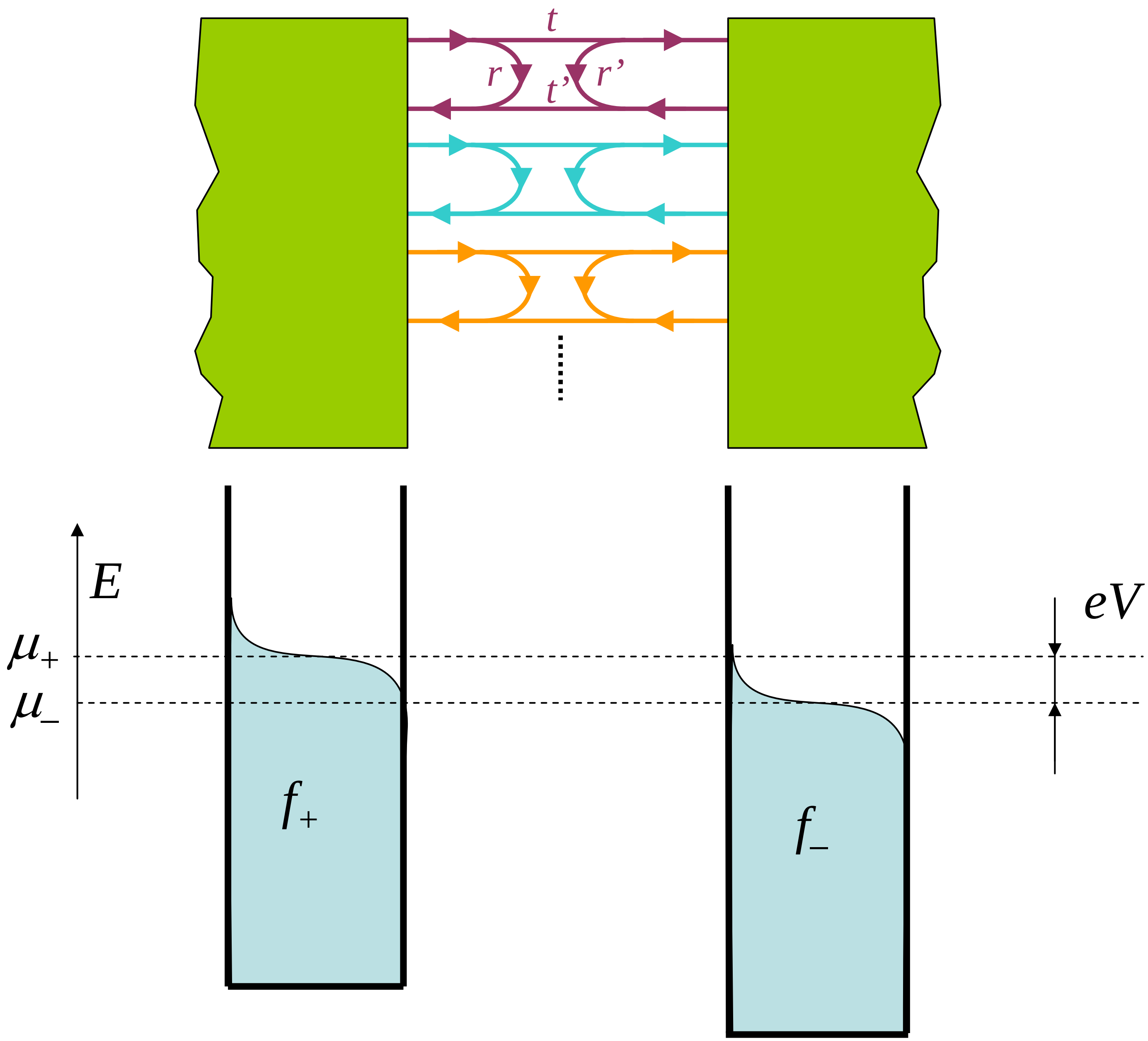
Purpose: provide groundwork for Landauer's approach of transport phenomena

THE MESOSCOPIC RESISTOR



Landauer reservoir: metallic electrode which injects into the quantum coherent region quasi-electron waves with well-defined chemical potential and temperature. It also accepts without reflection any quasi-electron waves and thoroughly recycles them. The Landauer reservoir is to Fermi waves what a black-body is to Bose waves.

Collection of independent channels



THE LANDAUER-BÜTTIKER FORMULA

$$I = I_+ - I_-$$

$$I_\pm = \frac{e}{h} \sum_m \int_{-\infty}^{+\infty} f_\pm(E) \left| t_m(E) \right|^2 \mathrm{d}E$$

$$f_\pm(E) = \frac{1}{1 + \exp \dfrac{E - \mu_\pm}{k_B T}}$$

$$\mu_+ - \mu_- = eV$$

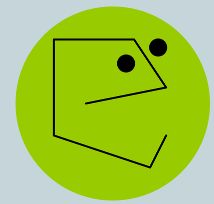
schizophrenic! why does it work?

RESISTANCE DETERMINED BY ELASTIC TRANSMISSION COEFFICIENT!

WHAT ABOUT JOULE HEATING?

QUESTION: What are the differences between a quasi-electron (a.k.a. dressed electron) and the usual (bare) electron?

PARTICLE IDENTIFICATION CARD



Last Name: Electron First name: Bare

Address: Vacuum Genre: Fermion

Occupation: Wave packet Lifetime: infinite

Average energy: $\hbar\omega$ Average momentum: $\hbar k$

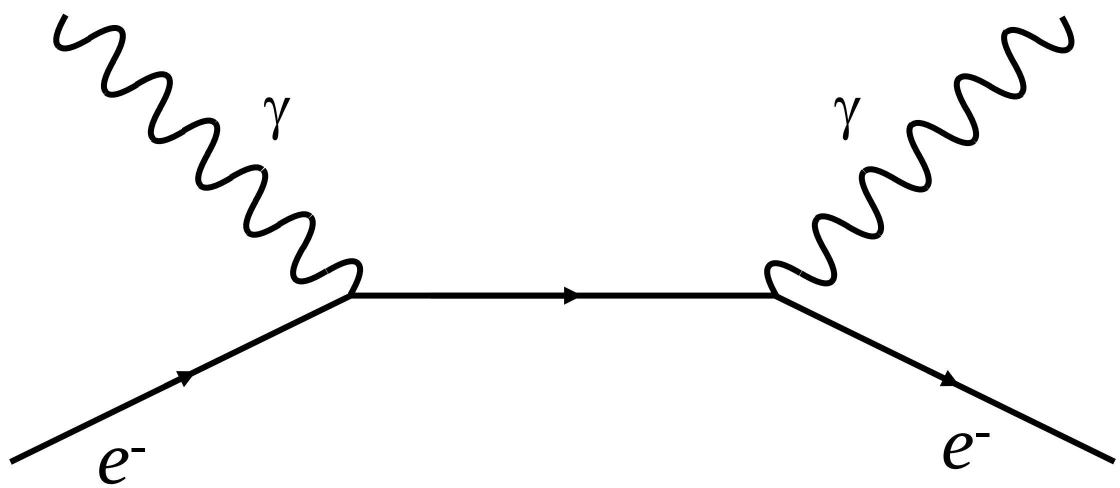
Velocity: $v=d\omega/dk$ Mass: $\hbar dk/dv=m_e$

Charge: -e

Spin: S=1/2 Magnetic moment: $g_e S\mu_B$

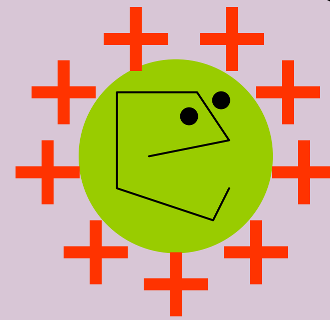
$$g_e = 2\left[1+\frac{\alpha}{2\pi}+O\left(\alpha^2\right)\right]$$

An example of a Feynman diagram
involving the usual electron and photon
of atomic physics
which propagate in vacuum



THE "ELECTRON" OF MESOSCOPICS

PARTICLE
IDENTIFICATION
CARD



Last Name: Electron	First name: Quasi
Address: Metal	Genre: Fermion
Occupation: Wave packet	Lifetime: finite, except @ k_F
Average energy: $\hbar\omega$	Average momentum: $\hbar k$
Velocity: $v=d\omega/dk$	Mass: $\hbar dk/dv=m_{eff}(k)$
Transverse charge: -e	Longitudinal charge: 0 $(q\rightarrow 0)$
Spin: S=1/2	Magnetic moment: $g_{eff}\, S\mu_B$

Definition of the longitudinal and transverse part of a field:

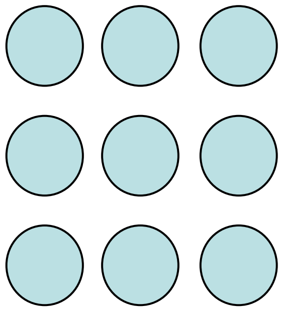
$$\vec{F} = \vec{F}_l + \vec{F}_t$$

$$\vec{\nabla} \cdot \vec{F}_t = 0$$

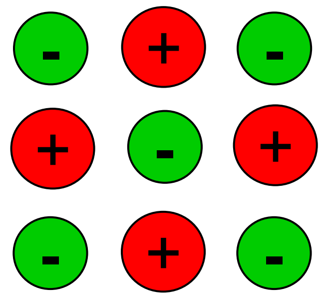
$$\vec{\nabla} \times \vec{F}_l = 0$$

The longitudinal and transverse charges are the sources of the longitudinal and transverse parts of the electrical field, respectively.

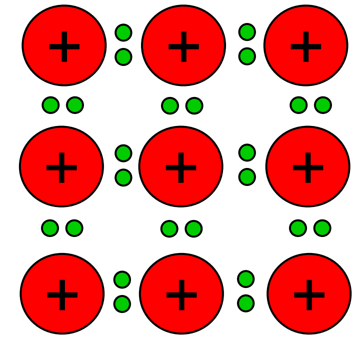
SOLIDS



van der Waals

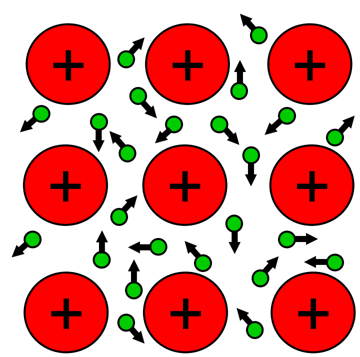


ionic



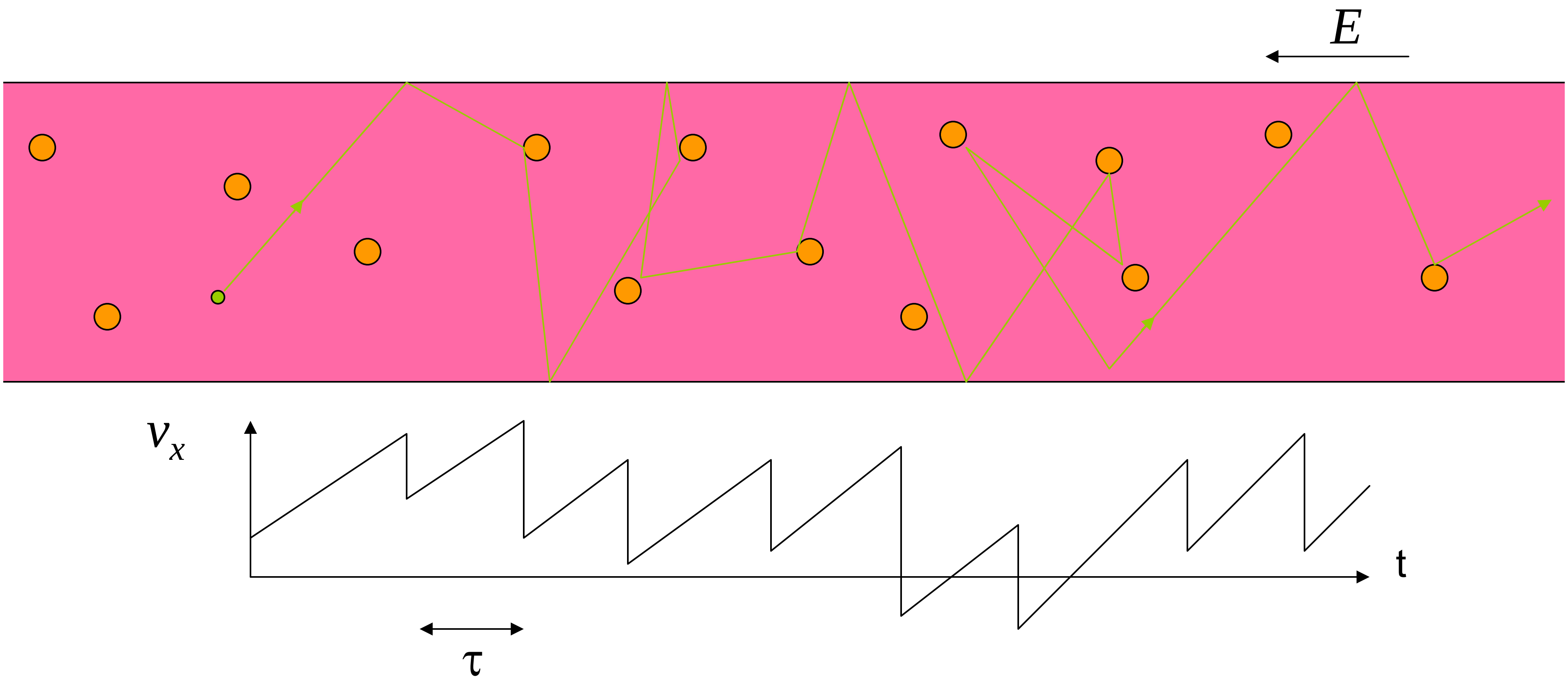
covalent

insulators



metal

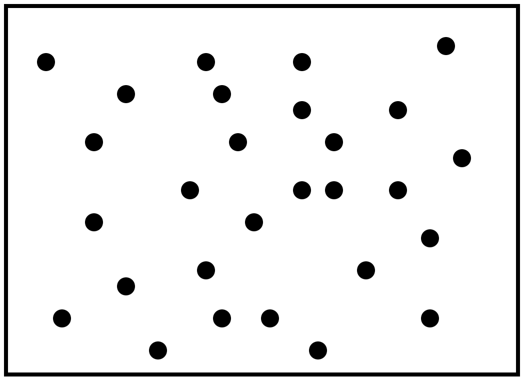
REVIEW OF DRUDE MODEL



$$\langle v_x \rangle = \frac{(-e)E}{m}\tau \qquad j = n(-e)\langle v_x \rangle \qquad \sigma = \frac{j}{E} = \frac{ne^2}{m}\tau$$

PROBLEM: ELECTRONS ARE FERMIONIC WAVES!

REVIEW OF FREE FERMI GAS MODEL



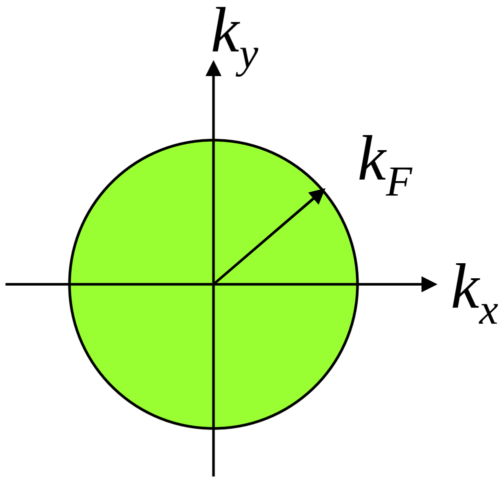
Box volume V

Nb of fermions N

Total energy E_K

Length scale a_0

Energy scale Ry



$$a_0 = 4\pi\varepsilon_0 \frac{\hbar^2}{m_e e^2}; \; Ry = \frac{m_e e^4}{2\hbar^2 \cdot 4\pi\varepsilon_0}$$

$$\frac{N}{V} = n = \frac{4\pi}{3}\frac{k_F^2}{(2\pi)^2} = \frac{1}{\frac{4\pi}{3}a^3}; \quad r_s = \frac{a}{a_0}; \quad k_F = \frac{1.92}{r_s a_0}$$

$$\frac{E_K}{N} = \frac{3}{5}\frac{(\hbar k_F)^2}{2m_e} = \frac{3}{5}E_F = \frac{2.22}{r_s^2}Ry$$

$$v_F = \frac{\hbar}{m_e}k_F = v_g\Big|_{E_F}$$

$$\frac{E_C}{N} = \frac{1}{r_s}Ry$$

OTHER PROBLEM: ELECTRONS INTERACT STRONGLY!

… BUT TOO STRONG INTERACTION KILLS INTERACTION…..

Acknowledgements : D. Esteve, H. Pothier, D. Stone
and C. Urbina